Models of Rising Active Region Flux Tubes

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Abstract. The buoyant rise of a magnetic flux loop originating from a single perturbed segment of a toroidal flux ring lying slightly beneath the base of the convection zone is studied by means of numerical simulations. We have considered flux loop evolution assuming both solid-body rotation, and differential rotation consistent with recent results from helioseismology. Our major conclusions are the following:
1) The latitudes of loop emergence are consistent with the observed butterfly diagram, assuming a dynamo wave propagating from 30 degrees latitude to the equator at the base of the convection zone. In the case of solid-body rotation, we require toroidal field strength $B_0 \gtrsim 60\ kG$ to avoid a significant equatorial gap, but if differential rotation is assumed, $B_0 = 30\ kG$ leads to an acceptable butterfly diagram.
2) The Coriolis force induced by the diverging east-west velocity near the loop apex acts to twist the loop as it rises and produces a tilt upon emergence, with the leading leg of the loop closer to the equator than the following. Our results agree qualitatively with similar calculations obtained recently by D'Silva and Choudhuri. For choices of toroidal field strength $30\ kG \leq B_0 \leq 90\ kG$, we find that computed tilt angles are consistent with both the magnitude and the latitudinal variation of observed active region tilt angles. In particular, the relationship between the tilt angle α and the active region latitude θ: $sin\ \alpha = 0.48\ sin\theta$, obtained from the observations by Wang and Sheeley, is closely matched by our simulation results for $B_0 = 50\ kG$ and flux $\Phi = 10^{22}\ Mx$.
3) From a simple force balance analysis, one can derive a scaling law for the tilt angle α in terms of the characteristic field strength B, latitude θ, and the total flux Φ of the loop: $\alpha \propto sin\theta\ B^{-5/4}\Phi^{1/4}$. We find that this

scaling relation describes most of our simulations reasonably well.
4) We find that the magnetic field in the leading leg of an emerging loop is approximately twice that in the following leg. We argue that this field strength asymmetry is the origin of morphological asymmetries in bipolar active regions, *i.e.* that the leading side is more compact, while the following side appears to be more dispersed and fragmented.

1. Introduction

The current consensus is that active region magnetic fields originate from the base of the solar convection zone (see e.g. the review by DeLuca and Gilman 1991); thus understanding the transport of magnetic flux from the bottom of the convection zone to the photosphere is essential for understanding active region evolution. In this paper, we use a numerical model for flux tube dynamics in 3 dimensions to study the emergence of active region magnetic fields. The model is based on the "thin flux tube approximation" for untwisted flux tubes described by Spruit (1981). We assume that an active region results from the emergence of a single flux loop, whose evolution is influenced by magnetic buoyancy, magnetic tension, Coriolis, and aerodynamic drag forces. The emerging flux loop forms after an initially toroidal flux ring with field strength B_0 and flux Φ lying at the base of the convection zone with initial latitude θ_0 is given an unstable perturbation. Further details of the flux tube model may be found in Fan, Fisher and DeLuca (1993, henceforth FFD) and Fan, Fisher and McClymont (1993, henceforth FFM).

In this paper, we describe results we have obtained with the numerical model which address the issues of where magnetic flux emerges, the origins of active region "tilt", and the observed differences in appearance between the leading and following polarity portions of an active region.

2. Latitude of Flux Emergence

Because the plasma within an initially toroidal flux ring will attempt to preserve its angular momentum, a rising flux loop tends to emerge somewhat further toward the poles (i.e. at higher latitudes) than its initial position, as is illustrated by the simulation shown in Figure 1. Earlier numerical simulations of flux tube dynamics (Choudhuri and Gilman 1987, Choudhuri 1989) found that the latitude of flux emergence was far greater than the latitude of the initial toroidal flux tube, to such an extent that it was difficult to reconcile these latitudes with the observed sunspot latitude zones. In contrast, we find that the latitude of flux emergence is increased only moderately from that of the initial flux ring. There are two important reasons for this difference: First, for non-axisymmetric flux tubes, the aerodynamic drag force will provide a torque which acts to increase the angular momentum of the emerging flux tube, decreasing the tendency for the tube to move poleward. Second, in our most recent calculations, only one loop of magnetic flux emerges to the photosphere. It is more realistic to assume that a single segment of a toroidal ring is perturbed into an unstable configu-

