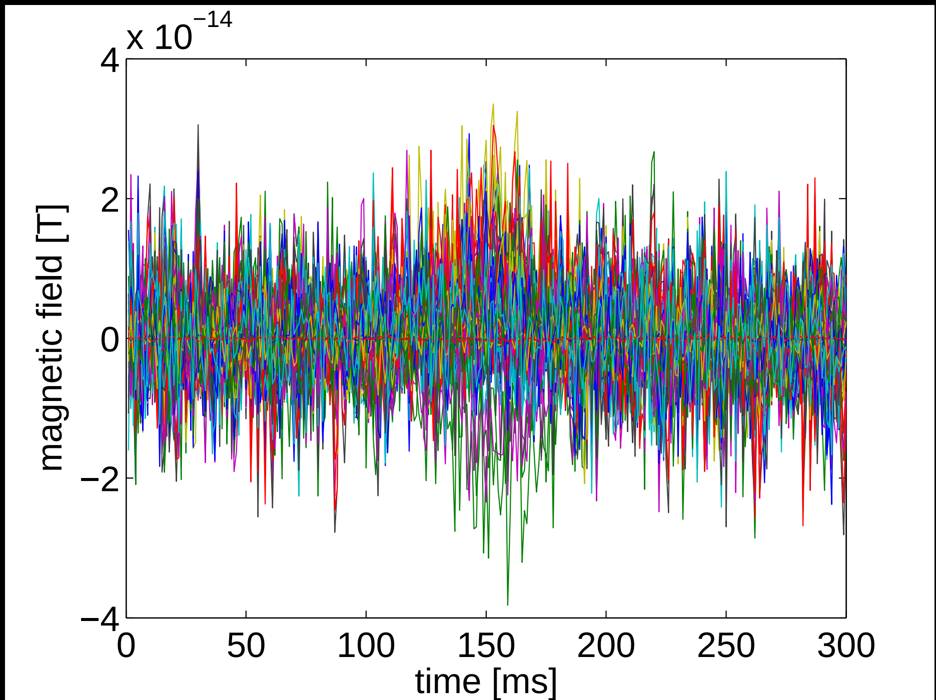
USING VISIBILITIES IN THE ELECTRON SPACE

Michele Piana
Universita' di Verona
The MIDA group

PROBLEM 1

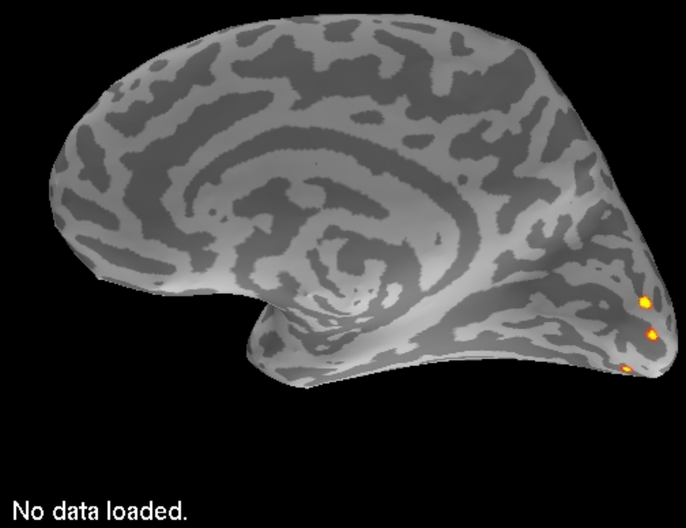
Suppose to be able to measure the magnetic field at all points outside a human brain looking at a meaningful visual message: how would the data look like?



- very low SNR
- fast oscillations over time

PROBLEM 2

Suppose to be able to image the activity inside a brain looking at a meaningful visual message: how would the image look like?



