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An Unambiguous Determination of the Propagation of a Compressional Pc 5 Wave

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During a 3-hour interval on August 21 and 22, 1978, a compressional Pc 5 event was observed by the ISEE-1 magnetometer and medium energetic particle experiment instrument. During the event the ISEE spacecraft was inbound near the equator and near the dusk meridian at distances between 11 and 7 R_e. The finite Larmor radii of the energetic protons allow us to determine unambiguously both the azimuthal and radial components of the phase velocity, and consequently the wave length and the azimuthal wave number. A $2n\pi$ ambiguity in evaluating the phase velocity is removed by finding a consistent and physically reasonable solution for different energy channels. As the spacecraft approached the plasmapause, a quasi-sinusoidal wave form having a 165-s period was observed; it was found to be propagating azimuthally westward with a phase velocity of about 33 km/s and an azimuthal wave number of 60. In the outer magnetosphere, more irregular wave forms were observed. These waves also propagated azimuthally westward and may possibly have had a small earthward (radial) component of phase velocity. To examine the effect of heavy ions on the estimated wavelength, we have derived the perturbation of differential flux by calculating the first-order perturbed phase space distribution of plasma consisting of two ion species. We found that, for the detector we used, the effect is very small even there is a significant fraction of heavy ions present. This is because for each energy channel, the detector response to the heavy ions is more sensitive to those with higher energy than those with the nominal proton energy, while the flux drops exponentially with the energy. In all cases the modulation amplitude of ion fluxes decreases with an increasing ratio of the ion Larmor radius to the wavelength.

INTRODUCTION

Since the late 1960s, efforts have been made to understand the structure of long-period magnetic pulsations ($T > 10$ s) in terms of resonant field line theory (see reviews by *Lanzerotti and Southwood* [1979]; *Hughes* [1982]; *Southwood and Hughes* [1983]). The field line resonance theory successfully accounts for many important features of observed long period pulsations which are believed to be transverse mode Alfven waves standing along magnetic fields. Harmonic oscillations of standing waves have been observed near the equatorial plane at the geostationary orbit [e.g., *Takahashi and McPherron*, 1982], on the ground [*Tonegawa et al.*, 1984], and along magnetic field lines (*Singer and Kivelson* [1979], *Singer et al.* [1982], and more recently, *Lin et al.* [1986]). The nodal structure of oscillating field lines has also been observed by the DE-1 satellite [*Cahill et al.*, 1986].

Recently, interest has focused increasingly on compressional waves which may be global oscillations of the magnetospheric cavity [*Kivelson and Southwood*, 1985*a*, 1986; *Southwood and Kivelson*, 1986; *Allan et al.*, 1985, 1986*a*, *b*], or caused by instabilities of inner-magnetospheric plasma, e.g., the drift mirror instability [*Hasegawa*,1969; *Lanzerotti and Hasegawa*, 1975]; the drift compressional instability [*Ng et al.*, 1984]; the instabilities in a high β plasma [*Pokhotelov et al.*, 1986], or by resonance with drifting particles [*Southwood*, 1980; *Kivelson and Southwood*, 1985*b*].

The characteristic phase relations between particle and magnetic signatures, wave polarizations, parallel and perpendicular wavelengths and wave phase velocity in the plasma rest frame differ for different types of theoretically predicted waves. Evidently, to determine which ULF wave generation mechanisms are significant in the magnetosphere, one must characterize the wave properties fully and compare the measurements with the theoretical predictions. For example, waves driven by Kelvin-Helmholtz instability at the magnetopause are expected to have phase velocities directed antisunward from the noon meridian. On the other hand, drift mirror waves in the afternoon sector are expected to have phase velocities oriented toward the noon meridian.

Wave polarization and phase relations can be established from single spacecraft measurements without difficulty. However, wavelengths and phase velocity in the plasma rest frame are hard to determine unambiguously. For example, measurements from three spacecraft noncolinear in a plane oblique to the magnetic field vector are needed to determine the direction of wave propagation from the phase delays in magnetic perturbations and such simultaneous measurements are rarely available. Even multipoint measurements remain ambiguous because phases are uncertain by integral multiples of 2π.

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In this paper, we will determine the perpendicular phase velocity of a compressional ULF wave by exploiting a technique that uses the finite Larmor radius effects in the signature of a multichannel energetic ion detector on a single spacecraft. We demonstrate that this technique determines unambiguously the propagation characteristics of the wave in two orthogonal directions in the plane perpendicular to the background magnetic field; our results remain valid even if heavy energetic ions with Larmor radii larger than proton Larmor radii are present in the plasma. We believe that the approach demonstrated in this paper will prove to be a powerful tool for the study of waves in the magnetosphere.

There have been many studies of compressional ULF waves, mostly at geostationary orbit. In addition to the earlier works on the storm-time compressional pulsations [e.g., *Brown et al.*, 1968; *Barfield and McPherron*, 1972; *Barfield et al.*, 1972), more observations and studies of compressional waves in various geomagnetic conditions have been made in recent years [*Su et al.*, 1977; *Barfield and McPherron*, 1978; *Kremser et al.*, 1981; *Higbie et al.*, 1982; *Nagano and Araki*, 1983; *Takahashi et al.*, 1985; *Takahashi and Higbie*, 1986; *Engebretson et al.*, 1986, and others], but there have been few cases where radial structure was observed. Two examples of the latter are *Kivelson et al.* [1984], and *Greenstadt et al.* [1986].

The event reported by *Greenstadt et al.* [1986] is an unusually clear example of storm-time compressional Pc 5 pulsations observed during an inbound, nearly equatorial pass of ISEE-1 and 2 in the dusk sector on August 21-22, 1978. Using magnetic observations from the ISEE-1 and 2 satellites, the authors studied wave propagation, polarization, and the relation of the pulsations to the interplanetary magnetic field and magnetic activity. Assuming that the phase delay between quasi-sinusoidal signals ($T = 165$ s) recorded at the two spacecraft represented only azimuthal propagation of the wave, the authors estimated that the azimuthal wave number was about 55. However, this result was ambiguous, because there is a $2n\pi$ ambiguity of phase. As well, the phase delay could have been modified by radial propagation of the wave. In this paper we will remove the above-noted ambiguity in the determination of the propagation properties of the wave by using the remote sensing capability of the energetic particle detector on board the ISEE-1 satellite.

The propagation direction and the azimuthal wave number of ULF waves have been determined for events observed at the geostationary orbit by several authors using data from multisatellite observations. *Hughes et al.* [1979], applying the coherence analysis technique, measured phase differences among magnetic signals recorded by three geostationary satellites; they were able to determine the propagation direction and estimate the wave number in the azimuthal direction. Unfortunately, no information concerning radial propagation could be obtained from these measurements. In deriving m values (azimuthal wave number), Hughes et al. selected phase differences lying between $-180°$ to $180°$, which gave consistent m values for different pairs of satellites. The authors stressed that these small azimuthal wave numbers implied much larger azimuthal phase velocities at the magnetopause than would be expected for waves generated by the Kelvin-Helmholtz instability.