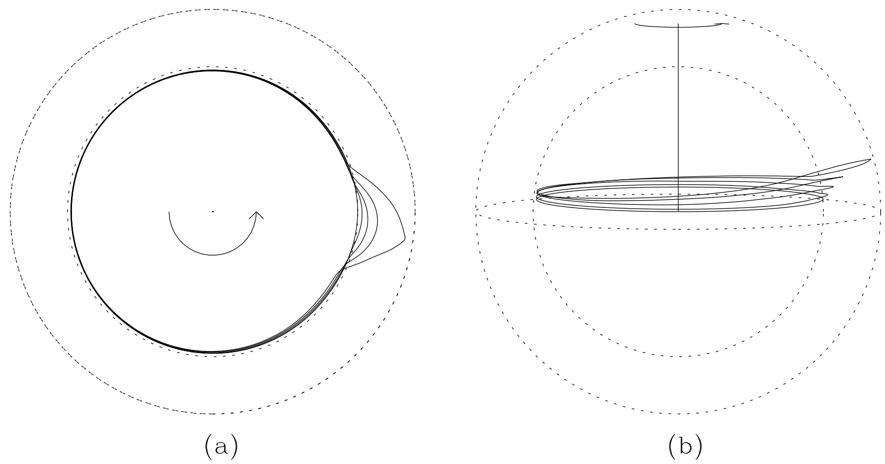

Figure 1. The configuration of an evolving flux ring perturbed from an initially toroidal configuration. The length scale of the perturbed segment is $\lambda = 4 \times 10^{10}$ cm; the flux $\Phi = 10^{22}$ Mx, and the initial field strength $B_0 = 3{\times}10^4$ G. The flux ring is viewed from the north pole and from 5 degrees above the equator in panels (a) and (b), respectively. The configurations of the flux ring at 5 instants during its evolution are shown as the solid curves. The inner and outer dotted circles mark, respectively, the base of the convection zone and the photosphere.

ration at any given time, and a single emerging loop arises from that segment. This reduces the amplitude of the counter-rotating flow, and the ensuing Coriolis force, as compared to the cases of azimuthal flux ring evolution (Choudhuri and Gilman 1987) or the simultaneous emergence of multiple unstable loops (Choudhuri 1989, also FFD) and results in a lower latitude of emergence. If solid body rotation of the sun is assumed, we find that a significant equatorial gap in the emergence of active regions will still occur if the initial field strength B_0 of the flux tubes near the base of the convection zone is less than roughly 60 kG. However, if we assume a convection zone differential rotation profile consistent with results from helioseismology (e.g. Brown *et al* 1989), the increased torque applied by the convection zone on the emerging flux loop means that an unacceptably large equatorial gap only occurs if $B_0 \lesssim 30$ kG. Further details on the latitude of emergence can be found in FFM.

3. Numerical Simulations of Active Region Tilt

It is well known that active regions emerge with a slight tilt relative to the azimuthal (E-W) direction (with the leading side of the active region emerging slightly closer to the equator than the following side), and that this tilt is in general an increasing function of latitude. D'Silva (1992) and D'Silva and Choudhuri (1993) have found from their numerical flux tube simulations that the amplitude and latitude variation of this tilt can be explained in terms of the

