

# MODELING OF “GRADUAL” SOLAR ENERGETIC PARTICLE EVENTS USING A STOCHASTIC DIFFERENTIAL EQUATION METHOD

M. Den<sup>1</sup>, T. Yoshida<sup>2</sup> and K. Yamashita<sup>3</sup>

<sup>1</sup>*Communications Research Laboratory (CRL), 4-2-1, Nukuiikita, Koganei, Tokyo, 184-8795, Japan*

<sup>2</sup>*Ibaraki University, 2-1-1, Bunkyo, Mito, Ibaraki, 310-8512, Japan*

<sup>3</sup>*Chiba University, 1-33 Yayoi, Inage, Chiba, 243-8522, Japan*

## ABSTRACT

We have modeled “gradual” solar energetic particle events through numerical simulations using a Stochastic Differential Equation (SDE) method. We consider that energetic particle events are roughly divided into two groups: (1) where the shock was driven by coronal mass ejections (CMEs) associated with large solar flares, and (2) where they have no related solar events apart from the CMEs. (The detailed classification of energetic particle events was discussed in our previous paper.) What we call “gradual” solar energetic particle events belong to the former group. Particles with energies greater than 10 MeV are observed within several hours after the occurrence of flares and CMEs in many gradual events. By applying the SDE method coupled with particle splitting to diffusive acceleration, we found that an injection of high energy particles is necessary for early enhancement of such a high-energy proton flux and that it should not be presumed that the solar wind particles act as the seed population.

## INTRODUCTION

Prediction of interplanetary shock wave passage is an important subject regarding space weather because the shock waves cause geomagnetic disturbances and accelerate particles. The most well known classes of solar particle events are impulsive events and gradual events (e.g., Reames, 1999). The particles classified in the former events are thought to be accelerated in the impulsive flare process because of the observed charge states of Fe. On the other hand, gradual solar energetic particle (SEP) events suggest that the origin of the energetic ions is coronal material, and that the CME-driven shock waves play an important role in the acceleration mechanism. In this paper, we study gradual SEP events and “SEPs” or “energetic particle events” mentioned in the following indicate gradual SEP events.

There are many variations in particle-flux behavior in energetic particle events; shock accelerated particle intensities mostly begin to rise in advance of shock passages with a duration ranging from several hours to several days, on the other hand, there are some events in which no enhancement of particle fluxes occur despite shock wave passages. The variance in the intensity flux of energetic protons has been studied by several groups. Differences in the energy range of accelerated protons lead to different proton flux profiles. For example, van Nes et al. (1984) studied proton energy spectra in the range of 35-1600 keV by surveying 75 interplanetary shocks, sub-divided shock events into four different groups, and analyzed the relation between the effect of shock acceleration mechanisms (diffusive or drift) and the shock strength. Kallenrode (1996) examined the intensity profiles of 5 MeV protons for 351 interplanetary shocks, divided them into three groups, and compared the features of the particle events with those of the shocks. Cane et al. (1988) and Reames et al. (1999) claimed that the intensity-time profiles of solar proton events (SEPs) depended on

the longitude of the source region on the Sun by studying the spatial distribution of typical events. Using the assumption that the nose of the shock is the region where acceleration is strongest, they explained the variation of the particle flux. However, this needs to be examined for consistency with regards to particle acceleration theories.

In our previous paper (Den et al., 2001, hereafter called Paper 1), we studied 68 shock accelerated energetic particle events with energies ranging from 47 keV to 4.75 MeV using particle data observed by the Electron, Proton and Alpha Monitor (EPAM) onboard the ACE spacecraft, and these data were detected from November 6, 1997 to July 10, 2000. We classified those events in four groups according to the flux variance, the characteristic event duration, and the maximum energy of the accelerated particles as follows. Type 1: duration time in advance of shock passage, which we call "the precursor time", was about one day and protons were not accelerated beyond 10 MeV. Type 2: solar energetic protons and interplanetary shock accelerated protons were mixed and the source CMEs were accompanied by large X-ray flares, so the associated shocks were strong ones and the maximum energy of accelerated particles was greater than 10 MeV. Type 3: the precursor time and total duration of the flux enhancement were very long, about 10 days. Type 4: There was no evident enhancement of particle flux despite an interplanetary shock passage having occurred. In Paper 1, we modeled typical events for Type 1 and 2 events through numerical simulations and obtained preliminary results.

