The Relationship Between Solar Radio and Hard X–ray Emission

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Abstract.

1. Introduction

From the beginning, the study of hard X–ray emission from the Sun has had a natural ally in solar radio emission (Kundu, 1961). The reason is straightforward: the electrons that produce hard X–ray emission, by definition, have energies of order 10 keV or more, and such energetic electrons are also very efficient emitters of radio emission in the solar corona. That they can produce radio emission by a number of different physical mechanisms, in contrast to the bremsstrahlung–dominated hard X–rays, means that the radio data provide a range of diagnostics that complement the hard X–ray measurements. Between these two wavelength ranges we should expect to be sensitive to most sources of energetic electrons, both thermal and nonthermal, in the solar corona.

In this article we will review developments in our understanding of the use of radio and hard X–ray emission since the launch of the Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI) satellite in 2002. In all cases discussed here, the hard X–ray emission arises from bremsstrahlung emitted when energetic electrons are accelerated by Coulomb forces in collisions with ambient ions, either in the chromosphere or in the corona. Inverse Compton emission is also a possible emission mechanism for the hard X–rays, but more extreme conditions are required and are not believed to be relevant for our discussion. Bremsstrahlung hard X–ray emission is proportional to the product of the nonthermal electron density and the ambient ion density.

2. Gyrosynchrotron emission from nonthermal electrons

2.1. Reconciliation of hard X–ray and microwave spectra

We follow the prescription of Hudson et al. (1978), based in turn on Brown (1971) and the Bethe–Heitler bremsstrahlung cross–section. Assume that the hard X–ray photon spectrum is cast into the form

$$\Phi(E_\gamma) = A_0 \left(\frac{E_\gamma}{E_0}\right)^{-\gamma} \quad \text{photons cm}^{-2}\ \text{s}^{-1}\ \text{keV}^{-1} \tag{1}$$

where E_γ is the photon energy, γ is the power–law slope of the photon spectrum, and A_0 is the normalization constant at a fiducial photon energy E_0 keV. In the case of the SPEX package of Richard Schwartz, E_0 = 50 keV for the broken power–law fit to the spectrum and the normalization obtained from the fit refers to this energy. Given this photon spectrum, the corresponding electron flux energy spectrum into the target is given by

$$\frac{d^2N(E)}{dE\,dt} = 3.28 \times 10^{33}\,\frac{A_0\,b(\gamma)}{E_{0,\text{keV}}}\left(\frac{E}{E_0}\right)^{-(\gamma+1)} \quad \text{electrons keV}^{-1}\ \text{s}^{-1} \tag{2}$$

where $b(\gamma) = \gamma^2 (\gamma-1)^2 B(\gamma-0.5, 1.5)$ and $B(x,y)$ is the beta function[1]. This parameter is of order 10 for normal conditions. The factor of E_0 appearing in the denominator here must be in units of keV, if we are interpreting the comment in Brown (1971, p. 498) following his equation (15) correctly. We use $N(E)$ to denote the total number of electrons at energy E in some unspecified volume: derivatives of this quantity with respect to volume and energy yield the volume number density and the energy distribution, respectively.

The electron flux itself cannot be compared with the microwave parameters, which involve a volume number density of electrons. To derive the latter from the electron flux energy distribution, we assume that it may be written

$$\frac{d^2N(E)}{dE\,dt} = A_X \, \mathrm{v} \frac{d^2N(E)}{dE\,dV} \tag{3}$$

where A_X is the area of the X–ray source and v is the downwards velocity of the electrons. This is the classic expression relating a density to a flux. A_X must be obtained from observations, and its determination relies on high–quality imaging data.

The downwards velocity v is treated as follows. For ultrarelativistic electrons it is assumed to be of order c with no energy dependence. For nonrelativistic electrons we assume, as an approximation, that the downwards component of motion carries one–third of the electron energy (equipartition between directions of motion). Thus we set

$$\frac{1}{2} m \mathrm{v}^2 = \frac{1}{3} E \tag{4}$$

which leads to

$$\mathrm{v} = c\sqrt{\frac{2}{3}\frac{E}{mc^2}} = 0.0361\, E_{\mathrm{keV}}^{0.5}\, c = 1.08 \times 10^9\, E_{\mathrm{keV}}^{0.5} \quad \mathrm{cm\ s^{-1}} \tag{5}$$

Substituting (2) and (5) into (3), we find the following expressions for the electron volume number density energy distribution in the nonrelativistic limit:

$$\frac{d^2N(E)}{dE\,dV} = 3.04 \times 10^{24} \frac{A_0\, b(\gamma)}{E_{0,\mathrm{keV}}^{1.5} A_X} \left(\frac{E}{E_0}\right)^{-(\gamma+1.5)} \quad \mathrm{electrons\ cm^{-3}\ keV^{-1}} \tag{6}$$

This formulae indicates that for a given photon power–law index γ, the electron energy power–law index δ in a thick target model is $\gamma + 3/2$ in the nonrelativistic case.

