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The effects of low frequency waves on ion trajectories in the Earth's magnetotail

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Abstract. The trajectories of ions in the Tsyganenko magnetic field model with a convection electric field have been compared to the trajectories when a perturbation electric field based on ISEE-1 and Geotail observations of lower hybrid frequency waves is included. The inclusion of waves dramatically modifies the characteristics of the ion trajectories. For example, initial conditions which result in adiabatic orbits in the absence of waves are often non-adiabatic when waves are included. In addition, both the regions of space accessible to the particles and the energization are changed. Although only individual trajectories were examined in this study, it is argued that the effect of the waves is to mix previously distinct regions of phase space.

I. Introduction

The nature of the motion of particles in the field-reversed geometry of the Earth's magnetotail has long been of interest to space physicists. Speiser [1965] provided the first description of the orbits as being adiabatic or non-adiabatic in a tail-like magnetic field, depending on their initial conditions. In the last decade, the nature of the trajectories has received much attention. Many early studies used a Harris or modified Harris magnetic field since this simplified model of the field allowed the use of powerful analysis techniques, including some devised for the study of chaotic systems. Chen and Palmedesso [1986] described the existence of three distinct types of orbits and the partition of phase space which resulted. Several researchers [Buchner and Zelenyi, 1987; Speiser, 1987] determined a parameter of adiabaticity (the ratio of the gyroradius to the magnetic field gradient scale length) which controls the transition to stochastic orbits. Lyons and Speiser [1985] calculated a collisionless conductivity due to the transient orbits. Comparisons between predicted ion distribution functions and those measured by particle detectors in the magnetotail were made by Chen *et al.* [1990]. More recently, studies utilizing the Tsyganenko magnetic field model [Tsyganenko, 1987] have been carried out including a determination of anomalous transport coefficients [Horton and Tajima, 1991] and a model for the production of both plasma sheet distribution functions from a mantle source population [Ashour-Abdalla *et al.*, 1990; 1991]. The effects of the magnetic field geometry on the nature of the particle orbits was discussed by Karimabadi *et al.* [1990] and Burkhart and Chen [1993]. The above studies assumed a time-stationary magnetic field and, in some cases, a constant electric field; perturbations in the electric and magnetic field were not included. Some research including perturbations has been performed. For example, the effects of an x-line geometry were examined by Martin [1986] and Speiser and Martin [1992]. Doxas *et al.* [1990] and Horton and Tajima [1990] performed studies including the tearing mode. Curran and Goertz [1989] included random fluctuations in both the electric and magnetic field. The issue of time dependence in the background magnetic field was addressed by Pulkinnen *et al.* [1991] and Chapman and Watkins [1993].

As our understanding of the waves in the magnetotail has increased, so have speculations about their possible importance for magnetotail dynamics. For example, based on the Gurnett *et al.* [1976] observations, Huba *et al.* [1981] suggested that the waves were lower hybrid drift waves and that they might provide the anomalous resistance needed for reconnection. Data from ISEE-1 and Geotail have extended the observations of wave electric fields to frequencies below the lower hybrid frequency and showed that large amplitude (10 mV/m - 50 mV/m) waves often occur [Cattell and Mozer, 1986; 1987; Cattell *et al.*, 1994]. Observational studies of longer period magnetic and electric fields in the magnetotail have emphasized their extremely variable nature [Cattell *et al.*, 1981; 1986; Cattell and Mozer, 1982, 1984]. In the case of the electric field, it has been shown that the amplitudes of perturbations are often one to two orders of magnitude larger than the background convection field. The perturbations occur over a range of time scales from <0.1s to tens of minutes and have a variety of scale sizes from <1 km to many R_E. Examples of such perturbations on time scales of minutes were shown in Fig. 2 of Cattell and Mozer [1986] which presented data at a radial distance of ~19 R_E during an event interpreted as the occurrence of reconnection near the satellite. Intense lower hybrid drift waves were also observed during this event with amplitudes large enough to provide the dissipation required for reconnection [Cattell and Mozer, 1986;1987]. Recent studies of the GEOTAIL data [Cattell *et al.*, 1994; Okada *et al.*, 1994] have provided additional information on the waves including the fact that they tend to occur in packets with durations of tens of seconds to a few minutes. Large spiky fields (up to 100 mV/m) are also commonly observed at the plasma sheet boundary [Cattell *et al.*, 1981; 1994] and at the current sheet [Cattell *et al.*, 1984; Mozer *et al.*, 1987].

