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ACCURATE DETERMINATION OF THE SOLAR PHOTOSPHERIC RADIUS

T. M. BROWN
High Altitude Observatory, National Center for Atmospheric Research,[1] 3450 Mitchell Lane, Boulder, CO 80301
AND
J. CHRISTENSEN-DALSGAARD[2]
Teoretisk Astrofysik Center, Danmarks Grundforskningsfond
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ABSTRACT

The Solar Diameter Monitor measured the duration of solar meridian transits during the 6 years 1981–1987, spanning the declining half of solar cycle 21. We have combined these photoelectric measurements with models of the solar limb-darkening function, deriving a mean value for the solar near-equatorial radius of 695.508 ± 0.026 Mm. Annual averages of the radius are identical within the measurement error of ±0.037 Mm.

Subject headings: astrometry — Sun: fundamental parameters — Sun: oscillations

1. INTRODUCTION

The Sun is the only star for which reasonably precise values of the mass, surface radius, and luminosity are known. The solar mass $M_\odot$ is known from planetary motion, with accuracy limited only by the uncertainty in the gravitational constant G. In principle, the solar radius can be obtained from direct optical measurement of the solar angular diameter, given the very accurate determinations of the mean distance between the Earth and the Sun. In solar modeling, the value $R_\odot = 695.99$ Mm (Allen 1973) has been commonly used. The models are calibrated to this photospheric radius, in the present Letter defined by the point in the atmosphere where the temperature equals the effective temperature, by adjusting some measure of the convective efficacy, such as the mixing length.

Recent accurate observations of solar *f*-mode frequencies from the Solar Oscillations Investigation/Michelson Doppler Imager instrument on the *Solar and Heliospheric Observatory* satellite (e.g., Kosovichev et al. 1997) have raised some doubts over this value of $R_\odot$. The frequencies of these modes are predominantly determined by $GM_\odot/R_\odot^3$. By comparing the observed frequencies with the frequencies of solar models calibrated to $R_\odot = 695.99$ Mm, Schou et al. (1997) and Antia (1998) concluded that the actual solar radius was smaller by about 0.3 Mm than the assumed radius of the model.

Other aspects of the modeling of the solar *f*-modes may affect their frequencies at this level (e.g., Campbell & Roberts 1989; Murawski & Roberts 1993; Ghosh, Antia, & Chitre 1995). Thus, it is obviously important to obtain independent verification of the proposed correction to the solar radius.

Indeed, there are significant uncertainties associated with the currently adopted radius value. These are related to the problem of the definition of the solar limb adopted in the radius determinations and the reduction of the measured value to the photosphere. It is not clear how the value quoted by Allen (1973) was obtained. However, it appears that the more recent determinations, which are generally consistent with Allen, in most cases refer to the inflection point of the solar limb intensity. According to solar atmospheric models, this corresponds to a height of about 0.3 Mm above the photosphere, thus perhaps accounting for the radius correction inferred from the *f*-mode frequencies.

The uncertainty in the precise definition of the measured values of the solar radius highlights the need to combine the observations with careful modeling of the quantity that is observed. Here we consider a long series of observations obtained with the High Altitude Observatory's Solar Diameter Monitor (Brown et al. 1982). This is based on a definition of the solar limb that minimizes the effect of seeing (Hill, Stebbins, & Oleson 1975). By combining daily data obtained over more than 6 years, extending between solar maximum and solar minimum, the possible effects of solar activity can be checked. The analysis of the data is carried out by means of a model of the solar limb intensity, following as closely as possible the actual procedure used in the reduction of the data and testing for the effects of seeing. In this way, we have eliminated several of the uncertainties affecting earlier determinations to arrive at what we believe to be an accurate measure of the solar photospheric radius. We note that the observations are limited to the near-equatorial solar diameter and, hence, contain no information about possible changes in the solar figure. However, the solar surface oblateness is so small (e.g., Hill & Stebbins 1975; Lydon & Sofia 1996) that our measured diameter can be taken as representative of the Sun as a whole.

2. OBSERVATIONS

The Solar Diameter Monitor instrument and its associated observing procedures were described in detail by Brown et al. (1982). It operated between 1981 August and 1987 December and consisted of a meridian-transit telescope arranged to allow the solar image to drift across a fixed detector package each day at local noon. A filter system confined the bandpass of the observed light to a 10 nm band near 800 nm. The horizontal (east/west in the sky) diameter was obtained by timing the passage of the solar limbs across each of two linear detector arrays aligned end-to-end. Each detector pixel subtended 1″ in the sky in the direction of the apparent solar motion and 80″ in the perpendicular direction. In addition, the vertical (north/south in the sky) diameter was measured, although less precisely. For the purposes of this Letter, we shall therefore consider only the horizontal diameter.