The ultrarelativistic limit is somewhat complicated and simple expressions do not seem to exist. In terms of the spectrum, $\mathrm{v} = c$ in (3) and no additional power of E is added when the electron flux is converted to a number density, i.e., if this were the only change in the ultrarelativistic limit we would find $\frac{d^2N(E)}{dE\,dV} \propto E^{-(\gamma+1)}$. In terms of the photon spectrum, this would actually imply that for a pure electron energy power law δ the bremsstrahlung photon spectrum would steepen in the ultrarelativistic limt compared to the nonrelativistic limit by 0.5 in the spectral index, i.e., $\gamma = \delta - 1$ instead of $\gamma = \delta - 1.5$. However, in practice the bremsstrahlung cross–section also changes in the ultrarelativistic limit, and the combined effect seems to be to flatten the photon spectrum above 500 keV by about 0.5, $\gamma = \delta - 2$. Vestrand (1988) notes that the probability of an electron–proton collision producing a photon with a large fraction of the electron energy increases in the relativistic

[1] Note that $B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$, $\Gamma(x+1) = x\Gamma(x)$ and hence $\gamma B(\gamma-0.5,1.5) = B(\gamma-0.5,0.5)/2$ since $\Gamma(0.5) = 2\Gamma(1.5)$.

limit, and further that electron–electron collisions play a more important role at high electron energies. McTiernan & Petrosian (1990) state that a rough integration of the product of electron flux and cross–section gives a flattening of the photon spectrum above 500 keV by $\log(E_\gamma/m_e c^2)$, although their calculation refers to fluxes rather than volume number densities. In any case, it does not seem to be readily feasible to carry out numerical calculations in the relativistic limit: the best one can do would appear to be to join the nonrelativistic limit smoothly onto a power law 0.5 flatter above 500 keV.

To derive the number density for the radio–emitting electrons we use the Dulk & Marsh (1982, see Dulk 1985 for corrected versions) approximation formulae. The quantity that is easiest to measure and least dependent on a specific geometrical model for the radio source is the brightness temperature at a point in the image, obtained at the highest frequency and spatial resolution possible. Other quantities, such as fluxes, require source areas to be known, and these are much more model–dependent than a localized brightness temperature. It is assumed that the radio spectral index α_r can be measured from two optically–thin frequencies. The expression for radio brightness temperature at a frequency f produced by an electron number density energy distribution

$$\frac{d^2N(E)}{dE\,dV} = N_r \, \frac{(\delta - 1)}{E_r} \left(\frac{E}{E_r}\right)^{-\delta} \quad \text{electrons cm}^{-3}\ \text{keV}^{-1} \tag{7}$$

(here N_r is the number of electrons per unit volume in the distribution above the fiducial electron energy E_r, taken to be 10 keV by Dulk & Marsh) in a magnetic field of strength B Gauss is

$$T_B = \kappa T_{eff} = 3.1 \times 10^{-0.53\delta} \sin^{-0.45+0.66\delta}\theta \left(\frac{f}{f_B}\right)^{-0.80-0.90\delta} \frac{N_r L}{B} \tag{8}$$

where θ is the angle between the magnetic field and the line of sight, L is the line–of–sight depth through the source at the point where T_B is measured, and $f_B = 2.8 \times 10^6 B$ Hz is the electron gyrofrequency. Expression (8) results from multiplying together the Dulk (1985) expressions for opacity κ and effective temperature T_{eff}. In the radio side of the analysis we proceed to use the expression (8) for T_B to determine the parameter N_r that appears in (7); note that N can also be derived directly from the X–ray expression (6) by equating it to (7), as presented below.