Given the large magnitudes of the perturbation electric fields and their ubiquitousness, it is likely that they modify the types of particle trajectories which occur in the magnetotail. Since the phase-space boundaries between different types of orbits are sharp, wave perturbations may be able to mingle orbits which were initially distinct. In this paper, the effects of waves near the lower hybrid frequency on ion trajectories are examined. The only previous work which explicitly addressed the effects of waves on particle trajectories in the magnetotail was the study by Holland and Chen [1991] who examined the effects of waves in a modified Harris magnetic field by including collisions modeled as random kicks in pitch angle and a change in energy between two possible states. They concluded that there were no significant alterations in the phase space structures. The study described herein includes a model of waves with a realistic spread in frequency and wavenumber. This allows resonant interaction between the waves and various particle frequencies (i.e., cyclotron, bounce) which would modify the ion trajectory in a very different way than a random kick. The numerical model utilized in this study and sample trajectories for several different initial conditions both in the absence of and in the presence of waves are described in Section II. Discussion, conclusions and plans for future studies are presented in Section IV.

II. Numerical Model and Examples of Trajectories

The calculations are made with a test particle code which uses the Runge-Kutta method to trace the motion of particles (electrons, H^+ and

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O^+ ions) in the Tsyganenko magnetic field model [Tsyganenko, 1987], with the optional inclusion of a duskward (y-component) convection electric field, and a model of the lower hybrid drift waves. It is based on a program developed to study the motion of ring current ions in the combination of the dipole magnetic field, the convection and corotation electric fields, and the electromagnetic field of Pc waves [Hudson *et al.*, 1991; Li *et al.*, 1993]. The original code was modified to use the Tsyganenko magnetic field model rather than a dipole magnetic field and to use a wave electric field based on lower hybrid waves.

The model of the wave field is based observations of Cattell and Mozer [1986] and Cattell *et al.* [1994] with the frequency, wavelength and amplitudes obtained from both ISEE-1 and Geotail data and the duration of the bursts determined from Geotail observations. The perturbation electric field is given by the sum of a selectable number of waves (up to 100) occurring in a box with the amplitude exponentially decaying in x, y and z. The decay constants (usually 5 R_E) and the center of the box (X = -25 R_E) are input parameters. The wave model is the following:

$$\bar{E}_w = \hat{y}\sum_{i=1}^{n} A_i \cos(k_\perp y - \omega_i t + \phi_i), \text{ with}$$

$$\omega_i = 0.6\omega_{lh}(1+r_i), k_\perp \rho_e = 0.6(1+s_i), A_i = A(1+t_i), \phi_i = p_i(2\pi),$$

where r_i, s_i, t_i are random numbers between -0.5 and 0.5 and p_i is a random number between 0 and 1. For n=1, the wave is monochromatic. In addition, the waves can be turned on for a specific period of time, ranging from a few wave periods to the entire time period of the run. This feature is designed to simulate the bursty nature of the observed waves. Particles were followed for ~5000 to 6000s.

An example of the simulation is presented in Fig. 1 which shows the trajectory of a proton with an initial energy of 100 eV and an initial pitch angle of 10° in the Kp = 1 Tsyganenko magnetic field with the convection E_y = 0.1 mV/m. The trajectory in the absence of waves is plotted in panels a and b which show, respectively, the projections in the x-z and the x-y plane. The trajectory plotted in panels c and d was calculated including the effect of a single, monochromatic wave with an amplitude of 10 mV/m turned on at the time indicated by the "o" and off at the time indicated by the "x" in Fig.1c. It is clear that the trajectories in the two cases are very different. In the absence of waves, the ion behaves adiabatically as can be seen most clearly in panel a which shows the combination of gyration, bounce, E x B drift and magnetic field gradient drift motion. In the case with waves, the ion is scattered in the current sheet region (panel c) and drifts much farther in the direction of the convection electric field resulting in increased energization (panel d). The ion energy increase is also indicated by the larger gyroradius in panel d compared to panel b. The interaction with the waves can have two effects. The first is the direct change of the energy and/or pitch angle of the ion by the wave. The second is that this change results in the ion entering the current sheet with a different pitch angle and energy than it would in the absence of waves. As a consequence, the ion may reside in the current sheet (region of weak magnetic field) for a longer time and, therefore, receive additional energization due to the acceleration along the convection electric field. When multiple, broader band waves are included, these effects are enhanced.