An automated guiding system assured that the solar disk transited the detector arrays centrally, so that a true diameter was measured. Each readout of a detector array yielded a sam-

[1] The National Center for Atmospheric Research is Sponsored by the National Science Foundation.

[2] Institut for Fysik og Astronomi, Aarhus Universitet, DK-8000 Aarhus C, Denmark.

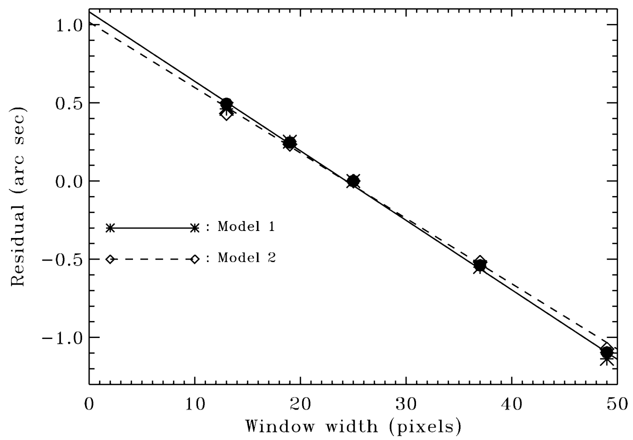

FIG. 1.—Observed and inferred diameters (i.e., separations between limb positions), as a function of window width. *Filled circles*, observed values; *stars*, calculated values for model 1; *diamonds*, calculated values for model 2. Seeing with an FWHM of 6″ was assumed. Also shown are the results of linear least-squares fits to the limb positions, defining the extrapolation to zero window width.

ple of the solar limb-darkening function; by reading the detectors at a 32 Hz rate, the instrument obtained samples at intervals comparable to the seeing-change time.

The instrument applied a real-time edge-finding algorithm and stored the resulting edge positions. This process was performed for the transits of both west and east solar limbs, so a transit duration could be measured. Ancillary quantities were also measured each day, including seeing and scattered-light parameters.

The edge-finding algorithm used was the finite Fourier transform definition (FFTD) described by Hill et al. (1975). The procedure involves forming the convolution of the observed limb-darkening function with a set of weights that are nonzero only within a certain window of width a. The edge was then defined to be the position of the center of the window for which the convolution crossed zero.

The FFTD has two important features. First, by a suitable choice of weights, one can eliminate the first-order sensitivity of the edge position to seeing, for some chosen width of the seeing point-spread function. Daytime seeing is commonly both poor and variable, and inflection-point definitions of the limb position are highly sensitive to this variability. Second, the FFTD depends on a free parameter, namely, the window width a. Applying the FFTD to the solar limb-darkening function yields diameters $D(a)$ approximately equal to

$$D(a) = D_0 - \alpha a, \tag{1}$$

where D_0 is the true angular distance between the nearly discontinuous intensity jumps at east and west limbs, and α is proportional to the intensity gradient in the last 25″ inside the limb and, hence, to the vertical temperature gradient in the photosphere above an optical depth of roughly 0.2. By measuring the diameter over a range of a and extrapolating the results to zero window width, one can obtain a value that is largely independent of changes in the slope of the limb-darkening function. Since this slope proves to vary significantly during the solar cycle, using multiple window widths is necessary to measure the slope and remove its effect from the raw diameter measurements.