Su et al. [1977] attempted to determine both the azimuthal and radial components of the propagation vector for a compressional wave at synchronous orbit. In studying a proton pulsation event observed on ATS-6, they examined the finite gyroradius effects that cause phase differences in particle flux oscillations among different energy channels, and between two oppositely oriented detectors. Su et al. were able to determine the propagation direction of the wave and to estimate the phase velocity both in the azimuthal and radial direction. Their analysis was not straightforward because their detectors were not strictly back-to-back, and the inferred phase velocities had rather large uncertainty (60–70 %).

The above two methods, comparing the data from pairs of geosynchronous satellites and making use of finite Larmor radius effects of energetic particles observed at a spacecraft, were also used by *Takahashi et al.* [1985]. They used data of tilt angle variation of the magnetic field from two geostationary satellites to determine the azimuthal phase velocity and wave number, while using the flux variations of energetic protons (105 to 187 keV) detected by a detector (LoP) on board the satellite 1979-053 to examine the radial component of the wave vector. Because the detector scans only the plane perpendicular to the radial vector, it cannot be used to determine the azimuthal phase velocity.

The method used in our study is essentially the same as the "finite Larmor radius effect" method mentioned above [*Su et al.*, 1977]. It was also discussed by *Kivelson and Southwood* [1983]. The idea is the following: a spinning detector with spin axis parallel to the ambient magnetic field samples distributions of particles whose gyrocenters fall on a circle at a distance R_L (the Larmor radius of particles) from the spacecraft. Back-to-back detectors (180° apart in spin) sample distributions of particles with gyrocenters separated by $2R_L$ and thus are affected by different parts of the wave structure. In a wave with wavelength comparable to the particle gyroradii, the dependence of flux wave phase on look direction, will occur, and fixed back-to-back detectors that measure particles at different phases of the wave may display oscillations with a lag between observations in the two detectors. In this study, we have used the ISEE-1 medium energy particle detector whose spin axis is approximately parallel to the ambient magnetic field to sample particles with gyrocenters separated in both radial and azimuthal directions, and determined the wave propagation in the two directions.

OBSERVATIONS

The compressional wave reported here was observed between 2145 UT, August 21, 1978, and 0100 UT of the next day. The ISEE-1 magnetometer record for the event is shown in Figure 1 in GSE coordinates. The spacecraft was traveling inbound at about 1700 local time from $L \sim 11$ to 7 near the equatorial plane. The waves have strong compressional components B_z and B_y; the y direction is close to radial in this case. During the first 2-hour interval, the wave form of the pulsations is quite irregular with spectra containing a major peak at 250 s and smaller peaks at about 165 s and 100 s

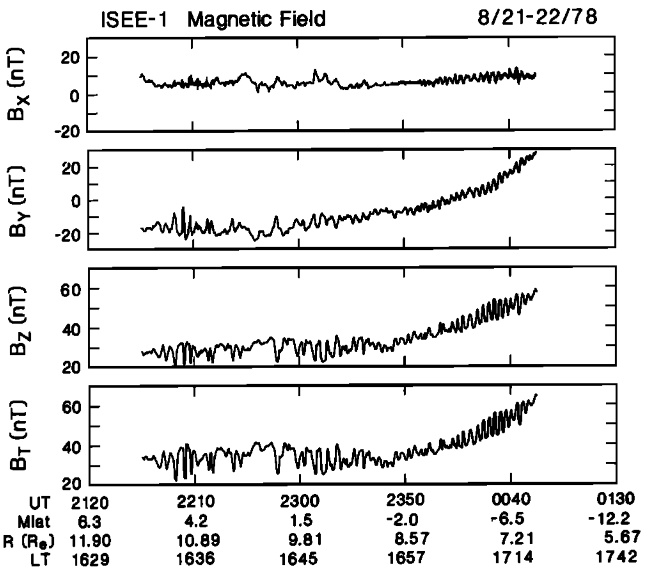

Fig. 1. A magnetic pulsation recorded by ISEE 1 on August 21 to 22, 1978. Three components B_x, B_y, B_z in GSE coordinates are shown. B_T is the total magnetic field. Spacecraft magnetic latitude, geocentric radial distance in Earth radii (R_e) and local time are provided.

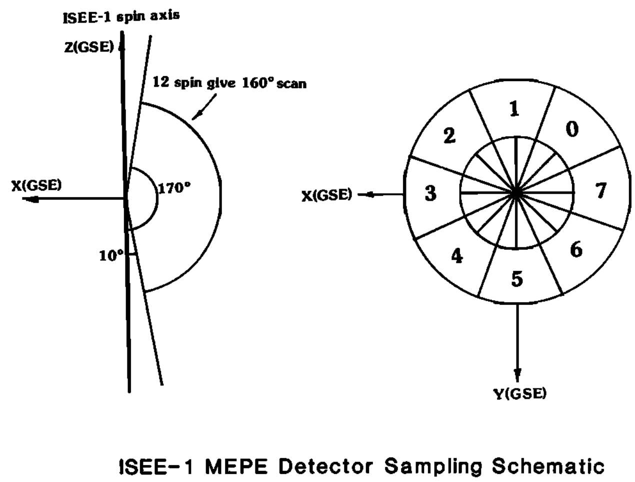

Fig. 2. MEPE instrument sampling schematic. See text for description.

[see Figure 5 of *Greenstadt et al.*, 1986]. Starting about 0100 UT, a quasi-sinusoidal wave developed and reached a maximum amplitude in B_z (and also in B_T) at about 0030 UT. The period of this wave was 165 s.

The energetic particle data we used were obtained by the medium energy particle experiment (MEPE) instrument on board the ISEE-1 satellite [*Williams et al.*, 1978]. The instrument measures both ion and electron fluxes over the respective energy ranges from 24 to 2081 keV for protons and 22.5 to 1200 keV for electrons. The data for this event were acquired in the low bit rate mode which provides the flux of protons and electrons in eight energy channels (see Table 1). The detector continuously scans in polar angle relative to the ISEE-1 spin axis which in turn is oriented normal to the ecliptic plane. This scan is synchronized to the satellite spin rate (~3 s/spin) and covers 160° (polar angle) in 12 spin periods (Figure 2). In low bit rate, samples are taken in eight azimuthal sectors each spin (one sample per sector). In figures discussed later in this paper, the count rate is actually the number of counts per readout interval (counts/sector). A three-dimensional sampling of 96 points is made over the unit sphere every 12 spins (~36 s). Each of these samples contains eight simultaneously observed energy channels of ions and electrons. Unfortunately, data for the two lowest energy channels of proton flux were not usable because of interference from the ISEE-1 electron density experiment.

The MEPE instrument does not distinguish among ion species. Although there may be a significant population of heavy ions in the storm time ring current (see, for example, review by *Williams* [1987]), the protons are essential in the bulk of the ring current. For example, recent observations of the storm-time ring current composition from the AMPTE-CCE satellite [*Gloeckler et al.*, 1985] showed that more than 70% of the number density of ring current ions (5 to 315 keV/e) at $L \sim 6$ to 7 consisted of protons. Thus in the following discussion we initially assume the observed ions are protons, and then later discuss the effects of the presence of other ion species.

