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Temperature anisotropy effects and the generation of anomalous slow shocks

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Abstract. Temperature anisotropy is a persistent feature of many space plasmas (e.g., the magnetosheath) and can result in significant modifications to the jump conditions at various discontinuities. The effect of anisotropy on the Rankine-Hugoniot (RH) relations is reexamined and the topological changes in the solutions are illustrated by means of a graphical technique. A class of slow shocks is found with quite unusual properties. In particular, there exists a class of nonswitchoff slow shocks which rotate the tranverse component of the magnetic field. To distinguish these new solutions from the usual solutions, they are termed anomalous slow shocks.

1. Introduction

In an isotropic plasma, it can be easily shown, using fluid theory (MHD) that there are three characteristic speeds (slow, intermediate and fast) which if exceeded result in formation of slow, intermediate and fast shocks in the flow, respectively [e.g., *Akhiezer et al.*, 1975]. There also exist nonshock discontinuities such as a rotational discontinuity (RD) which travels at the intermediate speed but aside from rotation of the transverse component of the magnetic field and bulk flow leaves the plasma unchanged. The presence of anisotropy is, however, a rule rather than exception and understanding the effects of anisotropy on the various discontinuities has become an important subject. An area of immediate relevance is that of Earth's magnetopause which separates the shocked solar wind (magnetosheath) from the magnetospheric plasma. The magnetosheath plasma is commonly found to have a temperature anisotropy.

The types of discontinuities possible in an anisotropic plasma have been discussed by several authors (e.g., *Lynn*, 1967; *Abraham-Shrauner*, 1967; *Chao*, 1970; *Hudson*, 1970; *Neubauer*, 1970). In an isotropic plasma, the Rankine-Hugoniot (RH) conditions are derived by assuming a simple polytropic law for the total pressure ($P = const. \times density^{\gamma}$) in the asymptotic upstream and downstream. In an anisotropic plasma, instead of one pressure, there are two pressures $P_{\|}$ and $P_{\perp}$, thus requiring two equations of state to achieve closure of the fluid equations. In the derivation of RH relations we eliminate the need for one of the equations of state by prescribing the anisotropy downstream and by using a simple polytropic law for the other equation of state (e.g., *Chao*, 1970; *Hudson*, 1971). This is a natural way to proceed because one is typically interested in matching the observed parameters of a discontinuity (both upstream and downstream) with predictions of RH.

Here, we reexamine the effect of anisotropy on the Rankine-Hugoniot relations by means of a graphical technique first introduced by *Hau and Sonnerup* (1989). The advantage of this technique is that the type and relationship between the various solutions can be immediately identified. We then find a new class of slow shocks which for an upstream intermediate Mach number of unity, have the same jumps in plasma parameters as the RD. The only distinction between these shocks and the RD is that the slow shock does not rotate the magnetic field. This branch of slow shocks extends to intermediate Mach numbers greater than unity where the flow speed can be superAlfvénic both upstream and downstream. Using the fluid characteristic speeds, we have verified that the upstream flow is superslow and the downstream flow is subslow, as expected for a slow shock. It is interesting to note that a slow shock of this type was recently reported in the observations of the magnetopause (*Walthour et al.*, 1994). At upstream intermediate Mach numbers below unity, these slow shocks exhibit even more bizarre behavior. The flow speed downstream becomes superAlfvénic and unlike any slow shock known, it rotates (reverses the transverse component of) the magnetic field.

In section 2 we discuss the effect of anisotropy on the RH relations. Discussion and summary of the results is given in section 3.

2. Rankine-Hugoniot Solutions

A useful way to visualize the various solutions of the RH relations is to plot the square of the upstream modified intermediate Mach number $M_{I_1}^2$ versus its value downstream $M_{I_2}^2$ (subscript 2). Here M_I refers to the flow speed in the shock rest frame divided by the modified intermediate speed which is given by $V_I = V_{Io}\sqrt{1-(\beta_{\|}-\beta_{\perp})/2}$, where V_{Io} is the intermediate speed in an isotropic plasma, and $\beta_{\|}$ and $\beta_{\perp}$ are the plasma beta in the direction parallel and perpendicular to the magnetic field, respectively.

This method was first used by *Hau and Sonnerup* (1989) and later by *Karimabadi* (1995) for an isotropic plasma. In order to generalize the result to the anisotropic case we proceed as follows.