The value of $D_0/2$ should not be confused with the solar

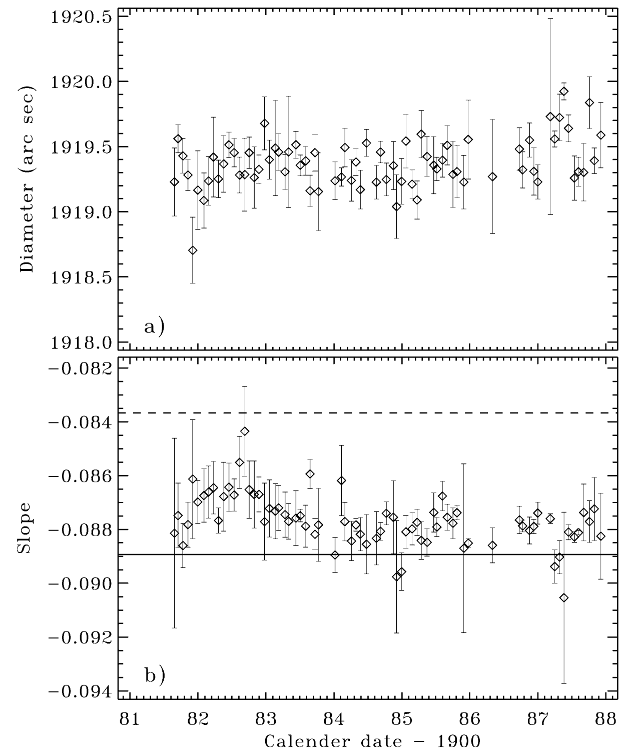

FIG. 2.—(*a*) Diameters extrapolated to zero window width, averaged over intervals of one Carrington rotation, shown as a function of calendar date. (*b*) The limb-slope parameter α, averaged in the same fashion as in (*a*). The solid and dashed horizontal lines indicate the slopes found for models 1 and 2, respectively.

radius as defined in § 1; it is instead a convenient observational quantity, constructed to be independent of the vertical temperature gradient in the upper photosphere. It differs from the correct radius by a constant (related to the arbitrariness inherent in any definition of the limb position) and possibly by time-dependent terms (related to temporal changes in the accuracy of the approximations underlying eq. [1]). The model results displayed in Figure 1 and described below are intended to estimate the value of the constant offset in radius and to indicate the likely importance of errors in equation (1).

Data were taken with the Solar Diameter Monitor on any day for which a successful observation seemed possible; many of the observations were therefore corrupted by clouds or (less often) instrument failures. Accordingly, we used several selection criteria to choose the observations to be used for the analysis, in the end retaining 550 of the original 986 daily measurements, each made with five different window widths. Finally, we corrected the diameter values for several geometrical sources of systematic error and projected all measurements to a standard seeing width of 6″, which is the most frequently observed value.

Figure 1 shows the unweighted average of these diameter values, reduced to a Sun-Earth distance of 1 AU and plotted against the FFTD window width. Also shown are the results of applying the FFTD to the seeing-blurred limb-darkening functions derived from two different model solar atmospheres (see below). The agreement between theory and observation is satisfactory, although residual differences affect the necessary extrapolation to zero a and are a significant source of systematic error.

Figure 2 shows the measured time series of diameter measurements D_0 (projected to $a = 0$ by linear extrapolation) and of α, averaged over Carrington rotation periods of 27.275 days. The error bars are standard deviations of the mean for each rotation, estimated from the dispersion of the daily measurements within that rotation. The scatter among the daily diameter measurements is about 0″.4 rms and arises mostly from time-dependent motions of the solar image related to atmospheric seeing. The diameter was essentially constant throughout the 6.3 yr observing period, aside from a possible but poorly sampled upturn of about 0″.1 during 1987. The limb-darkening function slope, however, varied with a timescale of a year or more over a total range of about 2.5%, being steeper at solar minimum than at maximum.

We estimate D_0 as the unweighted mean of the 550 daily values, with a random error equal to the standard deviation of the mean. These values are

$$D_0 = 1919''.359 \pm 0''.018. \qquad (2)$$

3. MODELING

As mentioned in § 1, the quantity to be measured is the radius of the surface at which the local temperature equals the solar effective temperature. Evidently, this radius is related to D_0 in equation (2), but the relationship is not a simple one; it depends on the radiation transfer in the outer solar atmosphere and on the behavior of the FFTD limb definition. To infer the correct radius from the observations, one must use a physically based model of the solar atmosphere to calculate the emergent intensity as a function of distance from the center of the solar disk and then compute the location on this brightness profile that would be identified as the edge by the FFTD. We calculated the limb intensity by integrating the equation of transfer along rays through an assumed spherically symmetrical solar atmosphere. Since the observations were carried out in a relatively narrow wavelength region around 800 nm, we considered simply the monochromatic continuum intensity at this wavelength.

We have used two models of the solar atmosphere. Model 1 was kindly computed by R. Medupe with the ATLAS9 code (Kurucz 1993). Model 2 was obtained as an average of a hydrodynamical simulation of convection in the upper part of the solar convection zone and lower atmosphere, as described by, e.g., Stein & Nordlund (1989), but with updated physics (Trampedach 1997); the average was performed at a constant monochromatic optical depth, at 800 nm. The opacity was computed from the ATLAS data in both cases. For model 1, the source function S_λ was obtained from the ATLAS code and hence allowed for mild departure from LTE. For model 2, LTE was assumed, so that $S_\lambda = B_\lambda$, the Planck function.

To simulate effects of seeing, we convolved the intensity with a Gaussian, with an FWHM specified in arcseconds and converted to linear distance at 1 AU. After this convolution, we integrated the intensities over pixels corresponding to 1″ at 1 AU, to match the observed intensities. We folded the pixel-weighted intensities with the FFTD weights over the five different windows described in § 2 and carried out the subsequent analysis to determine the limb position, through extrapolation to zero window width, as for the observations.