The well known acceleration mechanism of energetic particles associated with CME-driven shocks is the first-order Fermi acceleration (Axford et al., 1977, Blandford and Ostriker, 1978, Krymskii, 1977) and shock drift acceleration (Armstrong et al., 1977). However, there are few studies which show how these theories could explain the actual observations made regarding energetic events. Lee and Ryan (1986) solved analytically the time dependent cosmic ray transport equation by using some strong assumptions, (e.g., that the diffusion coefficient is self similar and proportional to  $r^2$ , where  $r$  is radial distance), which are not widely accepted. Zank et al. (2000) also studied shock acceleration, taking into account the dynamical evolution of a spherical shock wave. Their approach was semi-analytical: hydrodynamical quantities such as shock speed were calculated numerically, while the particle flux was obtained analytically through a simplification of the transport equation. Li and Zank (2002) claimed that diffusive shock acceleration is an overly simplistic mechanism for interplanetary shock waves because that theory assumes an infinitely extended diffusive region. They used a Monte Carlo simulation method to solve the Boltzmann equation to investigate the propagation of energetic particles when they escaped from the shock front.

In this paper, we extend Paper 1 and model a typical Type 2 event through numerical simulation. We solve the cosmic ray transport equation numerically without making any assumptions concerning the equation. Our calculation method is a stochastic differential equation (SDE) method, a kind of Monte Carlo simulation, coupled with particle splitting. We consider the possibility that the large X-ray flares that accompanied almost every Type 2 particle event play an important role as an injection mechanism, and perform simulations under the assumption that high energy particles act as seed populations.

We describe the observation of a typical energetic particle event in section 2, and discuss our modeling method and simulation results in section 3, and conclude in section 4.

## OBSERVATIONS

We used mainly the solar wind data and particle data observed by the SWEPAM, MAG, and EPAM teams of the ACE and also used the solar wind data obtained by the CELIAS/MTOF PM on the SOHO when there were data gaps in the ACE data. The GOES proton data provided useful information about protons with energies greater than 10 MeV, which we used to confirm the maximum energy of accelerated particles.

Here, we model the energetic particle event that occurred on 315 DOY, 2000. This was a typical Type 2 event because it was associated the M7.4 X-ray flare (Figure 1) and greater than 10 MeV protons were detected (Figure 2). Energetic particles increased at almost the same time as the flare occurrence (Figure 3). We considered the possibility that it would have been difficult for those particles to undergo diffusion process because there was insufficient diffusive acceleration time, and that some of the particles were scattered and propagated to the earth. Indeed, this should be confirmed through theoretical and numerical analyses that takes into account the time evolution and the distance dependence of the solar wind parameters and the

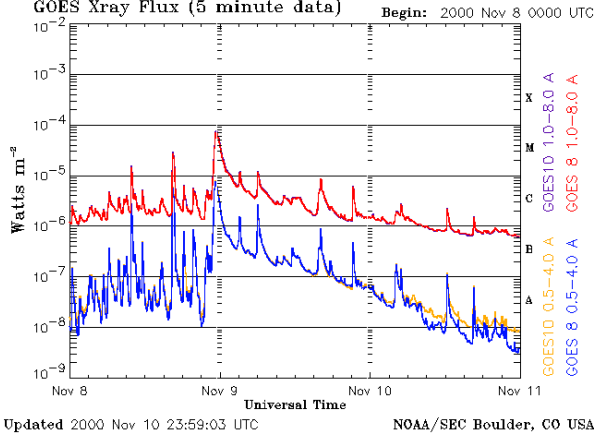


Fig. 1. X-ray flux observed by the GOES satellite for the period 2000 Nov. 8 (313 DOY) to 10 (315 DOY). The M7.4 X-ray flare occurred at 23:28 UT on Nov. 8. The source region is region 9213(N10W77). This plot is provided on URL <http://sec.noaa.gov/ftpmenu/plots/xray.html>.

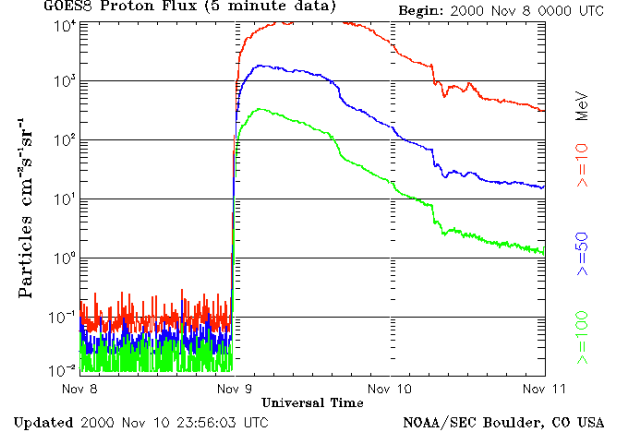


Fig. 2. MeV proton flux for the same period as Figure 1. A rapid increase of the proton flux after the flare can be seen, and the greater than 10 MeV event began at 23:50 UT. This plot is provided on URL <http://sec.noaa.gov/ftpmenu/plots/proton.html>.

