Characteristic speeds in high β isotropic/anisotropic plasmas

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Given the importance of linear mode properties (e.g., characteristic speeds) in identification/classification of discontinuities, a detailed comparison between the mode properties in fluid theory and kinetic theory in high β plasmas is carried out. It is found that conventional fluid theories of linear modes in both isotropic and anisotropic plasmas do not yield the correct mode properties, even in the long-wavelength limit. In particular, fluid phase velocities are very sensitive to the model and parameters (polytropic indices) employed. Because of this, fluid theory loses its predictive power. In linear kinetic theory, modes cannot be ordered according to their phase velocities. For instance, at small and moderate propagation angles, the slow/sound (S/SO) mode can have the fastest phase velocity. In such cases, a (quasiparallel) fast shock would be associated with the S/SO mode rather than the usual fast/magnetosonic (F/MS) mode. This has important implications for fast shocks. Since it is the F/MS rather than S/SO mode that connects to the whistler branch, low Mach number quasiparallel shocks associated with S/SO would not be expected to have a phase standing whistler wave train upstream, and their thickness is determined by dissipation rather than dispersion. The consequences of the kinetic mode properties are demonstrated via hybrid simulations (fluid electron, kinetic ions) using the quasiparallel shock as an example. © *1995 American Institute of Physics.*

I. INTRODUCTION

Much of the physics of discontinuities is based on the fluid characteristic speeds. Examples are their classification, the order in which they stand in the flow, or the type of wave train associated with the discontinuity. The characteristic speed is the phase velocity of a mode in the long-wavelength limit. The main appeal of fluid theory is that it yields simple analytical expressions for the characteristic speeds. With this simplicity, however, comes the price of less accuracy compared to kinetic calculations. Given the heavy reliance on the fluid characteristic speeds in studies and identification of discontinuities, we examine their validity in both isotropic and anisotropic but high β plasmas.

In the absence of anisotropy, it can be easily shown, using magnetohydrodynamic (MHD) theory that there are three characteristic speeds (slow, intermediate, and fast), which, if exceeded, result in the information of slow, intermediate, and fast shocks in the flow, respectively.[1] In MHD, even when the Hall term is included, the dispersion curves of the modes never intersect and modes have been named (slow, intermediate, and fast), based on the ordering of their phase velocities. A fourth mode, referred to as the entropy wave, is not associated with a propagating discontinuity and will not be discussed further. In the high β regime, however, kinetic dispersion curves can intersect and modes can no longer be classified according to their phase velocity. For instance, a slow/sound (S/SO) mode can have a phase velocity that is larger than the fast/magnetosonic (F/MS) mode. Since a fast shock is related to the fastest mode in the system, a fast shock would then be associated with the slow/sound, rather than the usual case of F/MS. As we show, this velocity inversion has important consequences for the physics of fast shocks.

We also consider the more complicated case of anisotropic plasmas. We point out that the presence of anisotropy is a rule rather than the exception in space plasmas. An area of immediate relevance is that of the Earth's magnetopause, which separates the shocked solar wind (magnetosheath) from the magnetospheric plasma. The magnetosheath plasma is commonly found to have a temperature anisotropy. The main complication arising from the presence of anisotropy is the fact that changes in anisotropy or equivalently, pitch angle scattering, are, by definition, a kinetic effect, and their proper description within fluid theory remains as a fundamental but as yet unresolved problem in plasma physics. Technically speaking, the closure of fluid equations requires the introduction of two equations of state, one for the parallel pressure and one for perpendicular pressure. In spite of much effort, no generally valid form for the two equations of states are known. This, in turn, implies that there are no unique fluid expressions for the characteristic speeds.

Most fluid studies of small amplitude waves in anisotropic plasmas have used the Chew–Goldberger–Low (CGL) equations of state.[2–5] However, the CGL limit is only expected to hold in a very limited cases from the outset. It breaks down when there exists a finite heat flux and/or the magnetic moment is not conserved. Thus, others have attempted more generalized models with a number of free parameters,[6] which include CGL as a limiting case. The appeal of these models is that they yield analytical expressions for mode properties, but the drawback is that they involve free parameters, and, in addition, one cannot be assured of the validity of the models *a priori*. This latter concern applies specially to high plasma β regimes, where fluid theory

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is known to break down.[4,7,8] Before abandoning the fluid characteristic speeds, it is worthwhile to determine their domain of applicability. To this end, we calculate the fluid characteristic speeds based on two different models and compare the results with linear kinetic calculations. The mode properties of kinetic theory are calculated, as outlined in Krauss-Varban *et al.*[8]

The outline of the paper is as follows. Section II contains a comparison of linear mode properties in fluid theory and kinetic theory for both isotropic and anisotropic plasmas. In Sec. III we demonstrate the implications of the kinetic phase velocity inversion for quasiparallel shocks via hybrid simulations (fluid electron, kinetic ions). Finally, a discussion and summary of the results is given in Sec. IV.