Our starting point is the one-dimensional time stationary nonviscous conservation laws which can be integrated once and written as:

$$[NV_x] = 0, \tag{1}$$

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$$[NV_x{}^2 + N\bar{v}^2 + \frac{1}{3}(\varepsilon + 0.5)\frac{B^2}{4\pi M} - \varepsilon\frac{B_x{}^2}{4\pi M}] = 0, \quad (2)$$

$$[\frac{1}{2}NV_x(V_x{}^2+V_z{}^2+ \Gamma\bar{v}^2)+\frac{1}{3}(\varepsilon + 2)V_x\frac{B^2}{4\pi M} - \varepsilon V_z\frac{B_xB_z}{4\pi M} - \varepsilon V_x\frac{B_x{}^2}{4\pi M}] = 0, \quad (3)$$

$$[V_xB_z - V_zB_x] = 0, \quad (4)$$

$$[NV_xV_z - \varepsilon\frac{B_xB_z}{4\pi M}] = 0, \quad (5)$$

Upstream we have chosen the fluid velocity to be along the shock normal, and the plane defined by $\vec{V}_1$ and $\vec{B}_1$ to be the (x, z) plane. In these equations, N is the plasma density, B is the total magnetic field, M is the ion mass, V is the flow velocity, $\bar{v}^2$ is the sum of the square of the thermal velocities of electrons and ions, $\Gamma = 2\gamma/(\gamma - 1)$, and $\varepsilon = 1 - (\beta_\parallel - \beta_\perp)/2$. We have also assumed that the total pressure $P = const. \times density^\gamma$. The brackets around each equation denote the difference between the upstream and downstream values for the enclosed quantities. These equations also apply to a system consisting of two fluids, with one fluid being isotropic and the second fluid being anisotropic. In such a case, the only difference is that the β's appearing in ε correspond only to the anisotropic fluid. At many discontinuities electrons achieve isotropization much more quickly than ions or simply remain more nearly isotropic. Moreover, such a model is useful for comparing the results with the hybrid simulations, where electrons are usually treated as an isotropic fluid whereas ions can be anisotropic. In what follows, we assume that the plasma consists of an isotropic population of electrons and an anisotropic population of ions with its anisotropy as characterized by the ε parameter.

A closer examination of these equations reveals that the number of equations is fewer than the number of unknowns, which can be traced back to the problem of closure of MHD equations in the presence of anisotropy. As we mentioned earlier, two equations of state describing the changes in the parallel and perpendicular pressures are needed. To overcome this problem, some authors have used the CGL (*Chew-Goldberger-Low*, 1956) theory to achieve closure (e.g., *Lynn*, 1967; *Abraham-Shrauner*, 1967). Since assumptions of CGL theory are violated at shocks and RDs (e.g., *Karimabadi*, 1995; *Krauss-Varban et al.*, 1995) we have taken a different approach used by *Chao* (1970) and *Hudson* (1971). Chao introduces the parameter ε which is chosen for the upstream and downstream plasmas. Once we choose a value for ε downstream, the number of unknowns is reduced by one in the equations above. This along with the assumption of a simple polytropic law for the total pressure enables us to solve the RH relations.

Transforming the above equations into the deHoffman-Teller frame and after some algebraic manipulations, one arrives at a fourth-order equation for M_{I_1} in terms of upstream quantities and M_{I_2}. The solution of M_{I_1} versus M_{I_2} are shown in Figure 1 for various combinations of $\varepsilon_1/\varepsilon_2$, where ε_1 and ε_2 refer to values of ε upstream and downstream, respectively. We also assume $\gamma = 5/3$. Figure 1 shows the changes in the topology of the solutions in cases where $\varepsilon_1 > \varepsilon_2$. Note that $\varepsilon > 1$ implies $T_\perp/T_\parallel > 1$. Other upstream plasma parameters are $\beta_e = 0.5$, $\beta_i = 0.1$, and a shock normal angle of $\theta_{Bn} = 60°$. Let us first consider the isotropic ($\varepsilon_1 = \varepsilon_2 = 1$) case. The solution consists of two branches, one is a curve (the outermost solid curve in Figure 1a) and the other is the diagonal line given by $M_{I_1} = M_{I_2}$. The only nontrivial solution along the diagonal line is the RD limit where the two branches meet. The only physical solutions are the ones lying at or above the diagonal line. The part of the curve below and above $M_{I_1} = 1$ corresponds to slow shock and intermediate shocks, respectively. The IS solutions are bounded on one side by the switchoff slow shock and an RD at the other side. Fast shocks occur at much larger Mach numbers and are not shown here. The downstream speed at the peak of the IS branch is equal to the sound speed. To the left of this peak, the downstream velocity is subslow (strong IS) and to its right it is superslow (weak IS).