- activations in the visual region
- activations around the visual region

SOME QUESTIONS

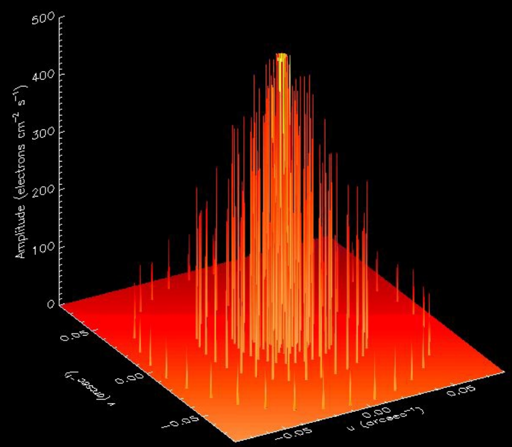
Q1: are devices providing biomagnetic data feasible?
A1: yes: use SQUIDs

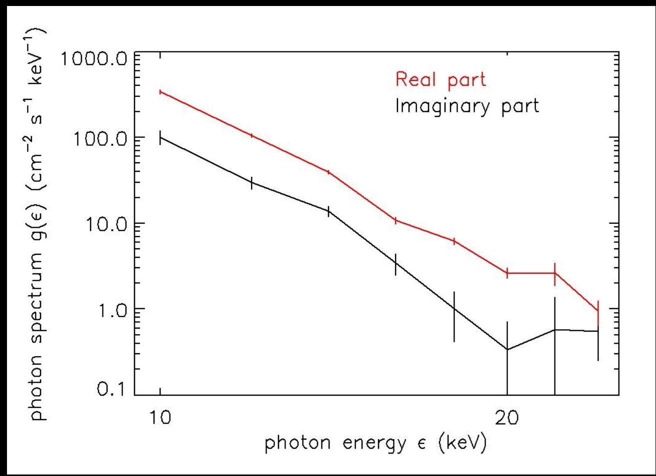
Q2: is it possible to infer information on the brain activity from biomagnetic data?
A2: yes: use math. In particular:

- model for the data formation (Maxwell's equations)
- assumptions on the solution (cortical constraint)
- smart method for data analysis (Bayesian tracking)

PROBLEM 1’

Suppose to be able to observe measurements of the spatial Fourier components of a radiation field emitted by a solar flare: how would the data look like?

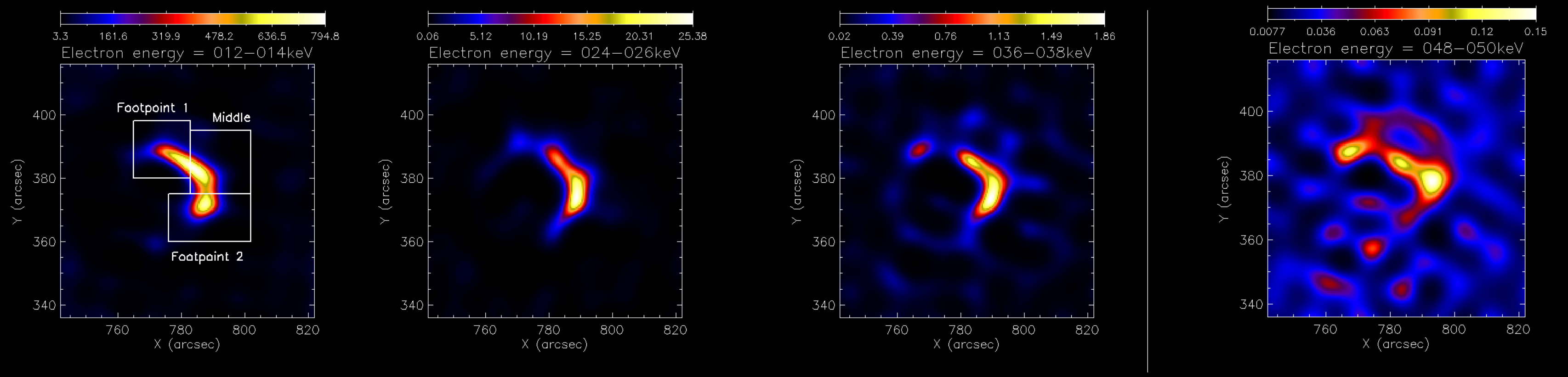




- sampling of the frequency space
- high DC component
- rapid deterioration with energy