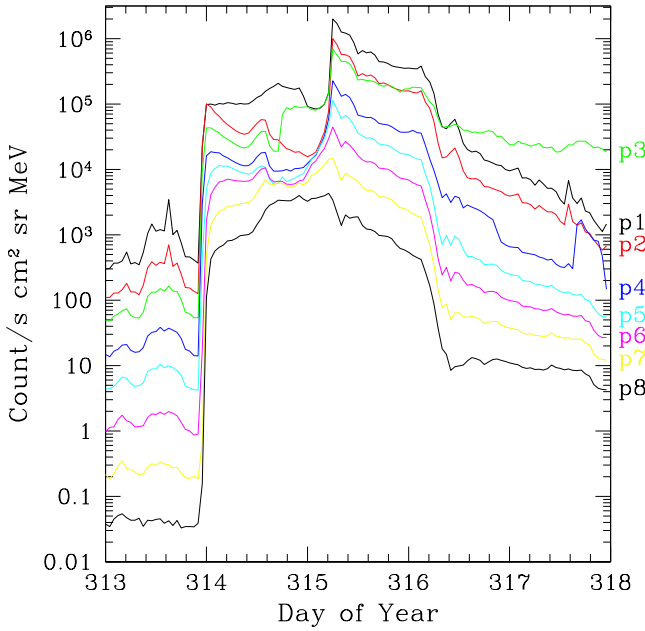


Fig. 3. Intensity-time profile of particles with energy channels, (p1 47-65 keV, p2 65-112 keV, p3 112-187 keV, p4 187-310 keV, p5 310-580 keV, p6 580-1060 keV, p7 1060-1910 keV and p8 1910-4750 keV), for the period Nov. 8 (313 DOY) to 12 (317 DOY). The shock wave passed on 315 DOY. These data are ACE Level 2 Data, and were obtained from <http://www.srl.caltech.edu/ACE/ASC/level2/index.html>.

proton mass, and  $c$  is the speed of light. Equation (1) is equivalent to following SDEs,

$$dr = (v_r + 2K/r)dt + \sqrt{2K}dW_r \quad (2)$$

shock structure.

## NUMERICAL SIMULATION METHOD AND RESULTS

We applied the SDE method coupled with the particle splitting to simulations of diffusive shock acceleration process. Numerical computations using SDEs are much easier than solving the original the cosmic-ray transport equation, because the SDEs are ordinary differential equations.

Assuming a spherically symmetric geometry, the Fokker-Planck form of the cosmic-ray transport equation is given by

$$\begin{aligned} \frac{\partial \phi}{\partial t} = & -\frac{\partial}{\partial r}(v_r + \frac{2K}{r})\phi + \frac{\partial^2}{\partial r^2}K\phi \\ & + \frac{\partial}{\partial u}\{\frac{1}{3}(\frac{\partial v_r}{\partial r} + \frac{2v_r}{r})\phi\}. \end{aligned} \quad (1)$$

Here, we introduce the quantities  $u = \ln(p/m_p c)$  and  $\phi = 4\pi r^2 p^3 f$ , where  $p$  is the proton momentum,  $f$  is the distribution function for protons,  $v_r$  is the radial velocity of the background flow,  $K$  is the spatial diffusion coefficient,  $m_p$  is the

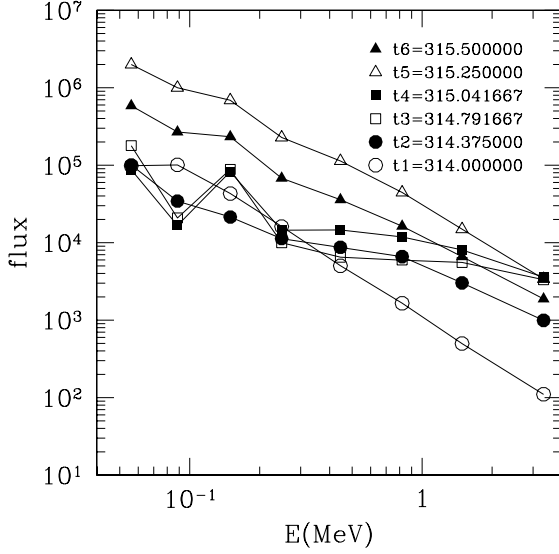


Fig. 4. Evolution of energy spectrum for the event on 2000 Nov 10 (315 DOY).