Figure 3 shows the time variation of particle fluxes measured by the MEPE instrument. Figure 3*a* displays the spin averaged flux of large pitch angle (80° to 100°) protons (scatter plot) in energy channels 3 to 6; Figure 3*b* displays analogous data for electrons in different channels. To emphasize the flux variation of greatest interest in this study, we filtered the data by taking running averages over 27 data points (81 s), and have superimposed the results (solid curves) on the same plots (Figures 3*a*, 3*b*).

In the smoothed particle flux data, there are obvious

TABLE 1. MEPE Energy Channels (Low Bit Rate)

Proton Channel	Energy Passband, keV	Center Energy, keV	Electron Channel	Energy Passband, keV	Center Energy, keV
1	24.0–44.5	34.25	1	22.5–39	30.75
2	44.5–65.3	54.9	2	39–75	57.0
3	65.3–95.5	80.4	3	75–120	97.5
4	95.5–142	118.75	4	120–189	154.5
5	142–210	176.0	5	189–302	245.5
6	210–333	271.5	6	302–477	389.5
7	333–849	591.0	7	477–756	616.5
8	849–2081	1465	8	756–1200	978.0

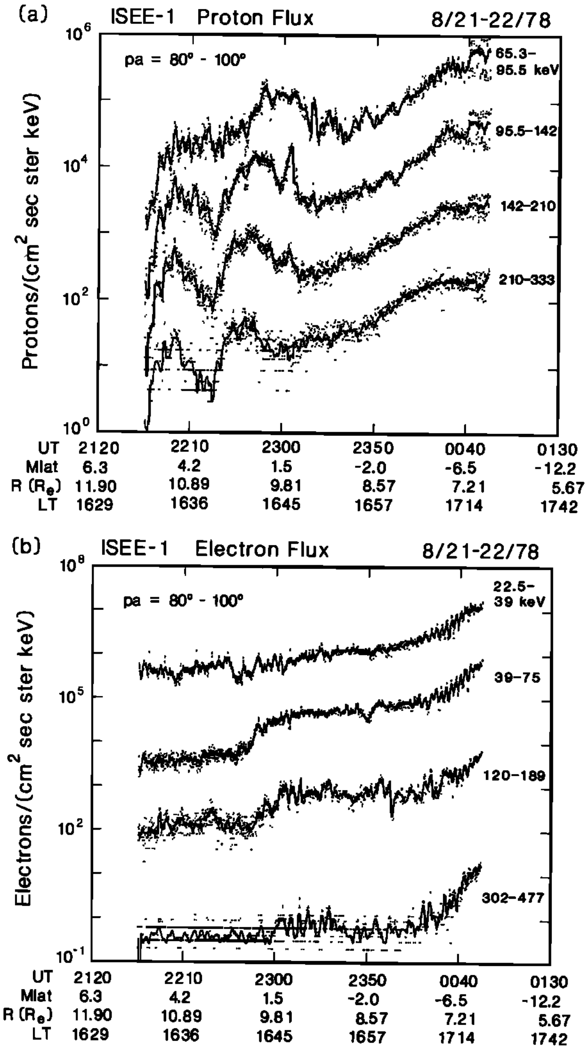

Fig. 3. (*a*) Spin averaged time variations of $90^\circ \pm 10^\circ$ pitch angle proton fluxes (scatter plots) for the Pc 5 event overplotted by 80-s running average curves (solid lines). Particle fluxes are plotted on a logarithmic scale. Only data of energy channels 3 to 6 are shown. Proton flux bursts starting near 0040 UT represent interference by the electron density experiment. (*b*) Same as 3*a* but for electron fluxes in channels 1 (multiplied by 10), 2, 4 (multiplied by 10), and 6.

variations that can be associated with the magnetic pulsations throughout the 3-hour interval. Both proton and electron data show that, after somewhat irregular oscillations in the first 2-hour period (2145 to 2400 UT, August 21), a quasi-sinusoidal wave developed and reached a maximum at about 0030 UT, just as is the case for the associated magnetic pulsations.

A comparison between power spectra of particle and magnetic pulsations is shown in Figure 4*a* for 80° to 100° pitch angle protons, and in Figure 4*b* for electrons. The spectra are for three successive 1-hour intervals. In the first hour (2145 to 2235 UT, Figure 4*a*), the proton spectrum exhibits a major peak at 250 s period which coincides with a peak at the same period in the magnetic spectrum. In the second interval (2300 to 2400 UT), the proton spectrum contains a broad peak at 250 s and a peak at 165 s which is similar to, but not identical to the magnetic spectrum. In the final interval (0000 to 0050 UT, August 22), when the quasi-sinusoidal wave occurred, the spectra of both particle flux and magnetic field display a pronounced peak at 165 s. There is also a minor peak at about 70 s period in the proton spectrum which was not observed in the magnetic spectrum. Similar correspondence between particle and magnetic spectra was also observed in the electron data (Figure 4*b*).

The above comparisons suggest that the dominant fluctuations of both proton and electron fluxes were produced by the compressional Pc 5 waves. The magnetic field oscillated in phase with the variation of spin averaged intensity of 90° pitch angle electrons while oscillating out of phase with the spin averaged intensity of 90° pitch angle ions (not shown).

ANALYSIS OF THE WAVE PROPAGATION

In this section we study the propagation of the Pc 5 wave in directions transverse to the main field by examining the wave form of proton flux variations at specific look directions relative to the spacecraft. As illustrated schematically in Figure 5, a detector looking radially (azimuthally) detects the flux of protons with gyrocenters at A and B (C and D) displaced from the satellite

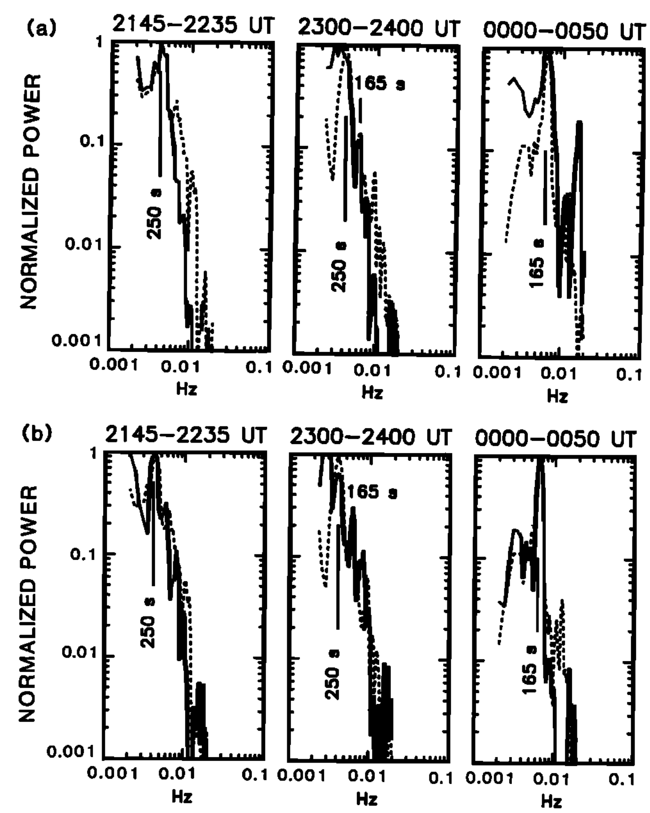

Fig. 4. Normalized power spectra of the smoothed particle data (solid lines) and the spectra of the total magnetic field B_T (dashed lines) for successive three 1-hour intervals: 2145 to 2235 UT, 2300 to 2400 UT, August 21, and 0000 to 0050 UT, August 22. (*a*) for $90^\circ \pm 10^\circ$ pitch angle protons in channel 3. (*b*) for $90^\circ \pm 10^\circ$ pitch angle electrons in channel 1. Before Fourier transformation, the data were detrended by subtracting 450 s running averages.