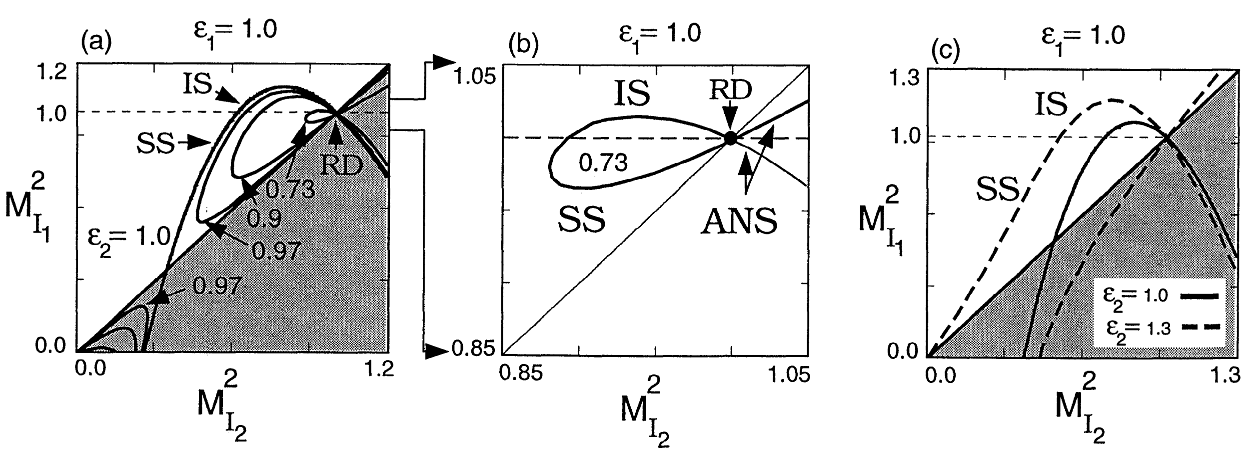

Figure 1. Rankine-Hugoniot solutions displayed as a plot of the square of upstream intermediate Mach number versus its value downstream. In all cases the upstream plasma is isotropic. (a) Downstream plasma is anisotropic with $T_\parallel/T_\perp > 1$. (b) Enlarged segment of (a) for $\varepsilon_2 = 0.73$. The new class of slow shocks are identified as ANS. (c) Downstream plasma is anisotropic with $T_\perp/T_\parallel > 1$.

As ε_2 is lowered below unity, the two branches (the slow shock branch and the diagonal line) merge, forming a loop that now connects the slow shock branch to the RD solution. In contrast to the isotropic case, for a given upstream Mach number there now exists two possible downstream states for the slow shock.

In order to examine this new branch more carefully we have enlarged the solution for $\varepsilon_2 = 0.73$ in Figure 1b. Comparison of the flow speed downstream of the RD with the slow characteristic speed reveals it to be subslow although it is superAlfvénic. This is because the downstream plasma has a large enough $T_{\|}/T_{\perp}$ so that the slow speed is larger than the intermediate speed. In passing, we point out that in deriving the linear mode properties (such as the characteristic speed) in an anisotropic plasma based on fluid theory, two equations of state describing the functional dependence of $P_{\|}$ and $P_{\perp}$ on other physical quantities are needed. This leads to some free range as to what model to choose and that in turn leads to model dependent expressions for the characteristic speeds. A detailed discussion of characteristic speeds based on various fluid theories and their comparison with the linear kinetic theory is given elsewhere (*Karimabadi et al.*, 1995). It suffices to say that characteristic speeds based on the simple polytropic law with $\gamma_{\|} = 1.6$, and $\gamma_{\perp} = 2.2$ yield fairly accurate estimates of the characteristic speeds in the parameter regime of interest here and will be used for calculation of characteristic speeds throughout the paper.

Inspection of the RH relations shows that there is an additional solution at the location of the RD in Figure 1b. This solution has the same jumps in the plasma parameters except for the sign of the transverse component of the magnetic field and is a nonswitchoff slow shock. The RD rotates the magnetic field whereas the slow shock does not. Thus, there are three solutions at $M_{I_1} = 1$. One is the usual switchoff slow shock and the other two are an RD and a nonswitchoff slow shock.