The results of applying the observational procedure to the computed pixel-averaged intensities for models 1 and 2, assuming 6″ seeing, are shown in Figure 1. The observed variation of diameter with window width and that calculated from the models agree well, especially for model 1. Of particular interest are the extrapolations to zero window width, corresponding to the observed limb position measured relative to the nominal photosphere of the models; we obtain 0.47950 and 0.51634 Mm for models 1 and 2.

We have tested the sensitivity of the results to various assumptions in the calculation. Replacing the true source function S_λ by B_λ for model 1 changes the limb position by much less than 0.001 Mm; thus, the assumption of LTE in model 2 is not a significant source of error. Using an assumed seeing of less than 6″ changes the limb position by less than 0.01 Mm, confirming the insensitivity of the FFTD to effects of seeing. However, the difference between the two models obviously remains a source of some concern.

4. RESULTS AND DISCUSSION

We adopted the modified IAU (1976) value of 1.4959787066×10^5 Mm (US Naval Observatory 1997) for the astronomical unit and adjusted this value by −4.678 Mm to account for the mean displacement between the telescope's noontime location and the Earth's center and by +0.449 Mm to account for the displacement of the Sun's center relative to the barycenter of the Earth-Sun system. This distance, combined with D_0 from equation (2), yields the Sun's apparent radius. Applying the model corrections described in the last section, we obtain

$$R_\odot = 695.5260 \pm 0.0065 \text{ Mm} \quad \text{for model 1,}$$

$$R_\odot = 695.4892 \pm 0.0065 \text{ Mm} \quad \text{for model 2.}$$

We estimate the modeling errors to be $1/\sqrt{2}$ of the difference between these estimates, or about 0.020 Mm. Based on the uncertainties in the geometric corrections that were made to the measured radius, we estimate the systematic errors in the measured value to be 0.015 Mm, or about twice as large as the random errors. Averaging our results for models 1 and 2 and adding the various error sources in quadrature, we arrive at our final estimate of

$$R_\odot = 695.508 \pm 0.026 \text{ Mm.}$$

The inferred solar photospheric radius is smaller by about 0.5 Mm than the normally used value of 695.99 Mm (Allen 1973). A review of recent observations was given by Schou et al. (1997), who concluded that these were consistent with an angular diameter of 1919″.26 ± 0″.2, corresponding to Allen's value of $R_\odot$. This is also consistent with the observed value obtained here (cf. eq. [2]). However, it appears that the observations considered by Schou et al. refer to the inflection point of intensity (or, in one case, to an FFTD determination) and hence do not contain the correction to photospheric radius. Such a correction, taking into account the observational characteristics, is an essential part of the radius determination.

Some confirmation of the reliability of the modeling comes from the comparison in Figure 2 of computed and observed slopes of the limb position as function of the scan widths. Nevertheless, it is striking that, as indicated by the difference between models 1 and 2, the major uncertainty in $R_\odot$ appears to come from the modeling. Indeed, it is evident that the real solar atmosphere is substantially more complicated than the one-dimensional model resulting from the ATLAS code or the mean model obtained from the hydrodynamical simulations. A more accurate determination of the radius correction can probably be obtained from a detailed calculation of the limb intensity, taking into account the inhomogeneous nature of the relevant layers, on the basis of the simulations. However, such an investigation is beyond the scope of the present Letter.

We find no significant variation in the observed diameter during the observation period (cf. Fig. 2); annual averages of the radius for the years 1981–1987 all agree within their measurement errors of ± 0.037 Mm. These limits are substantially smaller than diameter changes reported previously for the same interval of time (e.g., Ulrich & Bertello 1995; Laclare et al. 1996) but are in agreement with measurements by Wittman (1997). On the other hand, the limb-position slope shows fairly substantial variations. We also note that during solar maximum, the daily slope values tended to be highly variable as well as small in magnitude; this suggests that the long-term variation may result from localized activity-dependent features such as faculae. It is plausible that the previously inferred variations in solar diameter with solar activity is in fact a reflection of such variations in the limb-darkening slope.

It is interesting that the value of $R_\odot$ obtained here is somewhat smaller than that inferred from the solar *f*-mode frequencies, indicating additional contributions to the differences between the observed and model values of these frequencies. This issue, and the effects of the reduction of the model radius on the helioseismically determined structure of the solar interior, will be considered elsewhere. We note, however, that Antia (1998) and Schou et al. (1997) found significant effects on the helioseismically inferred sound speed from corresponding radius changes.

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