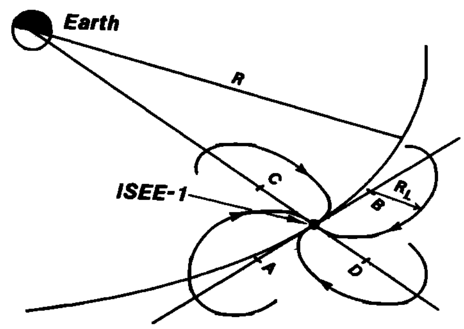

Fig. 5. Schematic diagram showing the gyromotion trajectories of protons seen by the detector looking in radial and azimuthal directions in the equatorial plane. R_L is the Larmor radius of the particle, and R is the radial distance of the spacecraft from the Earth.

in the azimuthal (radial) direction. By measuring the phase delay between the wave forms of flux variations taken in two opposite look directions, and knowing that the corresponding particle gyrocenters are separated by twice the Larmor radius of the particles, we can determine propagation direction and phase velocity of the wave. Only particles whose gyroradii are comparable to the wavelength can be used in this kind of study.

Interval 0000 to 0100 UT, August 22

We first use the above technique to examine the quasi-sinusoidal wave ($T = 165$ s) appearing in the last hour of the event. Figures 6*a* and 6*b* show the count rate variation of $90° \pm 10°$ pitch angle protons in the 65.3 to 95.5 keV band (channel 3) and the 95.5 to 142 keV band (channel 4), respectively, when the detector was looking earthward (solid curve), and outward (dashed line). Thus the solid curve (the "westside wave") represents the flux variation of protons with gyrocenters to the west of the spacecraft (point A in Figure 5), while the dashed line (the "eastside wave") gives the flux variation of protons with gyrocenters to the east of the spacecraft (point B in Figure 5). The plotted data were detrended by subtracting a 450 s running average.

Figure 6 shows that there is a time shift τ between the eastside wave and the westside wave. The time shift indicates that the propagation of the wave has an azimuthal (east-west) component. The actual time delay between the two sinusoidal wave forms could be $\tau + nT$, where T is the wave period and n is an integer that can be greater than or equal to zero (implying westward propagation of the wave), or less than zero (implying eastward propagation). The phase shift is equal to the difference of the wave phases at the two opposite gyrocenters [*Kivelson and Southwood*, 1983], and thus the phase velocity of the wave can be calculated by

$$V_P = \Delta R/(nT + \tau) \qquad (1)$$

and the wavelength is $\lambda = V_P \cdot T$, where ΔR is the distance between the two gyrocenters. The integer n corresponds to the number of wavelengths contained between the two gyrocenters. The value of τ can be obtained by doing cross-correlation analysis of the two wave forms under investigation.

Formula (1) shows that an apparent time shift τ may correspond to a family of possible phase velocities and thus wavelengths. Simultaneous measurements of ion fluxes in several channels allow us to remove this uncertainty by finding a unique wavelength consistent with all measurements. As an example, we analyze the two wave forms in Figure 6*a*. They are well correlated, with a correlation coefficient as high as 0.91 and an apparent time delay $\tau = 51$ s. For a nominal 80.4 keV energy of the protons in this channel (the center of the channel), and the observed 50 nT magnetic field (the average field value for the interval), the Larmor radius R_L is 820 km. Taking $n = 0$ (implying azimuthally westward propagation of the wave), and $\Delta R = 2R_L$, we calculated the phase velocity of the wave as $\sim$ 32 km/s, and the wavelength $\lambda \sim$ 5300 km. A similar calculation (taking $n = 0$) for channel 4 data in Figure 6*b* with $\tau = 63$ s gives about the same phase velocity (32 km/s) and wavelength (5216 km). There are other solutions that may also suit the observations of the two

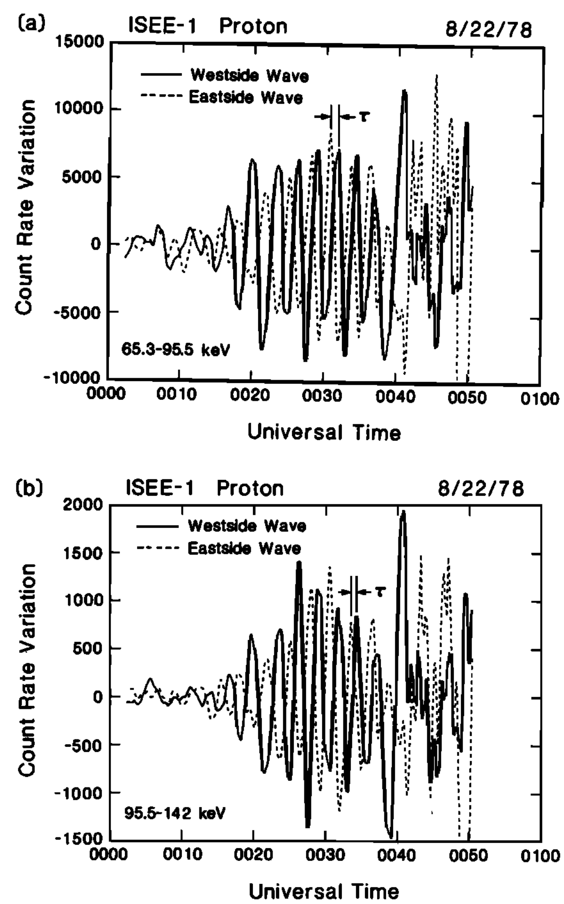

Fig. 6. (*a*) The variation of count rate (counts/sector) of $90° \pm 10°$ pitch angle protons (65.3 to 95.5 keV) with gyrocenters at the east side (dashed line) and the west side (solid line) of the satellite for the interval 0000 to 0050 UT, August 22, 1978. τ is the time delay between the two wave forms. Irregular fluctuations after 0040 UT result from interference with the electron density experiment. (*b*) Same as 6*a* but for channel 4 (95.5 to 142 keV).