II. LINEAR MODE PROPERTIES IN THE LONG-WAVELENGTH LIMIT: KINETIC VERSUS FLUID DESCRIPTION

The linear mode properties are important in understanding shocks and discontinuities. In fact, the connection between linear mode properties and shock waves have been utilized in at least four different fashions. (1) According to classical shock physics, a necessary condition for formation of a shock is for the flow speed to exceed the long-wavelength phase velocity (characteristic speed) of the mode. Thus, the knowledge of the characteristic speed is important in classifying the various types of discontinuity. (2) The order in which various discontinuities stand in the flow is typically discussed in terms of upstream and downstream speeds relative to the fluid characteristic speeds. For instance, the flow speeds upstream and downstream of an isotropic slow shock are superslow (but sub-Alfvénic) and subslow, respectively. Thus, a rotational discontinuity (RD) that propagates with the intermediate speed can form and remain upstream, but not downstream of a slow shock. (3) The correlation between the magnetic field and density across the shocks are usually explained in terms of the linear waves. For instance, the density and magnetic field are anticorrelated for the slow mode in isotropic plasmas. This is consistent with the fact that the total magnetic field decreases but density increases across a slow shock. So it is important to determine the correlation between density and magnetic field for each mode and to see if this correlation changes for different parameter regimes. (4) The dispersion and damping of a mode at a finite wavelength is used to determine whether the mode can steepen. For instance, Hada and Kennel[9] argued that a slow shock cannot be formed in the high β solar wind because of the large damping rate of the slow mode.

We emphasize that shocks are nonlinear entities and care must be taken in using linear theory to explain their behavior. Furthermore, the formation of a shock depends strongly on the boundary conditions. One way to form a shock is if the flow encounters an obstacle. Such a situation is different from a shock formation process in which a linear wave grows in amplitude and eventually steepens. The latter process depends on the mode properties of the mode at the wavelength comparable to that of the linear wave. In the former case the shock properties are determined by those of the long-wavelength limit. The two shock formation processes do not always yield the same answer. For instance, the slow mode is heavily damped at finite wavelength in a high β plasma, and thus a slow shock cannot be formed by the steepening of a linear mode in such a plasma. At the same time, a slow shock can be formed in a high β plasma if the flow encounters an obstacle.[10] From the foregoing and given the lack of a suitable nonlinear analytical theory, it is clear that the question of existence of a particular type of shock has to ultimately be answered by simulations. Linear theory can, however, provide some level of understanding of an otherwise untractable problem.

Before discussing the anisotropic case, we contrast the isotropic mode properties based on fluid and kinetic theories in the long-wavelength limit.

A. Isotropic plasma

The shock classification based on MHD characteristic speeds has gained general acceptance, and the order in which various discontinuities stand in the flow is often discussed using the fluid theory. Ideal MHD lacks any dispersion and yields only the phase velocity of a mode. In order to determine other shock properties such as its polarization, the sense of correlation between the magnetic field and density, as well as the location and the types of coherent waves that can be generated by the shock, Hall MHD theory[11] has often be used. By retaining the Hall term a characteristic scale (ion inertial length) is introduced in the fluid equations, which gives rise to dispersion of the modes. In this section, we compare and contrast linear mode properties at the long-wavelength limit based on Hall MHD and linear kinetic theory.

We start by reviewing linear mode properties in Hall MHD.[8,12] The fast mode (F) always has a positive correlation between the magnetic field (B) and density (N), is right handed and has a phase velocity that increases as a function of wave number k. In the limit of $k\to 0$, the phase velocity approaches the MHD fast speed. The intermediate mode (I) has a phase velocity that approaches the MHD intermediate speed $V_A \cos\theta$ and the sound speed as $k\to 0$ and $k\to\infty$, respectively. The intermediate mode is different from the fast and slow (S) modes in that its dispersion and polarization change as a function of plasma β. Depending on whether β is smaller or larger than $(2/\gamma)\cos^2\theta$, the mode is left handed with positive correlation between B and N and ordinary dispersion or is right handed, with anticorrelated B and N and anomalous dispersion (i.e., its group velocity is larger than its phase velocity). Finally, the slow mode is always left handed, has an anticorrelated B and N, and its phase velocity approaches the MHD slow speed as $k\to 0$ and zero as $k\to\infty$. In Hall MHD, the three modes never intersect at finite wave number k and the phase velocities satisfy the condition $V_S < V_I < V_F$.

In the low β regime, Hall MHD provides a reasonable approximation to the mode properties in the long-wavelength limit. However, for β in the range of 0.1–0.5 some of the mode properties based on fluid theory start to deviate from kinetic theory, and for higher β fluid theory fails altogether (see below). Note that plasmas with $\beta>0.5$ are common place in space physics. Plasmas downstream of nearly

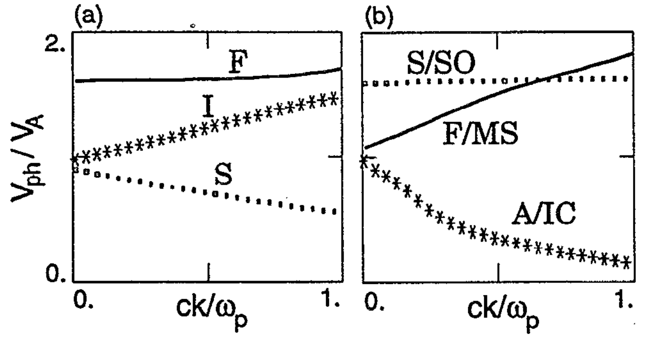

FIG. 1. Phase velocity of the three modes versus wave number in (a) the Hall MHD and in (b) kinetic theory.

switchoff shocks, strong intermediate shocks, and fast shocks are generally high β, even if the upstream plasma has low β.