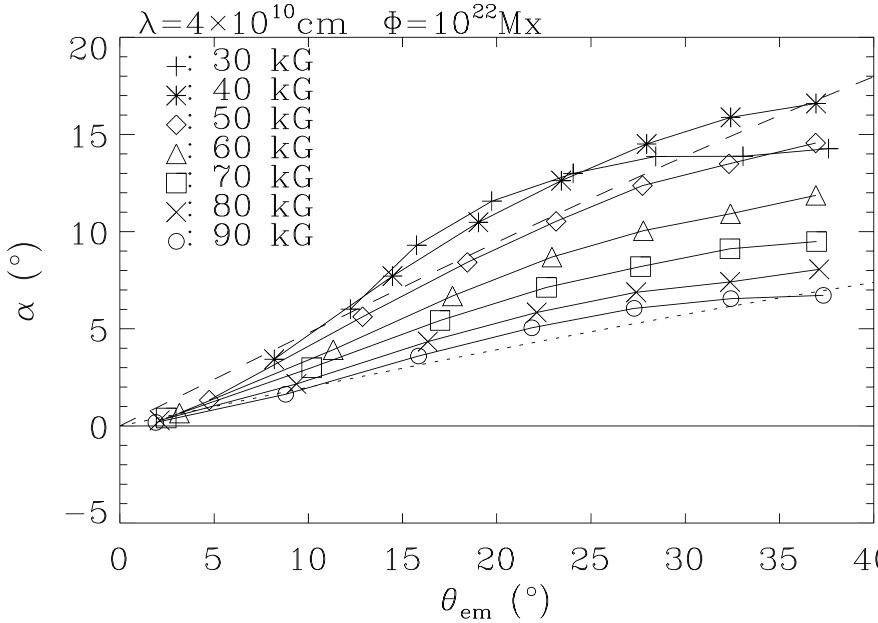

Figure 2. The tilt angle α of emerging loops as a function of emerging latitude θ_{em}, computed from simulations with different values of B_0. The dashed line corresponds to the linear least square fit obtained from analysis of magnetogram data by Wang and Sheeley (1991). The dotted line corresponds roughly to the observed variation of tilt of sunspot groups with latitude.

Coriolis force acting on the horizontally diverging flow which occurs in an emerging flux tube, provided that the magnetic field strength is roughly 100 kG. From our own numerical calculations (see Figure 2), we find that a somewhat lower value of the toroidal magnetic field strength provides a match with the observations, but we agree with D'Silva and Choudhuri that the field strength which is consistent with the observations is well above the estimated equipartition field strength near the base of the convection zone. Our numerical simulations also show that the tilt is a function of Φ, with $\Phi = 10^{21}$ Mx loops significantly less tilted than the 10^{22} Mx cases shown in Figure 2. In an effort to understand the physics underlying the systematic variation of tilt with B_0, Φ and θ_{em}, we have developed a simple, qualitative theory in which we derive a scaling law for the tilt angle α in terms of the above quantities. This scaling law is outlined below.

4. The Physical Origins of Active Region Tilt

It is the Coriolis force *gradient* across the loop apex that acts to twist a rising flux tube, and this gradient is proportional to the divergence in the horizontal component of motion in the loop. If $B_0 \gtrsim 3 \times 10^4$G, an emerging flux loop results

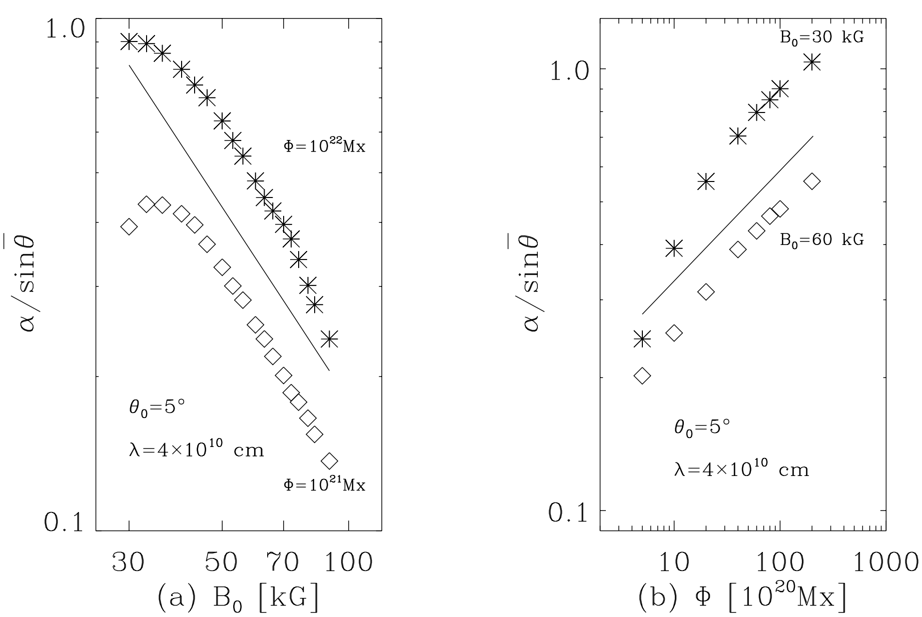

Figure 3. The variation of $\alpha/sin(\bar{\theta})$ with (a) B_0 and (b) Φ, obtained from the simulations, in comparison with the scaling relation (the straight lines) $\alpha/sin(\bar{\theta}) \propto B_0^{-5/4}$ and $\alpha/sin(\bar{\theta}) \propto \Phi^{1/4}$. Here $\bar{\theta} = (\theta_{em} + \theta_0)/2$ approximates the mean latitude of the loop apex during its passage through the convection zone.