TABLE 2. Possible Wavelengths of Azimuthal Propagation of the 165 s Wave as Determined From Proton Fluxes for Interval 0020 to 0040 UT, August 22, 1978

Channel Pair	ΔR, km	Correlation Coefficient	Lag (τ), s	λ_1, km ($n = 0$)	λ_2, km	n
6w–6e	3011	0.78	84	5915	−402	−8
5w–3e	2032	0.85	66	5079	−442	−5
5w–4e	2208	0.82	66	5520	−394	−6
5w–5e	2424	0.68	66	6061	−432	−6
5w–6e	2718	0.73	87	5155	−420	−7
4w–3e	1815	0.87	63	4754	−393	−5
4w–4e	1992	0.81	63	5216	−431	−5
4w–5e	2208	0.77	63	5783	−393	−6
4w–6e	2501	0.76	84	4913	−456	−6
3w–3e	1638	0.91	51	5302	−444	−4
3w–4e	1815	0.86	54	5546	−388	−5
3w–5e	2032	0.73	54	6208	−435	−5
3w–6e	2325	0.76	78	4918	−421	−6
average				5413	419	

Negative values of λ and n represent eastward propagation of the wave.
In the calculations we have used $B = 50$ nT, wave period $T = 165$ s, and $L = 8$.

channels. For example, taking $n = -4$ for channel 3, we have $\lambda = -444$ km, while taking $n = -5$ for channel 4, we have $\lambda = -431$ km. The negative sign here denotes eastward propagation of the wave. This means that an eastward propagating wave with a wavelength of about 440 km may also produce ion flux oscillations with apparent time delays seen in Figures 6*a* and 6*b*.

To determine unambiguously the propagation direction and the wavelength of the wave, we have taken advantage of simultaneous multichannel measurements of ion fluxes with the MEPE instrument. Data of channels 3 to 6 were used in this study (data for higher energy protons in channels 7 and 8 were not usable owing to very low count rates). Since each channel provides measurements of wave phases at two gyrocenters of ions at both sides of the spacecraft, 4 energy channels then provide information of wave phases at eight effective measurement points aligned azimuthally provided the detector was looking in radial direction. We have applied cross-correlation analyses to every possible pair of proton flux waves measured at two effective measurement points located at different sides of the spacecraft. For each pair of flux waves, the time delay τ between them was determined, and the phase velocities of the wave and the wavelengths corresponding to various $|n|$ were calculated. From solutions thus obtained we select the ones that appear consistent with all measurements. The results are shown in Table 2, where we list only those calculated from pairs of well-correlated fluxes (correlation coeffecient ≥ 0.68). The effective measurement point of each flux oscillation in Table 2 is denoted by the channel number followed by w (the flux of protons whose gyrocenters are west to the spacecraft) or by e (the flux of protons whose gyrocenters are east to the spacecraft). For example, "5w–3e" in the first column in Table 2 represents the cross-correlation study between fluxes of protons in energy channel 5 with gyrocenters at west of the spacecraft and those in channel 3 with gyrocenters at east of the spacecraft.

Table 2 shows that there are two possible wavelengths which seem to be consistent with all pairs of measurements. One is about 5000 km with average value $\overline{\lambda} = 5413$ km, for which $n = 0$ which implies westward propagation of the wave; the other is about 400 km ($\overline{\lambda} = 419$ km) with negative n, which implies eastward propagation of the wave. The standard deviation for the above averages is 468 km for 5413-km solution (8.7%) and 23 km for 419 km solution (5.5%). There are other solutions corresponding to even smaller wavelengths that may fit all the observations, but they were discarded since those wavelengths are too small compared to the proton gyroradii for the proton flux to be modulated by the wave field.

Using the above two possible wavelengths, we simulated the time variation of fluxes at the 8 effective measurement points x_i as

$$j_i(x_i, t) = a_i \cos 2\pi(x_i/\lambda \pm t/T) \tag{2}$$

where the amplitude a_i is taken as unity, and the wave period $T = 165$ s. The origin for measurements of the 8 points x_i was chosen at the point one Larmor radius of the channel 3 protons to the west of the spacecraft. The model wave forms are compared to the real flux data in Figure 7. The amplitudes of the flux data have been normalized for this comparison. From Figure 7*a* we see that the wave models (solid lines) with $\lambda = 5413$ km agree well with the principal feature of the real data (dashed lines) at all effective measurement points, but the model with $\lambda = 419$ km (Figure 7*b*) is not consistent with all observations; there are obvious phase discrepancies between the models and the data at some measurement points.

The above studies show that in azimuthal direction the wave was propagating westward with a wavelength of about 5400 km and a phase velocity of ~33 km/s. Since the spacecraft was at about 8 R_e away from the earth when the sinusoidal wave was observed, the wavelength gives an azimuthal wave number $m \sim 60$.

We now consider the flux variations seen for fluxes

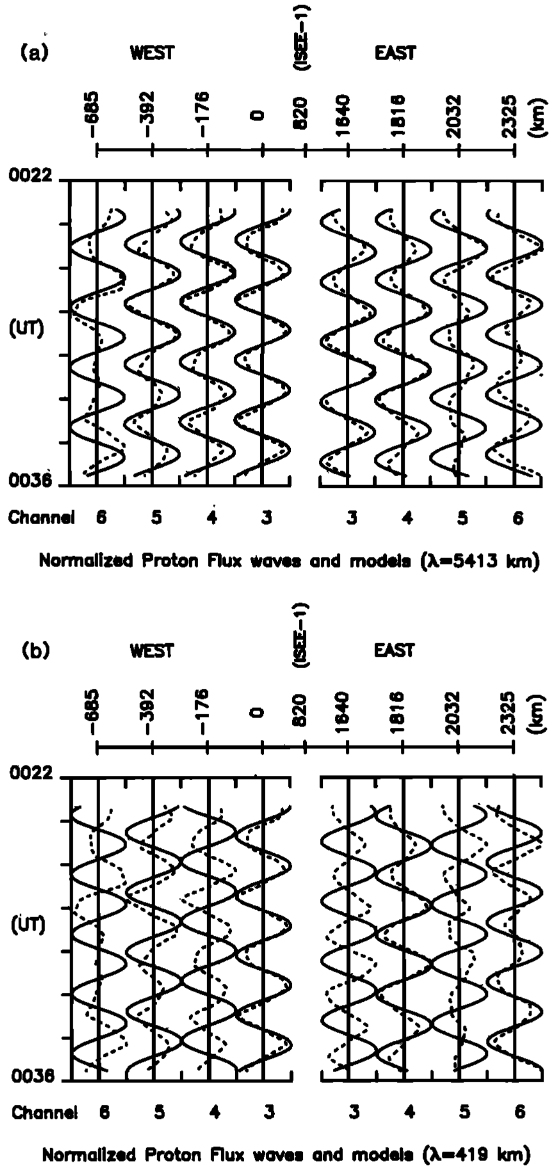

Fig. 7. Comparison of normalized observed proton flux oscillations (dashed lines) at 8 effective measurement points with modeled flux waves (solid lines): (*a*) for model waves with 5413-km wavelength propagating westward, and (*b*) for model waves with 419-km wavelength propagating eastward.

with gyrocenters separated in the radial direction (corresponding to points C, D in Figure 5). Figures 8*a* and 8*b* show the count rate variations of $90^\circ \pm 10^\circ$ pitch angle protons in channels 3 and 4 when the detector was looking eastward (solid curves, correspond to the flux variations for gyrocenters at the point radially earthward of the spacecraft, designated as the "inside wave") and looking westward (dashed line, corresponding to the "outside wave").