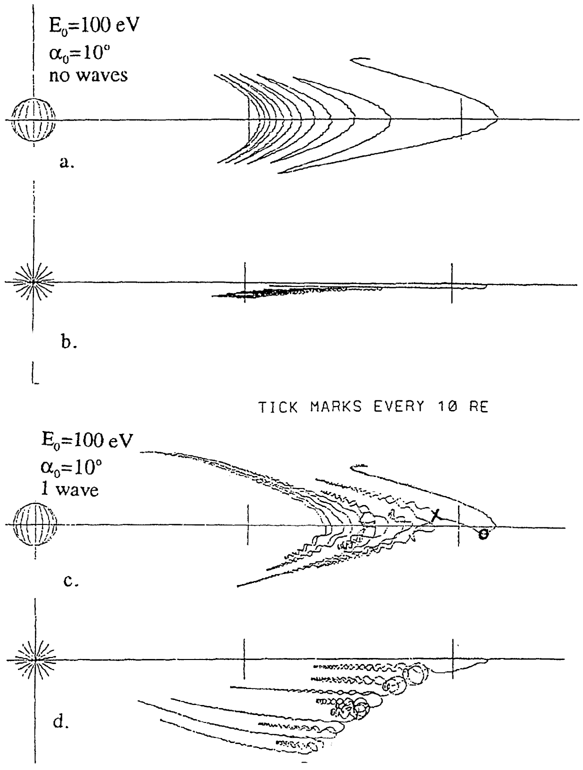

Fig. 1. The trajectory of a proton with initial energy of 100 eV and an initial pitch angle of 10°. No waves: (a) x-z plane; (b) x-y plane. One wave: (c) x-z plane; (d) x-y plane.

The trajectories of a proton with an initial energy of 300 eV and an initial pitch angle of 40° are shown in Fig. 2. The case in panels a and b had no waves. The case plotted in panels c and d and the one in panels e and f included the effects of 10 waves with total rms amplitude of 10 mV/m. The waves were turned on between the "o" and the "x" marked on the x-z projections. In contrast to the lower initial energy and smaller initial pitch angle case (Fig. 1), the ion behaves non-adiabatically in the absence of waves, executing the type of motion described by Ashour-Abdalla *et al.* [1991] and others. When waves are included and turned on prior to the encounter with the current sheet (panel c and d), the ion still behaves non-adiabatically near the current sheet but the details of the orbit are quite different. The ion shown in Fig. 2a remains trapped near the current sheet for a distance of ~ 4 R_E in X before being ejected whereas the ion pictured in Fig. 2c is trapped in the current sheet for less than 1 R_E. In addition, it experiences a second period of non-adiabatic motion closer to the earth and drifts only slightly farther duskward. In the case where the waves were turned on at a later time, the ion trajectory in the x-z projection is quite complex, with multiple scatterings within the current sheet, until it reaches X=~12 R_E. The ion reaches an X location much closer to the Earth than in the cases shown in panels a and c. The x-y projection is also very different (panel f). The ion drifts almost 10 R_E in the duskward direction and is energized much more than in the cases presented in panels b and d.

These examples illustrate the importance of including realistic perturbation wave fields. The character of the trajectory (adiabatic vs. non-adiabatic), ion energization, and the regions of accessible space are all modified. In the first example presented (Fig. 1), the waves changed the initial pitch angle and energy with which the proton entered the current sheet resulting in an extended residence time in the weak magnetic field region and additional energization in comparison to the trajectory in the absence of waves. The trajectory in the absence of waves was adiabatic, while parts of the trajectory in the presence of waves were non-adiabatic. In the second case (Fig. 2), the effect of waves was either to rapidly eject the ion from the current sheet and later to cause chaotic behavior at distances closer to the Earth or to cause chaotic behavior over most of the trajectory, depending on the time period during which the waves were turned on.

III. Discussion and Conclusions

In this paper, we have examined the effects of lower hybrid drift waves on the trajectories of ions in the Tsyganenko Kp=1 magnetic field model. Examples of difference in the trajectories in Kp=5 compared to Kp=1 magnetic field models were presented by Hudson *et al.* [1995]. As discussed above, the previous study [Holland and Chen, 1991] which modeled the effects of waves on the structure of the ion trajectories in the Earth's magnetotail, concluded that waves with reasonable amplitudes do not strongly modify the observed non-adiabatic features. Since our studies indicate that realistic waves can dramatically change the individual ion trajectories, we will briefly discuss the differences between these two approaches.

Holland and Chen [1991] examined the effects of waves on particle trajectories in a modified Harris magnetic field by including collisions