in a diverging horizontal flow field, and the Coriolis force gradient acts to twist the tube into an orientation consistent with the observed tilt. The Coriolis force gradient f_c for a loop arising from an initially azimuthal ring can be written as $f_c \sim 2\rho\omega sin\theta(< v_\phi >_l - < v_\phi >_f)/\Delta s$, where $< v_\phi >_l$ and $< v_\phi >_f$ are the averaged values of the azimuthal component of the velocity in the leading and following legs of the loop, and Δs represents the separation scale length of the loop legs.

By carefully examining the results of our numerical simulations, we find that if the loop is emerging slowly, f_c is opposed primarily by a magnetic tension gradient, whose amplitude can be estimated as $f_t \sim 4B_0^2\alpha/(4\pi\Delta s^2)$, where α is the "tilt" angle of the apex of the loop relative to the azimuthal direction, and where we have assumed the length of each loop leg is roughly equal to the separation scale length Δs between the two loop legs. It is reasonable to assume that the separation velocity $< v_\phi >_l - < v_\phi >_f \propto v_r$, where v_r is the loop rise speed. We can then estimate v_r by noting that the rise speed is determined by a balance between the buoyancy and aerodynamic drag forces, resulting in $v_r \propto B_0^{3/4}\Phi^{1/4}$ (see §3.1 of FFD). Using these assumptions and approximations, and equating f_c to f_t we find the scaling relation

$$\alpha \propto sin\bar{\theta}\ B_0^{-5/4}\ \Phi^{1/4}\ , \tag{1}$$

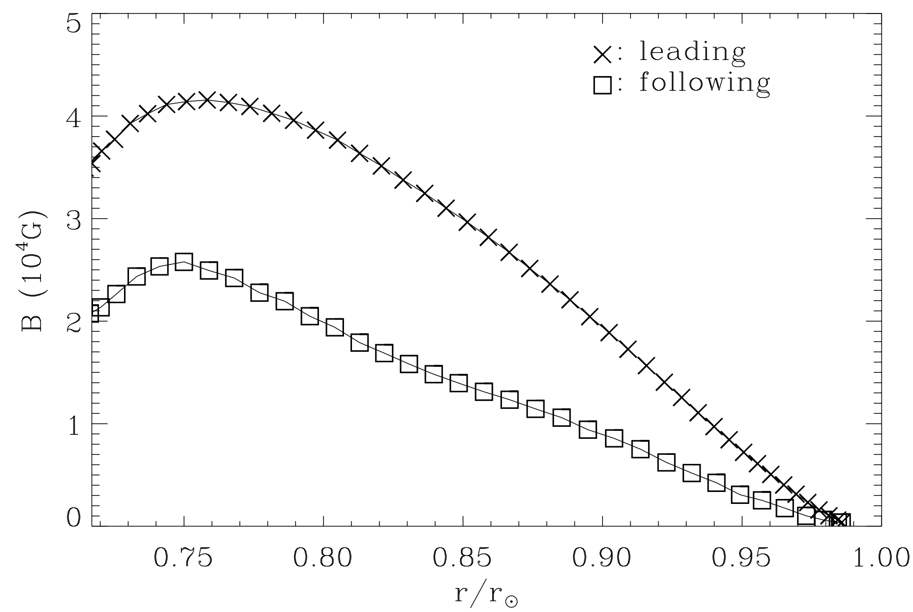

Figure 4. The variation of magnetic field strength along the emerging loop, as a function of depth in the convection zone. The field strength of the leading leg of the loop is approximately twice that of the following leg at any given depth.

where $\bar{\theta}$ represents an average of latitude over the loop. If the loop is emerging rapidly, our simulations show that f_c is balanced primarily by a gradient in the aerodynamic drag force rather than the tension force. However, one can show through a similar (but more tedious) force balance calculation that equation (1) still holds in that case. Thus, we expect equation (1) to be valid over a wide range of conditions, and a comparison of equation (1) with results from our numerical simulations (see Figure 3) shows that for $B_0 \gtrsim 30$kG, this is generally true.