The correlations between inside and outside wave forms in both Figure 8*a* and 8*b* are high (> 0.90) when the wave was fully developed around 0030 UT, and show no time delay. This implies that the quasi-sinusoidal wave does not propagate radially. Similar analysis of channels 5 and 6 shows very poor correlation between particle fluxes inside and outside the spacecraft (the correlation coefficient is 0.25 for channel 5 and 0.55 for channel 6).

Similarly, we may also determine the phase velocity of the wave in other directions perpendicular to the ambient magnetic field by analyzing the 90° pitch angle ions measured in other pairs of sectors. Since we have already obtained velocity components in two orthogonal direction, the tranverse propagation of the wave is completely determined. The velocity component parallel to the ambient magnetic field cannot be determined by this "finite Larmor radius" method.

Interval 2200 to 2400 UT, August 21

We now apply the above analysis to the fluctuations that were observed in the outer magnetosphere during the 2 hours, 2145 to 2400 UT, August 21. Figure 9*a* shows the count rate variations of the protons in channel 3, for the westside (solid line) and eastside (dashed line) of ISEE-1. We can see a clear phase difference (clearest in the first hour, from 2145 to 2300 UT) indicating westward propagation of the wave. A cross-correlation between the two wave forms in the first hour gives a correlation coefficient of 0.88 at a lag of 60 s with the eastside wave leading the westside wave. Since the wave

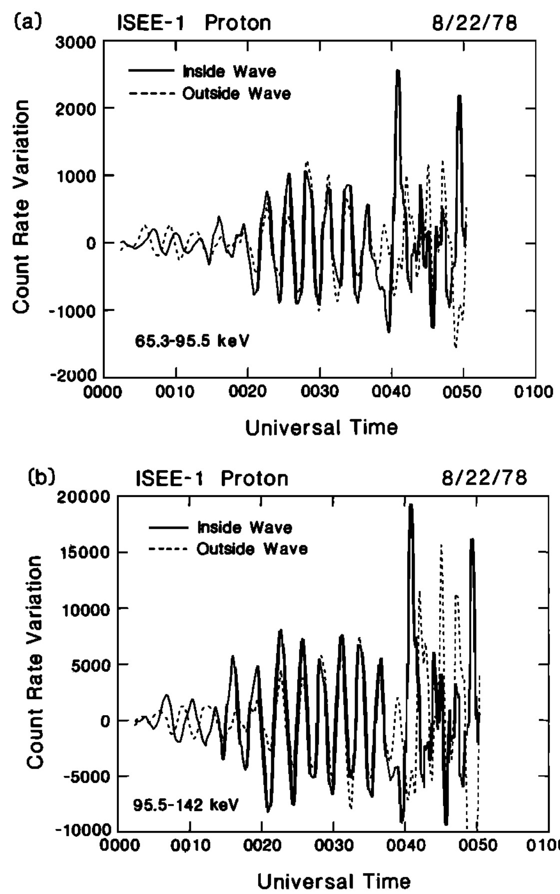

Fig. 8. (*a*) Same as Figure 6*a* but for protons whose gyrocenters are radially inside (solid line) and outside (dashed line) of the spacecraft position. No time delay is seen between the two wave forms. (*b*) Same as 8*a* but for channel 4 protons (95.5 to 142 keV).

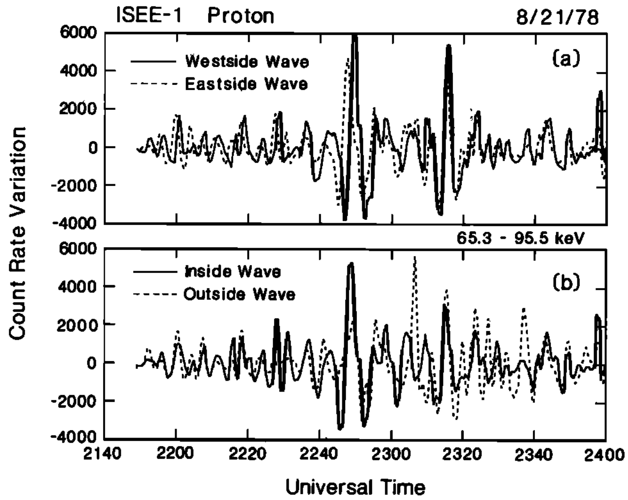

Fig. 9. (a) Same as Figure $6a$ but for the interval 2150 to 2400 UT, August 21, 1978. (b) Same as $9a$ but for the waves seen radially inside (solid line) and outside (dashed line) of the spacecraft position.

form is rather irregular, there is no $2n\pi$ ambiguity as in the case of the sinusoidal wave discussed previously. Taking the average magnetic field for this time interval as 34 nT and $L \sim 11$, we calculated the proton gyroradius and the phase velocity $V_P \sim 40$ km/s, giving an m value of 44 for waves of 250-s period. The same calculation was done for other proton channels in the interval 2155 to 2230 UT. The results are listed in Table 3. The results differ from channel to channel, with m numbers ranging from 20 to 44. However, phase differences in all channels consistently indicate westward propagation of the wave (the waves at eastside of the spacecraft lead the waves at west in phase). The disagreement between calculated phase velocities for different channels may be caused by the broad band nature of the wave. Protons in different energy channels may respond strongly to different wave frequencies; thus the calculated phase velocities may not be the same for all energy channels.

For the second interval, 2300 to 2400 UT, the correlations between proton flux variations at two points separated azimuthally are generally bad. The only channel that gives a significant correlation (0.88) is channel 3 with a lag time of 15 s, also indicating westward propagation. With the average magnetic field equal to 32 nT and $L = 9.3$, this time delay gives an even higher velocity (170 km/s) and lower wave number.

Propagation in the radial direction was also examined for the first 2-hour interval. The correlations between two wave forms separated radially were poor in all channels (less than 0.65) as shown in Table 4. To illustrate this point a comparison is made in Figure $9b$ for channel 3 protons. Table 4 suggests, however, that the flux variation during the first interval (2155 to 2230 UT) may have a radially earthward component of propagation (the "outside waves" lead the "inside waves" in all channels). Since the correlations are relatively low, the quantitative results are probably not significant. In the second interval (2310 to 2350 UT), the correlations are poor, and no consistent direction of propagation can be determined from the lag times.

EFFECTS OF HEAVY IONS

Since the determination of the wavelength depends on the gyroradii of the energetic ions, the presence of heavy ions which have gyroradii different from those of protons may require some modification in the calculation of the wavelength. In this section we will discuss the problem by considering that the plasma contains two ion species only: protons and another species of heavy ions.

The energy response of the MEPE instrument changes with ion species. For example, particles detected in channel 3 with nominal center energy of 80.4 keV could be 80.4 keV protons, or 92.9 keV helium ions, or 153 keV oxygen ions. Thus the differential flux observed by the detector consists of the contribution of protons with nominal energy and that of the heavy ions with higher energy.

The measured perturbation of differential flux in a two-ion species plasma can be expressed as

$$j^1 = j_1^1 + j_2^1 = (2W_1/M_1^2)f_1^1 + (2W_2/M_2^2)f_2^1 \qquad (3)$$

where f is the phase space density of the ions, M and W are the mass and the energy of the ions, respectively. Superscripts 1 represent the first-order perturbation, while subscripts 1 and 2 denote the proton and the heavy ion, respectively.