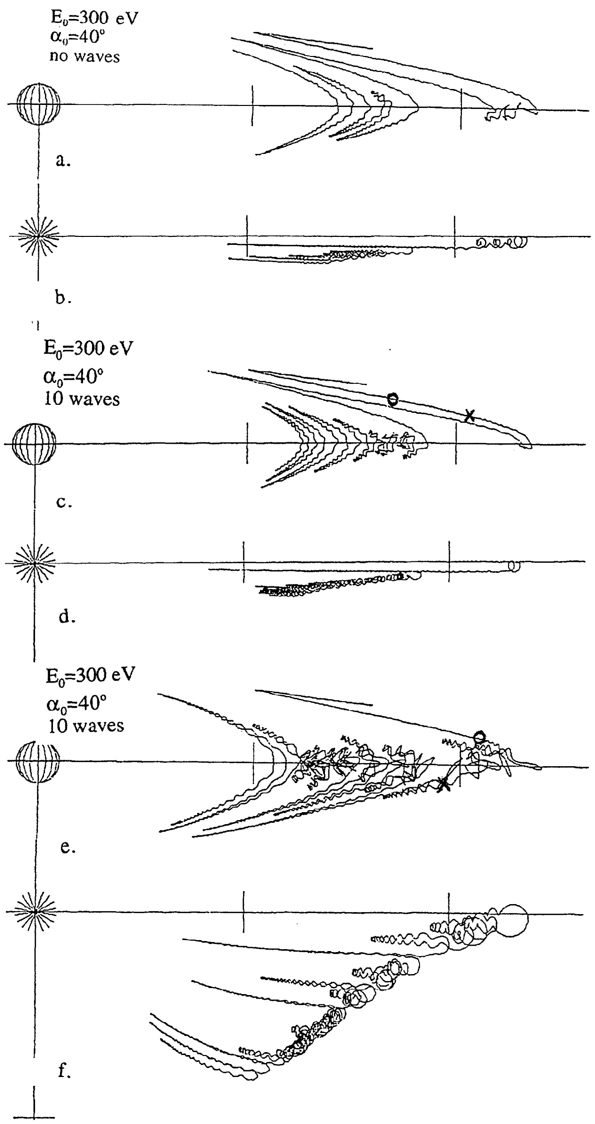

Fig. 2. The trajectory of proton with an initial energy of 300 eV and an initial pitch angle of 40°. No waves: (a) x-z plane; (b) x-y plane. Ten waves on between "o" and "x": (c) x-z plane; (d) x-y plane. Ten waves on between "o" and "x": (e) x-z plane; (f) x-y plane

which were modeled as random "kicks" in pitch-angle and a change in energy between two states. They showed that this type of collision did not significantly alter the coherence factor which measures the transition of particles across the phase space boundaries separating the various classes of orbits [Chen and Palmedesso, 1986] although individual orbits in the stochastic region of phase space were modified. They concluded that the electric field amplitude required to destroy the resonant structure via collisions is well above any measured wave amplitudes. There are two explanations for the differences between our conclusions and those of Holland and Chen. The first is that the observed amplitudes (up to 50 mV/m) are, in fact, large enough to cause transitions between resonance peaks for small N as can be seen by examining their Table 1. The second is that an ion interacting with a broad spectrum of wave turbulence is subject to a varying acceleration which changes its phase space trajectory in a way which is distinctly different from random small amplitude scattering kicks and a change between only two possible energy states. Since the bounce and gyrofrequencies change as the particle migrates in phase space in the realistic background field, resonance between harmonics of the bounce and/or gyrofrequencies and the wave frequency may provide a viable mechanism for an effective interaction between the waves and the particles. In the study described herein, the particle motion was followed in fields which include a broad spectrum of waves based on the characteristics of the waves observed on ISEE-1 and Geotail. The resulting particle trajectories were dramatically different from those obtained in the absence of waves. These differences occur both due to the direct modification of the ion pitch angle and energy by the waves and due to the change in the energy and pitch angle of the particle when it enters the weak magnetic field region.

Modifications of ion trajectories due to the interaction with lower hybrid waves have also been observed in lower hybrid heating experiments. Karney [1978] found that, for a wave field with a given k (perpendicular to the background magnetic field B_0), gyrofrequency, Ω, and amplitude, E, significant heating and stochastic behavior occurred when $\alpha=(kE)/(\Omega B_0)>1$. For the parameters used herein, $\alpha=5(E/B_0)$ where E is in mV/m and B_0 is in nT. For typical values of E=10mV/m and B_0=10nT, $\alpha=5>1$, so stochastic behavior would be expected.

Although we have only described changes in individual trajectories, a more formal argument which indicates that changes in the phase space structures should be expected when waves are added to the system is the following: The system studied by Chen and Palmadesso [1986] and Holland and Chen [1991] has two degrees of freedom and the Hamiltonian is constant. The particle trajectories are confined to three dimensions and the surface utilized for the Poincaré surface-of-section plots is two dimensional. When a time-dependent electric field is added to the system, the system can be thought of as having 2 1/2 degrees of freedom, and the Hamiltonian is no longer constant. The particle trajectories are in four dimensions and the surface required for a surface-of-section is three dimensional. The organization of phase space is, therefore, likely to be qualitatively different. An example of this can be seen in a system with a time varying magnetic field studied by Chapman and Watkins [1993] who concluded that orbits can no longer be classified as stochastic or integrable for all times.

In this study, only individual ion orbits have been examined. Future studies will include diagnostics to follow distributions and to examine changes in both energy and pitch angle of ensembles of particles. In addition, the effects of electric field perturbations with longer time scales, the effects of varying source locations, and electron and oxygen ion trajectories will be examined.

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