The merging of the slow shock branch and the RD requires two conditions. First, the RD has to be compressional since the slow shock is compressional in this case. Secondly, the flow speed has to be superslow upstream and subslow downstream, although the flow speed remains the same as the intermediate speed across the solution. The first condition is satisfied in Figure 1b because the density jump at the RD is given by $\varepsilon_1/\varepsilon_2$ and thus for $\varepsilon_1 > \varepsilon_2$, the RD is compressional. The second condition requires the downstream plasma to have a large enough $T_{\|}/T_{\perp}$ so that the slow speed becomes larger than the intermediate speed.

Another difference between the isotropic case and the anisotropic case in Figure 1b is that the flow speed downstream of the weak IS is subslow in the anisotropic case whereas it is always superslow in the isotropic case.

Next, we examine the solutions in Figure 1b which have $M_{I_2} > 1$. In the isotropic case, all solutions below the diagonal line are unphysical but as it turns out that is not the case in the anisotropic case. Let us first consider the continuation of the slow shock branch to $M_{I_1} > 1$ immediately following the RD limit. Examination of the jump conditions, flow speeds and entropy lead us to the conclusion that part of this curve corresponds to physical solutions. As we now demonstrate, these are anomalous slow shocks. Since the upstream and downstream flow speeds are both superAlfvénic in this case, they cannot be intermediate shocks. In the upstream region, the plasma is isotropic so a superAlfvénic flow is also superslow. The downstream speed is subslow although it is superAlfvénic. As in the case of the RD, this is possible because the large temperature anisotropy downstream is such that the slow speed is larger than the intermediate speed. The superslow flow speed upstream and subslow flow speed downstream are consistent with a slow shock interpretation. The jumps in plasma quantities are also consistent with that of a slow shock as density increases and the magnetic field drops across it. The question of entropy change is more difficult to address as no unique entropy condition may be established without a detailed knowledge of the shock structure (e.g., *Neubauer*, 1970). But if we assume that the definition of entropy (S) remains similar to that in the isotropic case, we find that the entropy increases across these solutions. We refer to such solutions as anomalous slow shock (ANS in Figure 1b) because unlike the usual slow shocks, the upstream and downstream flow speeds are superAlfvénic. As we follow the curve to higher Mach numbers, at one point the solutions become unphysical as the flow speed becomes superslow downstream, the entropy change becomes negative, and the magnetic field increases.

Yet more bizarre solutions are those that lie along the continuation of the IS branch beyond the RD limit and to the regime where $M_{I_2} > 1$ (Figure 1b). The consideration of the jumps and the flow speeds reveal surprising results. The plasma density, temperature and entropy increase and magnetic field decreases across these solutions, much like a slow shock. The flow speed is superslow upstream and subslow downstream. This is also consistent with a slow shock. However, unlike a usual slow shock the magnetic field rotates across the discontinuity. Another strange property of these solutions is the fact that the flow is subAlfvénic upstream but superAlfvénic downstream. This is also unlike any other known discontinuity. Using the above considerations, we conclude that the above solutions are indeed physical, albeit their strange properties consists of a mix of slow shock and intermediate shock. Whether there exist other conditions that would make these solutions unphysical and/or whether they can be formed in a kinetic plasma are two important topics left for future research. Given the fact that the flow is superslow upstream and subslow downstream we classify these solutions as slow shocks. However, in order to distinguish them from the usual slow shocks, we again refer to them as anomalous slow shocks. Note that for the RD in Figure 1b, the flow is superslow upstream and subslow downstream and in that sense it can also be thought of as an anomalous slow shock.

Next, we consider a case where $\varepsilon_1 < \varepsilon_2$. Figure 1c shows the changes in the solutions for an isotropic upstream but anisotropic downstream with $\varepsilon_2 = 1.3$ ($T_{\perp}/T_{\|} > 1$). The other plasma parameters are $\beta_e = 0.2$, $\beta_i = 1.0$, $\theta = 60°$, and $\omega_p/\Omega = 4000$, where ω_p is the plasma frequency and Ω is the ion gyrofrequency. Aside from the fact that the range of solutions for intermediate shocks ($M_{I_1} > 1$) as well as slow shocks has increased, there are no other significant changes as compared to anisotropic plasma in this case. Note that unlike the case where $\varepsilon_1 > \varepsilon_2$ (Figures 1-b), the slow shock branch does not connect to the RD solution. This is due to the fact that the plasma density decreases across the RD in the present case, whereas density increases across a slow shock. Thus, the two solutions remain separated as in the isotropic case. The solution curve that lies above the diagonal line and extends to $M_{I_1} > 1.3$ is characterized by superAlfvénic flow speeds upstream and downstream. Since the slow speed is slower than the intermediate speed in this case, the flow speed remains superslow across the solution. We thus conclude that solutions along this branch are unphysical.