The first-order perturbed particle distribution f^1 can be obtained by integrating the unperturbed particle distribution function f^0 in the Vlasov equation along the unperturbed particle trajectory [*Su et al.*, 1977]. Following a similar derivation in the appendix of Su et al. and integrating their expression (A3), we found that the first order perturbation f^1 has a form as

$$f^1 \propto V_\perp J_1(kR_L)(\partial f^0/\partial W)e^{i(\pm kR_L - \omega t)} \qquad (4)$$

where $V_\perp$ is the velocity of the gyromotion of the parti-

TABLE 3. Properties of Azimuthal Propagation of the 250-s Wave as Determined From Proton Fluxes for Interval 2155 to 2230 UT, August 21, 1978 ($B \sim 34$ nT, $L \sim 11.9$)

Channel Pair	ΔR, km	Correlation Coefficient	Lag (τ), s	V_P, km/s	λ, km	m
3w−3e	2410	0.88	60	40	10041	44
4w−4e	2929	0.71	63	47	11622	37
5w−5e	3566	0.83	60	59	14856	29
6w−6e	4428	0.63	51	87	21708	20

TABLE 4. Cross Correlation Between Proton Flux Waves at Opposite Points in the Radial Direction for the Interval 2155 to 2230 UT, August 21, 1978

Channel Pair	Correlation Coefficient	Lag (τ), s
3 in–3 out	0.65	25
4 in–4 out	0.57	30
5 in–5 out	0.48	21
6 in–6 out	0.49	81

cles which is assumed to be much larger than the drift velocity of the particles, R_L is the gyroradius, ω is the wave frequency which is much smaller than the gyrofrequency, k is the wave number and J_1 is the Bessel function of the first kind. The plus or minus sign apply to effective measurements points at east and west sides of the spacecraft which is located at the origin. In the derivation we have assumed that the wave propagates purely in the azimuthal direction, as in the case of 165-s wave studied, and we neglected the drift velocity of the ions which is much smaller than the velocity of the gyromotion.

Assuming that the unperturbed differential fluxes of protons and heavy ions have the same spectrum

$$j^0 = cW^{-\kappa}$$

where c and κ are positive constants, and that the ratio of the unperturbed phase space density of the two components $\epsilon = f_2^0/f_1^0$ is independent of energy, we may substitute the expression (4) for the proton and for the heavy ion into equation (3) and, get a simple form of the perturbed differential flux:

$$j^1 \propto D_\pm \cos(\pm kR_{L1} - \omega t + \gamma_\pm) \tag{5}$$

where

$$\begin{aligned} D_\pm &= (A^2 + B_\pm^2)^{1/2} \\ A &= J_1(kR_{L1}) + B_1 \cos[k(R_{L2} - R_{L1})] \\ B_\pm &= \pm B_1 \sin[k(R_{L2} - R_{L1})] \\ B_1 &= \epsilon(W_2/W_1)^{-\kappa-1/2}(M_2/M_1)^{-5/2} J_1(kR_{L1}) \\ \tan\gamma_\pm &= B_\pm/A \end{aligned} \tag{6}$$

In the derivation, we used the relations

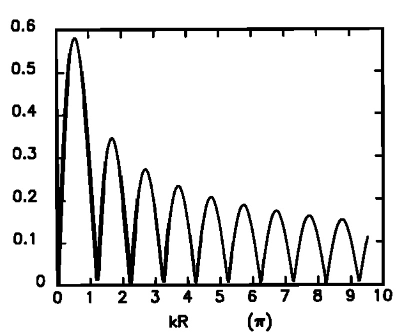

Fig. 10. The dependence of the amplitude factor D on kR_L in proton plasma ($\epsilon = 0.$).

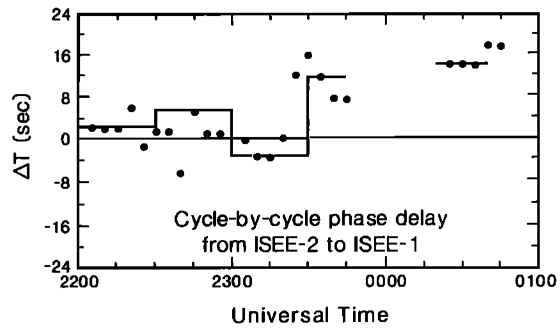

Fig. 11. Cycle-by-cycle time delay from magnetic signal of ISEE-1 to that of ISEE-2 [after *Greenstadt et al.*, 1986].

$$\partial f^0/\partial W = cM^2(-\kappa - 1)W^{-\kappa-2}$$

and

$$\partial f_2^0/\partial W = \epsilon(\partial f_1^0/\partial W).$$

Expression (5) show that the phases of the ion flux oscillation at points $\pm R_{L1}$ are

$$\phi_\pm = \pm kR_{L1} - \omega t + \gamma_\pm$$

and the phase difference between the fluxes at two opposite gyrocenters is then

$$\Delta\phi = \phi_+ - \phi_- = 2kR_{L1} + \gamma_+ - \gamma_- = -\omega(\tau + nT). \tag{7}$$

In the absence of heavy ions, $\epsilon = 0$ and thus $\gamma_\pm = 0$; equation (7) then reduces to equation (1). In fact, it can be seen from (6) that B_1 and thus γ are usually negligible even there is a significant fraction of heavy ions present ($\epsilon \neq 0$), because $W_2 > W_1$, $M_2 > M_1$, and κ is much larger than 1 (in this event, $\kappa \sim 6$). Thus our previous analyses that assume proton plasma only are justified.

Equation (5) shows that the amplitude of the flux modulation is proportional to a factor D which is Bessel-function-like ($\sim J_1(kR_{L1})$). Its envelope decays with kR_{L1} as shown in Figure 10, i.e., the modulation of ion fluxes by a wave tends to decrease in amplitude with a decreasing ratio of the wavelength to the Larmor radius of the ions.

COMPARISON WITH THE MAGNETIC FIELD ANALYSIS

The above analysis of energetic particle data has shown that the Pc 5 wave in this event propagated azimuthally westward. In the outer magnetosphere, where a broad band wave was observed, the wave may have a radially earthward component as well. Near the outer edge of the plasmapause ($L \sim 8$ in this event, as determined by *Greenstadt et al.* [1986], and also by *Fraser et al.* [1986]), the wave became nearly monochromatic ($T \sim 165$ s at around 0030 UT, August 22) and propagated purely westward with phase velocity ~ 33 km/s and wave number about 60.

To compare the above conclusions to the results obtained in the previous study of magnetic field alone, we reproduce a diagram summarizing the phase delay analysis of *Greenstadt et al.* [1986] in Figure 11. The figure shows the cycle-by-cycle phase delay between signals from ISEE-2 and ISEE-1. Δt in the figure is the lagtime of the signal recorded in ISEE-1, following behind ISEE-2 inbound. During the event ISEE-2 was 390 km west of ISEE-1, so positive Δt in Figure 11 corresponds to westward propagation of the wave, assuming the time delay results only from azimuthal propagation of the wave. Figure 11 shows predominantly westward propagation ($\Delta t > 0$) except for the interval 2300 to 2330 UT, when the results of the correlation study of proton data are poorly determined. Greenstadt et al. estimated the wave number as 55 for the 165-s quasi-sinusoidal wave which is close to our result.