For the radio approach we use

$$\frac{f}{f_B} = \frac{10^9 f_{\rm GHz}}{2.8 \times 10^6 B} = 357 \, \frac{f_{\rm GHz}}{B} \tag{9}$$

(where B is everywhere in units of Gauss) and find

$$T_B = 2.81 \times 10^{-2.00-2.83\delta} \sin^{-0.45+0.66\delta}\theta \, f_{\rm GHz}^{-0.80-0.90\delta} \, N_r L B^{-0.20+0.90\delta} \tag{10}$$

Inverting this expression to obtain N_r and substituting into (7) we find

$$\frac{d^2N(E)}{dE\,dV} = 35.3\,(\delta-1)\,10^{3.83\delta-1} \, \frac{T_B}{LB^{-0.20+0.90\delta}} \, \sin^{0.45-0.66\delta}\theta \, f_{\rm GHz}^{0.80+0.90\delta} \, E_{keV}^{-\delta} \tag{11}$$

in units of electrons cm^{-3} keV^{-1}. The quantity T_B is measured; θ is usually assumed to be close to 90° since this maximizes the gyrosynchrotron emissivity; L can be estimated from the maps; and since $\alpha_r = 1.20 - 0.90\delta$ is the radio flux spectral index, the value of δ obtained from the hard X–ray spectrum can be independently checked from the high frequency radio spectrum. The main remaining unknown in the equation is B, which appears as a very high power. In principle this too can be determined using the location of the radio peak frequency, which determines B via the following gyrosynchrotron approximation

$$f_{peak} = 2.72 \times 10^{3.00+0.27\delta} \sin^{0.41+0.03\delta}\theta\,(N_r L)^{0.32-0.03\delta}\,B^{0.68+0.03\delta} \tag{12}$$

Typically a peak frequency of 10 GHz corresponds to $B \approx 600$ G.

Expressions (6) and (11) are two separate expressions for a volume density energy distribution and can be compared. When the spectral indices disagree by a large amount, as is often the case, then it is usually assumed that they refer to different energy ranges: the optically–thin microwaves are more sensitive to electrons above 500 keV, while hard X–rays are usually dominated by electrons below 500 keV.

One complication for the comparison that needs to be borne in mind is that the number density appearing in (6) refers to the density at the footpoints just above the chromosphere, while the radio number density is often at a different location in the loop. Implicit is also the assumption that the pitch angle distribution is everywhere isotropic, corresponding to the fast pitch angle scattering limit; an anisotropy in the pitch angle distribution will affect the comparison.

We may now derive an expression for the radio flux from an integrated hard X–ray spectrum. Given the radio brightness temperature, in this case the Dulk and Marsh approximation (8), we obtain the total radio flux from the standard expression (e.g., Dulk, 1985, equation 14):

$$S = \frac{k_B f^2}{c^2} \int T_B \, d\Omega \tag{13}$$

where the integral is over the solid angle Ω subtended by the radio source on the sky. We may replace the solid angle element $d\Omega$ with dA/d^2, where A is the actual physical area of the source on the sky and d is the distance of the source (in this case, d is one astronomical unit, 1.5 10^{13} cm). From (8) we may write

$$S = \frac{k_B f^2}{c^2} \int 2.81 \times 10^{-2.00-2.83\delta} \sin^{-0.45+0.66\delta}\theta\, f_{\rm GHz}^{-0.80-0.90\delta}\, B^{-0.20+0.90\delta}\, N_r \frac{L dA}{d^2} \tag{14}$$

Now we note that $\int N_r\, L dA$ is the integral of the number density per unit volume (above the reference energy E_r) over the volume of the source, i.e., it is the total number of electrons in the emitting volume, N_{tot}. Further, 1 solar flux unit (sfu) is 10^{-19} in cgs units. Combining the physical constants and setting the angular factor to unity as an approximation (e.g., for $\theta = 70^\circ$ and $\delta = 3.0$, $\sin^{-0.45+0.66\delta}\theta = 0.9$), we find

$$S_{sfu} = 1.9 \times 10^{-28.0-2.83\delta}\, N_{tot}\, f_{\rm GHz}^{1.20-0.90\delta}\, B^{-0.20+0.90\delta} \tag{15}$$

2.2. SPECTRAL COMPARISON

2.3. Morphological comparison

3. Millimeter– and submillimeter–wavelength emission from flares

4. Low–frequency radio emission and hard X–rays

4.1. Radio bursts and energy release in flares

4.2. Hard X-rays from Type III radio bursts

4.3. Hard X-rays from reverse drift bursts

4.4. Radio emission from CMEs

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