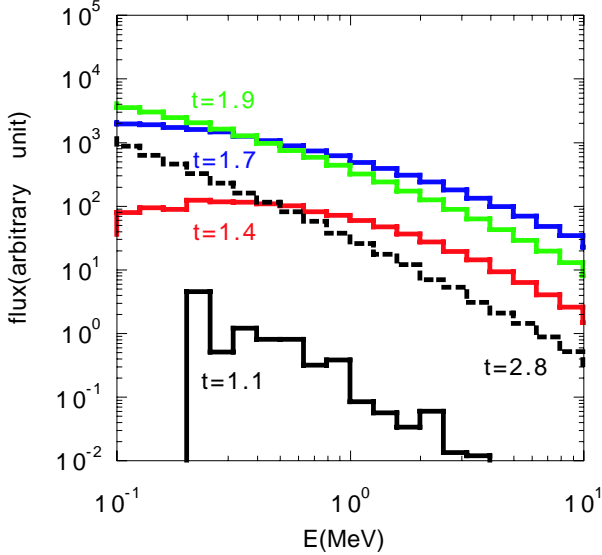


Fig. 5. Simulated evolution of the energy spectrum for model A obtained by the particles around 1 AU.

and

$$du = -\frac{1}{3}\left(\frac{\partial v_r}{\partial r} + \frac{2v_r}{r}\right)dt, \quad (3)$$

where  $dW_r$  is a Wiener process given by the Gaussian distribution

$$P(dW_r) = (2\pi dt)^{-1/2} \exp(-dW_r^2/2dt). \quad (4)$$

The integration of the SDEs is achieved through a simple Euler method. Particle splitting is essential to achieve a wide dynamic range for the energy attained by accelerated particles (Yoshida and Yanagita, 1994, MacKinnon and Craig, 1991, Achterberg and Krulls, 1992, Krulls and Achterberg, 1994). We set splitting surfaces  $u_i$  in energy space with an equal spacing in logarithmic scale. Each time an accelerated particle hits the surface  $u_i$ , the particle is split into

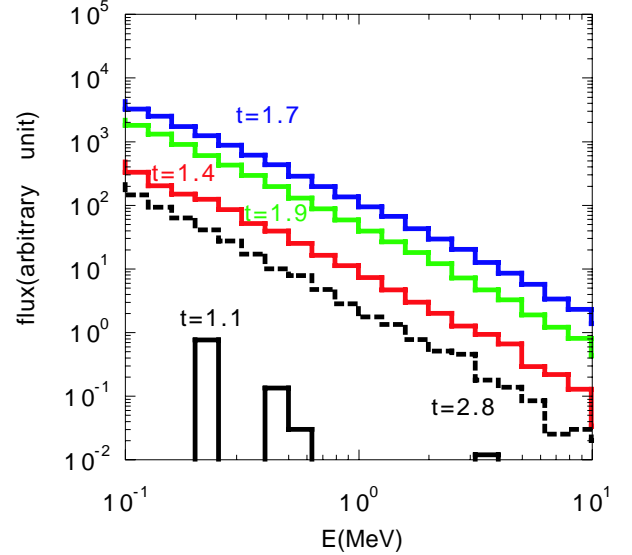


Fig. 6. Simulated evolution of the energy spectrum for model B obtained by the particles around 1 AU.

$w$  particles with the same energy and spatial position which the particle has attained. The statistical weight which is needed to calculate the final spectrum of the particles is decreased by a factor of  $w$  in each splitting. In our simulation we take  $w = 2$  as a choice. This particle splitting method is permitted, because the process which we are considering is Markovian.

It is well known that many elements are involved in the particle acceleration in interplanetary space such as a finite diffusive region, the time and spatial dependence of the magnetic field, the shock structure, and so on. However, we assumed a simple model as a starting point for our simulations so that we could clarify the effect of each element on the acceleration mechanism.