We emphasize that the above topological changes shown in Figures 1a-c only occur if ε_1 is different than ε_2. If $\varepsilon_1 = \varepsilon_2$, it does not matter whether the plasma is isotropic ($\varepsilon = 1$) or not, the solution curves would have the same shape as the isotropic case. In other words, it is the jump in the anisotropy across the discontinuities that results in the above changes.

Another point of importance regarding RH solutions shown above is the question of their existence in a kinetic plasma. Whether a given solution can form kinetically depends on the detailed structure of the discontinuity and has not been addressed here. For instance, one may find a solution based on RH relations for a slow shock having a large change in the anisotropy across it. But whether such a change can be accommodated kinetically requires knowledge about the structure of the slow shock and other physics not included in the RH relations. Questions regarding the kinetic existence of each of the above solutions can currently only be addressed by means of simulations and should be decided on a case by case basis.

FInally, we have examined the changes to the RH solutions as a function of the polytropic index γ. In most cases the jump conditions are not sensitive to the exact value of γ.

3. Discussion and Summary

The RH relations are widely used to identify various discontinuities in observational data as well as in simulations. Given the fact that space plasmas are often anisotropic, we reexamined the modifications to RH relations due to the presence of anisotropy. We identified a new class of slow shocks, which due to their unusual properties we refer to as anomalous slow shocks (ANS). These shocks can form only if $\varepsilon_1 > \varepsilon_2$ and behave differently depending on whether the upstream intermediate Mach number (M_{I1}) is below, equal to, or larger than the intermediate Mach number. For $M_{I1} > 1$, the flow speed is superAlfvénic both upstream and downstream. Recall that for a usual slow shock the slow speed is subAlfvénic both upstream and downstream. For $M_{I1} = 1$ and $M_{I2} = 1$ there are two solutions, which have the same jumps in the plasma quantities, except one solution rotates the transverse component of the magnetic field and the other does not. The former is identified as an RD and the latter as a nonswitchoff slow shock. Note, however, that the coplanar RD solution can also be thought of as a slow shock since its upstream and downstream flow speed are superslow and subslow, respectively and it connects to the slow shock branch that exists for $M_{I1} < 1$ but $M_{I2} > 1$. Finally for $M_{I1} < 1$, ANS has a superAlfvénic downstream speed and unlike all other slow shocks it rotates the magnetic field. In all three cases, the flow speed is superslow upstream and subslow downstream. One possible formation site of the above class of slow shocks is at the magnetopause as part of the reconnection geometry. Since the magnetopause structure during reconnection events involves a rotation of the transverse component of the magnetic field, ANS with $M_{I1} < 1$ could affect the necessary field rotation but if instead an ANS with $M_{I1} > 1$ is formed, the presence of an RD or an intermediate shock would be required to rotate the field. A likely candidate for the anomalous slow shock with $M_{I1} > 1$ at the magnetopause was recently reported by *Walthour et al.* (1994). The flow speed was measured to be superAlfvénic both upstream and downstream of the structure that they identified as a slow shock, but unlike the cases considered here, the downstream state is fire-hose unstable ($\varepsilon_2 < 0$). Since the intermediate speed becomes zero at the marginal fire-hose condition ($\varepsilon = 0$), any finite flow speed would clearly be superAlfvénic in such cases. As a result, for $\varepsilon_2 < 0$ only one type of anomalous slow shock which has a superAlfvénic downstream speed is possible. In this sense, the slow shock solutions with fire-hose unstable downstream states are a special case of the anomalous slow shocks discussed here. The question of existence and stability of shocks with $\varepsilon_2 < 0$ is an important issue that needs to be addressed via kinetic simulations. With the current interest in the magnetopause, the study of effects of anisotropy on discontinuities has gained a new impetus. Our finding of a new class of slow shocks adds further importance to this subject.

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