The phase velocity and the wavelength can be estimated roughly from Δt in Figure 11 and the azimuthal separation of ISEE-2 and 1, $s = 390$ km. Around 0030 UT, August 22, Δt has an almost constant value of 12 s. If we rule out the possibility of having $2n\pi$ more (or less) in the phase difference, based on our study of particle data, we obtain a phase velocity $V_P = s/\Delta t = 32.5$ km/s, and the wavelength $\lambda = V_P T = 5362$ km. The results are quite consistent with that obtained from particle data.

From data of the first hour (2200 to 2300 UT) in Figure 11 we obtain smaller time delays (less than 8 s) implying a somewhat larger phase velocity. These results also agree qualitatively with those of proton data analysis, but from the proton data we are able to distinguish the wave propagation in the azimuthal and the radial directions, while the analysis of the magnetic measurements simply assumes a purely azimuthal propagation.

We have also applied the finite gyroradius technique to proton data from the ISEE-2 medium energetic particle instrument WAPS (T. Fritz, private communication, 1986) for the 165-s wave in this event. In most of the 12 energy channels of WAPS proton data (proton energy ranges from 25 to 800 keV), we have confirmed azimuthally westward propagation of the wave. Data in 9 out of 12 channels give quite consistent results with calculated m value ranges from 50 to 60, and give an average of $m = 57$. Thus the ISEE-2 observation supports the conclusions based on the analysis of ISEE-1 data and of two spacecraft delay measurements.

SUMMARY AND DISCUSSION

The oscillation of differential fluxes of energetic ions modulated by a ULF wave often shows a phase shift between measurements in back-to-back detectors with look directions perpendicular to the ambient magnetic field (the MEPE instrument on board the ISEE-1 spacecraft may serve as such detectors). In a plasma of a single ion species, the phase difference is caused by displacement of the effective measurement positions by one ion gyroradius to each side of the detector. Because of the periodicity of the wave phase, the observed phase shift may correspond to a family of possible wavelengths. Simultaneous measurements of the flux modulation in different energy channels, which is equivalent to probing the wave phase at several positions, may allow one to single out a unique wavelength consistent with all the measurements.

To examine the effect of heavy ions on the estimated wavelength, we have derived the perturbation of differential flux by calculating the first-order perturbed phase space distribution in a plasma consisting of two ion species. We found that for a detector with characteristics like those of the ISEE-1 medium energy particle experiment the effect is very small even when there is a significant fraction of heavy ions present, assuming that the unperturbed differential flux of the energetic ions has a spectrum as $j^0 = cW^{-\kappa}$. This is because the detector responds to the heavy ions with higher energy than the nominal proton energy for each energy channel, whereas the flux drops exponentially with the energy. In all cases the envelope of the modulation amplitude of ion fluxes decreases with an increasing ratio of the ion Larmor radius to the wavelength.

We have applied the above theory to a study of a compressional Pc 5 wave observed on August 21 to 22, 1978, and determined unambiguously the propagation properties of the waves in both the azimuthal and the radial directions. The quasi-sinusoidal wave ($T = 165$ s) in this event propagated azimuthally westward with a phase velocity of about 33 km/s, a wavelength of ~ 5400 km, and an m number of 60. This result is in good agreement with that of magnetic field study. The waves in the outer magnetosphere, which were more irregular, also propagated westward with a somewhat smaller wave number, but they may have a component of radially inward propagation, too.

The westward propagation of the waves rules out the possibility of the waves' having been generated by Kelvin-Helmholtz instability at the magnetopause, since such waves would propagate antisunward from the noon meridian, i.e., eastward in the afternoon sector. The fact that the irregular waves in the outer magnetosphere have a radially earthward phase velocity component may be a clue that there is a wave source at the magnetopause, but the nature of such a source is not understood. The theory of the global excitation of compressional waves [e.g., *Allan et al.*, 1985; *Kivelson and Southwood*, 1986] is not applicable to this event, as discussed in *Greenstadt et al.* [1986], because the observed azimuthal wave number is too large.

Our waves seem similar to those classified as "out-of-phase events" by *Kremser et al.* [1981]. These waves were observed around dusk during geomagnetically disturbed periods and propagated westward; the ion intensity oscillated 180° out of phase with the magnetic variation. Kremser et al. suggested that a likely source of the waves is the drift mirror instability [*Hasegawa*, 1969]. The instability requires an anisotropy of hot proton pressure, $\beta_\perp > \beta_\parallel$ (β is the ratio of the plasma pressure to the magnetic pressure). For our case, this requirement cannot be tested with the available data of protons with energy higher than 65 keV, but the pitch angle distributions (not shown) of the ions in all channels we used peaked at 90° pitch angle, consistent with the anisotropy required. The propagation direction of drift mirror waves depends on the direction of the pressure gradient. The 165-s sinusoidal wave occurred at the outer edge of the plasmasphere, where the plasma density gradient is radially inward. The energetic ion fluxes also show an inward gradient when the wave was observed (see Figure 3*a*). The gradient implies a net westward drift of ions and is consistent with the observed propagation direction of the wave. The azimuthal wave number predicted by the theory is given by

$$k_\perp = \frac{(2\Delta)^{1/2}}{3} R_L$$

where $\Delta = \frac{3}{4}\{\beta_\perp(T_\perp/T_\parallel - 1) - 1\}$, and $T_\perp$ and $T_\parallel$ are the perpendicular and parallel temperatures of the hot ions, respectively. If we take a typical value of Δ as 0.2 [*Hasegawa*, 1969], then the observed wave number for $\sim$ 5400 km wavelength requires R_L = 180 km, i.e., the Larmor radius for 90° pitch angle protons of 4 keV, which is a reasonable energy for ring current protons.

There is another possible mechanism for generating waves which may be consistent with our observation. Large amplitude ULF waves can be generated through resonance with particles which bounce and drift through the wave field (see, for example, *Southwood* [1980]; *Kivelson and Southwood* [1985*b*]). The wave frequency ω must satisfy the resonance condition

$$\omega - m\omega_d = N\omega_b$$

where ω_d and ω_b are the angular drift and bounce frequency, respectively, and N is an integer. Our event has a large m and small ω which are favored for this kind of instability [*Southwood*, 1980]. Since the event has significant transverse components around the magnetic equator, one expects it to have antisymmetric structure along field lines (i.e., N is an odd number). Thus if we take $N = 1$ in the above resonance condition, then two groups of particles may resonate with the observed wave (T = 165 s, m = 60 at L = 8): high-energy protons of $\sim$ 255 keV which satisfy

$$m\omega_d \simeq \omega_b$$

and low-energy protons of ~4 keV which satisfy

$$\omega \simeq \omega_b$$

More complete particle data (in the energy range lower than that used in this study) will be needed to cast further light on the source or sources of the waves. Since the main purpose of this study is to demonstrate a powerful method in investigating wave propagation by making use of the remote sensing capability of energetic particle detectors, more detailed analysis on wave generation mechanism is left for future work, but the above brief discussion shows that an unambiguous determination of the propagation properties of a wave contributes critical information to wave studies.

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