Our modeling assumptions were that the space was spherically symmetric, and that the shock velocity, the compression ratio of the shock front, and the diffusion coefficient were constant and uniform. For the shock parameters, we set the shock speed  $V_s = 852$  km/s and the compression ratio  $r_n = 3.0$ ; these values were obtained from observational data at 1AU and for the diffusion coefficient,  $K = 2.7 \times 10^{19}$  cm<sup>2</sup>/s, which was set by fitting the diffusion length of the observed proton flux to the theoretical solution obtained by Blandford and Ostriker (1978). The shock wave started at  $r_s = 0.1$  AU where  $r_s$  is the radial distance from the Sun and we set  $t = 0$  at that time.

Injection is an issue that is not well understood. As explained in the previous section, large X-ray flares were associated with Type 2 energetic events, so we considered how the flare process contributed to particle ejection into the interplanetary space in which the particles were accelerated by shock waves. After trial simulations with various values of the ratio of impulsively injected particles to continuously injected ones, the energies of injected particles, and the distance from the sun where the particles were injected impulsively, we found the following model, called model A, could fit the observation. In model A, 98% of injected particles with an energy of 150 keV were input impulsively when the shock front arrived at  $r = 0.32$  AU, and we considered the “flare particles”, while 2 % of the particles with an energy of 50 keV were input continuously. It should be noted that detailed numerical values used for simulations such as “98%” and “0.32 AU” have not rigid meaning and that the order of those values has physical sense.

To clarify the effect of high energy injection particles, in model B all injection particles were low energy particles, set to 50 keV, and were continuously input.

Figure 4 shows an energy spectrum obtained from observational data regarding the particle flux. Some notable features can be seen in the time evolution of the spectrum. The spectrum became flatter immediately after the source CME occurred, and then became steep, i.e., “soft” after the shock passed. Our simulation results are shown in Figures 5 and 6 for models A and B respectively. Note that the simulated energy spectrum was plotted from particles close to 1 AU, not from the entire space. The observed data were detected at one point, e.g., the L1 point for the ACE data. Hence we should use the simulated particle data placed around 1 AU when comparing results with the observations. (We discuss detailed difference between the simulation result plotted from the particles close to 1 AU and that plotted from the particles in the entire space in the other paper (Den, et al., these proceedings).) The spectrum for model A is similar to the observational one, that is, the spectrum is flat just after the X-ray flare and the source CME having occurred, and then it evolves to be in proportion to a power law which theoretical models of diffusive shock acceleration suggest (Blandford and Ostriker, 1978). On the other hand, for model B, the spectrum evolves to become slightly harder, which indicates that the seed particles with an energy of 50 keV were accelerated by the shock wave diffusively and that our numerical simulations were correct. However, model B cannot explain the flat spectrum feature seen in the observation. Our simulation results thus indicate that considerable amount of the particles with an energy of a few hundred keV already exist in the middle of 1 AU before the shock wave passes. We think that these seed particles with a few hundred keV are ejected in the process of large X-ray flares which are highly correlated with the source CMEs and are re-accelerated by the shock waves. We conclude that the impulsive injection is an important consideration and may be necessary for Type 2 events.

## CONCLUDING REMARKS

Through numerical simulations, we have modeled typical events classified as Type 2 events in which a maximum energy of protons is greater than 10 MeV and obtained the energy spectrum. Those protons were presumably accelerated in a flare process and were re-accelerated by CME-driven shock waves. We considered two injection models: in model A (corresponding to Type 2 events), 98% of the particles were impulsively injected with an energy of 0.15 MeV when the shock front arrived at 0.32 AU, and the remaining particles were continuously injected with an energy of 0.05 MeV; in model B (corresponding to Type 1 events) all particles were injected continuously with an energy of 0.05 MeV. Our simulation results show that impulsive injection leads to a flatter spectra, while 100% lower-energy continuous injection models cannot explain such spectral flatness, and that considerable amount of the particles with an energy of a few hundred keV already exist in the middle of 1 AU before the shock wave passes. We consider that those impulsively injected particles were ejected in the flare process, and our simulation results are consistent with the result of statistical analysis obtained in Paper 1, that is, high correlation between the source CMEs for Type 2 events and large X-ray flares. Hence we conclude that the impulsive injection is important and may be necessary for Type 2 events.

The simulation method used in this study was a simple one; i.e., our method was essentially one-dimensional, so the shock drift acceleration could not be included, and our assumptions of constant and uniform shock speeds and diffusion coefficients are not valid. The effect of dynamical behavior of those parameters is investigated at present. Finally, turbulence, or self-excited waves in a magnetic field caused

by wave-particle interaction, and a particle transport process were not taken into account. The existence of turbulence is essential for shock acceleration, and we must study the possibility that self-excited waves are closely related to Types 3 and 4 events. Use of a fully self-consistent multidimensional simulation method can clarify the acceleration mechanism and particle propagation process of events for each event type. Our modeling method seems to be a good starting point for this approach.