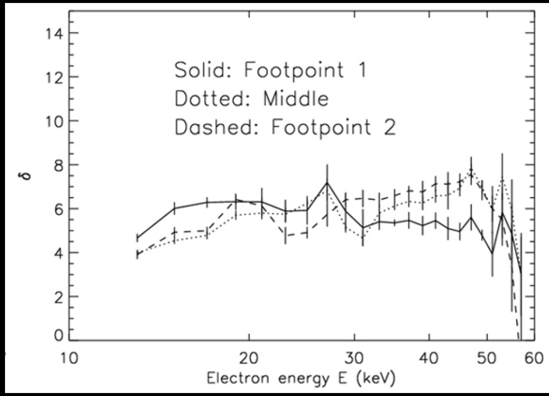
PROBLEM 2'

Suppose to be able to image the distribution of accelerated particles at different energies during a flaring event: how would the image look like?



- more electron than count energies

- coherency between spectral and spatial information



SOME QUESTIONS AGAIN

Q1: are devices providing spatial frequency information on the emission feasible?

A1: yes: use RHESSI

Q2: is it possible to infer spatio-spectral information on the physics in situ from these Fourier components?

A2: yes: use math. In particular

- model for the data formation (collisional bremsstrahlung)
- assumptions on the solution (smoothing constraint in the energy direction)
- smart method for data analysis (inverse problems theory)

A SIMPLE SYLLOGISM

Given that:

the relation between the photon flux spectrum and
the electron flux spectrum is linear, and

the relation between photon visibilities and photon flux
at each energy is linear

It follows that:

the relation between the photon visibility spectrum and
the electron visibility spectrum is linear

REMARK

This imaging-spectroscopy approach (Piana et al 07) is extremely flexible:

- the model can be generalized to more complicated (and realistic) conditions: anisotropies, ee-bremsstrahlung recombination

- more constraints can be inserted into the analysis procedure: statistical characteristics of the noise, dynamical range of the spectral and spatial distributions of the electrons

GENERAL SCHEME

- A set of physical hypotheses leads to formulas relating some parameters of the flaring region

- These parameters can be empirically estimated from the electron maps

- If the formulas are satisfied for the empirical values of the parameters, the hypotheses in the set are reliable

EXAMPLE - I

The centroid is a good parameterization of the flare location

Photon space:

$$N_c^{\varepsilon} = \frac{\int_0^{\infty} N(s) I(\varepsilon, s) ds}{\int_0^{\infty} I(\varepsilon, s) ds}$$

Electron space:

$$N_c^{E} = \frac{\int_0^{\infty} N F(E, N) dN}{\int_0^{\infty} F(E, N) dN}$$

EXAMPLE - I

Xu et al 08:

Point-source injection + Coulomb collisions:

$$N_c^{\varepsilon} = \left(\frac{\varepsilon^2}{K}\right)\frac{\delta-2}{(\delta-3)(\delta-4)}$$

Collisional target:

$$N_c^{E} = \left(\frac{E^2}{K}\right)\frac{1}{(\delta-3)}$$

$\Rightarrow$ the centroids in the photon and electron maps are related by

$$N_c^{\varepsilon} = \frac{\delta-2}{\delta-4}\left(\frac{\varepsilon}{E}\right)^2 N_c^{E}$$

EXAMPLES - I

Hypotheses:

H1: Coulomb collisions

H2: point-source injection

Formula:

$$N_c^{\varepsilon} = \frac{\delta - 2}{\delta - 4}\left(\frac{\varepsilon}{E}\right)^2 N_c^E$$

Parameters (empirical): the centroids can be determined from the photon and electron maps or (better) directly from the visibilities (see next talk)

Remark: if this formula is validated by the geometrical parameters determined from the photon and electron maps, the target density can be determined by simple numerical differentiation

CONCLUSIONS

- Thanks to RHESSI, imaging spectroscopy in the electron space is feasible

- Thanks to visibilities, in order to perform imaging spectroscopy in the electron space you do not have to go through imaging spectroscopy in the count/photon space

- The mathematics making imaging spectroscopy in the electron space feasible is based on some assumptions on the model and on some (natural) constraints on the solution