However, Figure 3 also indicates that equation (1) begins to break down as B_0 approaches 30kG, and simulations for smaller values of B_0 show that a qualitatively different behavior occurs. Small values of B_0 result in small tilts, and in some cases, negative or even anti-Hale tilts. The reason for this is that for small values of B_0, the rapid expansion of the rising loop results in a strong sucking of mass toward the apex, and this motion can overwhelm the separation velocity of the loop legs themselves. The Coriolis force gradient then acts on a converging, rather than a diverging flow pattern, and counteracts (and in some cases overcomes) the tendency for the loop to be tilted in the "normal" direction. We find that for $B_0 \lesssim 20$kG, loops tend to emerge with a negative tilt angle α.

5. Morphological Asymmetries of Active Regions

Figure 4 shows the magnetic field strength as a function of depth in the solar interior for a flux loop which has risen from the base of the convection zone to the photosphere. Note that the field strength in the leading leg of the loop is roughly twice that of the following leg at the same depth in the interior; note that this is true at almost all depths. This interesting phenomenon is a result of the Coriolis force acting on the plasma in the most rapidly rising part of the emerging flux loop.

As the loop rises, the Coriolis force attempts to preserve the angular momentum of the rising plasma, which then acquires an azimuthal velocity in the counter-rotating direction (as seen from the sun's rotating coordinate system). Because this azimuthal velocity is greatest at the loop apex, and essentially nonexistent at the base of the loop (because the loop is not rising there), the net result is an evacuation of plasma out of the leading leg of the loop, through the apex and into the following leg of the loop. By the time the loop apex breaks through the photosphere, there is a significant asymmetry in the density and gas pressure between the two legs in the loop. Because of the requirement of pressure balance across the flux tube, the total (gas plus magnetic) pressure inside the loop must balance the external gas pressure, which depends only on depth. Therefore, the decrease of gas pressure in the leading leg of the loop is exactly compensated by an increase in the magnetic pressure. This asymmetry in the field strength between the leading and following legs of rising flux loops is one of the most robust results of our simulations; it holds in almost every case we have studied.

We believe that the magnetic field strength asymmetry provides a key to understanding many of the observed differences in the morphology of leading and following polarities of active regions. For example, the more fragmented appearance of the following side of an active region can be understood in terms of a reduced ability of the weaker magnetic fields there to resist turbulent motions from convection and drag; the same argument explains why the leading side of sunspot groups frequently consists of just one sunspot, while the following side consists of a collection of small spots and pores.

6. Summary

We have outlined some of our recent results obtained from numerical simulations of flux tube dynamics; these results are described in further detail in FFD and FFM. We find that many of the observed properties of active regions, including their latitudinal distribution on the disk, their orientation, and their morphological character can be explained in simple terms by the dynamics of flux loops emerging through the solar convection zone.

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G. Van Hoven: (1) What erupting field strength comes from $\sim$ 50kG at the base of the convection zone? (How can this vary in the "thin" tube approximation?) (2) Is the flux tube twisted? (3) Are convective driving motions included in your model?

R. Stein: Convection has strong downdrafts. How will these effect the flux tubes? will they reduce the buoyant rise?

P. Gilman: You must be able to estimate the field strength at the surface for a given field at the base. What do you get?

D. Rust: Have you considered the effects of thermal shadows on the buoyancy of magnetic flux tubes?

G. Fisher: We find that the field strength at the apex is typically several hundred Gauss when the apex reaches the photosphere. The field strength varies in accordance with solutions of the momentum and induction equations inside the tube, plus the conditions of pressure balance across the tube and flux conservation. See FFD for further details on how the field strength is determined in the simulations. The flux tubes are currently assumed to be untwisted, although a graduate student at Berkeley is working on developing a formalism for twisted flux tubes. At present, the only external flow field we have included so far is differential rotation; we plan to include simple models of convective motions in our dynamics equations in the near future. We have thought about implementing an additional force term to account for "thermal shadows", but at present we believe that thermal shadows will not be important unless the thickness of the flux tube is comparable to a pressure scale height near the base of the convection zone.

S. Koutchmy: There are many bipolar small regions outside sunspot regions, like those revealed by X-ray bright points/dark HeI 10830 points, especially in coronal holes and/or at high latitudes (polar faculae, etc.), as well as those observed in the active chromospheric network. Would you suggest these bipolar regions are also formed at the bottom of the convection zone?

G. Fisher: That's a really good question, and I don't know the answer.