## ACKNOWLEDGEMENTS

We would like to thank the ACE EPAM, SWEPAM and MAG teams and the SOHO CELIAS/MTOF PM team for providing their data and the GOES team for providing their plots.

## REFERENCES

- Achterberg, A., and W. M. Krulls, A Fast Simulation Method for Particle Acceleration, *Astron. and Astrophys.*, **265** L13-L16, 1992.
- Armstrong, T. P., G. Chen, E. T. Sarris, and S. M. Krimigis, Acceleration and Modulation of Electrons and Ions by Propagating Interplanetary Shocks, *Study of Traveling Interplanetary Phenomena*, 367, D. Reidel, Hingham, Mass, 1977.
- Axford, W. I., E. Leer, and G. Skadron, The Acceleration of Cosmic Rays by Shock Waves, *Proc. 15th Int. Conf. Cosmic Rays*, **11**, 132-137, 1977.
- Blandford, R. D., and J. P. Ostriker, Particle Acceleration by Astrophysical Shocks, *Astrophys. J.*, **221**, L29-L32, 1978.
- Cane, H. V., D. V. Reames, and T. T. von Rosenvinge, The Role of Interplanetary Shocks in the Longitude Distribution of Solar Energetic Particles, *J. Geophys. Res.*, **93**, 9555-9567, 1988.
- Den, M., T. Yoshida, and K. Yamashita, Particle Acceleration in Interplanetary Shocks: Classification of Energetic Particle Events and Modeling, in *Solar and Galactic Composition*, edited by Wimmer-Schweingruber, R.F., 323-328, American Institute of Physics, St. Louis, 2001.
- Den, M., T. Yoshida, and K. Yamashita, Enhancement in Low-energy Region of a Proton Flux Associated with Interplanetary Shock Waves, *submitted to Adv. Space Res.*
- Kallenrode, M.-B., A Statistical Survey of 5-MeV Proton Events at Transient Interplanetary Shocks, *J. Geophys. Res.*, **101**, 24393-24410, 1996.
- Krymskii, G.F., A regular mechanism for the acceleration of charged particles on the front of a shock wave, *Sov. Phys. Dokl.*, **22**, 327-328, 1977.
- Krulls, W. M., and A. Achterberg, Computation of Cosmic-ray Acceleration by Ito's Stochastic Differential Equations *Astron. and Astrophys.*, **286**, 314-327, 1994.
- Lee, M. A., and J. M. Ryan, Time-dependent Coronal Shock Acceleration of Energetic Solar-Flare Particles, *Astrophys. J.*, **303**, 829-842, 1986.
- Li, G., and G. P. Zank, Energetic particle acceleration and transport at coronal mass ejection-driven shocks, *J. Geophys. Res.*, **108**, in press.
- MacKinnon, A. L., and I. J. D. Craig, Stochastic Simulation of Fast Particle Diffusive Transport *Sol. Phys.*, **251**, 693-699, 1991.
- Reames, D. V., L. M. Barbier, and C. K. Ng, The Spatial Distribution of Particles Accelerated by Coronal Mass Ejection-driven Shocks, *Astrophys. J.*, **466**, 473-486, 1996.
- Reames, D. V., Particle Acceleration at the Sun and in the Heliosphere, *Space Sci. Rev.*, **90**, 413-491, 1999.
- van Nes, P., R. Reinhard, T. R. Sanderson et al., The Energy Spectrum of 35-1600 KeV Protons Associated with Interplanetary Shocks, *J. Geophys. Res.*, **89**, 2122-2132, 1984.
- Yoshida, T., and S. Yanagita, A Stochastic Simulation Method for Particle Acceleration and Non-Thermal Photon Emission in Astrophysical Processes, *Prog. of Theor. Phys.*, **92**, 1217-1222, 1994.
- Zank, G. P., W. K. M. Riefe, and C. C. Wu, Particle Acceleration and Coronal Mass Ejection-driven Shocks: a Theoretical Model, *J. Geophys. Res.*, **105**, 25079-25096, 2000.

E-mail address of M. Den den@crl.go.jp

Manuscript received 02 December, 2002; revised 31 March, 2003; accepted 31 March, 2003