The discrepancies between fluid and kinetic predictions is best illustrated by way of an example. Figures 1(a) and 1(b) show the phase velocities for the three modes based on Hall MHD and kinetic theory, respectively. The plasma parameters are $\beta_i=2.5$, $\beta_e=0.5$, and $\theta=15°$. In the Hall MHD case, the modes are ordered according to their phase velocities, $V_F>V_I>V_S$, and the three modes never intersect [Fig. 1(a)]. In the kinetic case, however, the modes can intersect,[7,8] as is shown in Fig. 1(b). For this reason, modes in the kinetic theory cannot be classified according to the ordering of their phase velocities, rather they are classified based on their physical properties. In Fig. 1(b), we have labeled the three modes Alfvén/ion cyclotron (A/IC), F/MS, and S/SO, respectively. The name convention[8] is chosen so that the first letter corresponds to the low β limit of the mode and the remaining two letters correspond to more general properties of the mode. For instance, the slow/sound (S/SO) is the slow mode in the low β regime, but it is essentially a sound mode that is slightly modified by the presence of the ambient magnetic field.

Ignoring the names of the modes for the moment, we consider the three phase velocities at $k=0$ in Figs. 1(a) and 1(b). Of the three fluid phase velocities, only one matches a kinetic phase velocity. This mode is the intermediate mode, which has a phase velocity of $V_A \cos\theta$, both in the fluid and kinetic limit. In contrast to the fluid case, the other two kinetic modes have phase velocities larger than the A/IC. Thus, except for the A/IC mode, fluid theory does not yield the correct phase velocities of the modes in the long-wavelength limit. The discrepancy between fluid and kinetic theory becomes even more pronounced at finite k. For instance, the phase velocity of A/IC and intermediate mode match at $k=0$, but the phase velocity of the intermediate mode increases as a function of k [Fig. 1(a)], whereas the phase velocity of the A/IC mode decreases with increasing k [Fig. 1(b)].

Aside from the phase velocity, there are significant differences in other predicted mode properties in fluid versus kinetic theory. This is shown in Table I, which summarizes the mode properties for the above parameters based on fluid and kinetic theories. Although the actual entries in this table would change for different parameter regimes, it is representative of the types of problems encountered by using fluid theory. As we mentioned above, a number of mode properties such as compressibility, damping, and polarization, need to be considered before a mode can be classified in the kinetic regime. The A/IC mode is classified by the fact that its phase velocity is given accurately by the intermediate speed in the long-wavelength limit. Furthermore, its magnetic fluctuations lie mainly out of the plane of **k** and the external magnetic field $\mathbf{B}_0$. The classifications of S/SO and F/MS are less straightforward. The S/SO is characterized by a large damping (highest damping of all modes), since for $T_e \lesssim T_i$, its phase velocity is comparable to the ion thermal speed. In addition, it is the most compressible of all modes (Table I), and behaves like a sound wave slightly modified by the presence of the magnetic field. The F/MS is characterized by the fact that it is always right handed and continues to the whistler branch. Note that for small propagation angles and high β, the fast shock should be associated with the S/SO mode (which is the fastest mode), rather than the usual F/MS mode. This has important implications for the structure of the quasiparallel shock, as we show in Sec. III.

A careful examination of Table I reveals that in addition to phase velocities, fluid theory does not yield a correct prediction for other mode properties either. For instance, the slow mode has a negative correlation between B and N, whereas S/SO can have a positively correlated B and N if $\beta_e \sim \beta_i$ (Table I). Recall that the density increase and magnetic field decrease across a slow shock is attributed to the fact that for the slow mode B and N are anticorrelated. Whether a slow shock can still be formed when B and N are correlated is not known at the present time.

TABLE I. Kinetic versus fluid theory of linear modes.

Mode		Polarization		Characteristic speed		B and N correlation		Compressibility	
Fluid	Kinetic	Fluid	Kinetic	Fluid	Kinetic	Fluid	Kinetic	Fluid	Kinetic
Slow	S/SO	L	~linear/R	0.9464	1.624	−	− (+ for $\beta_e\sim\beta_i$)	10^{-2}	$\sim 2\times10^2$
Intermediate	A/IC	R	L	0.9659	0.9657	−	+ (− for large θ)	10^{-3}	$\lesssim 10^{-4}$–10^{-3}
Fast	F/MS	~linear/R	R	1.6138	1.057	+	+	6.0	0.1

B. Anisotropic plasma

In this section we derive the linear mode properties in the long-wavelength limit for waves propagating in a homogeneous anisotropic plasma. We consider two general classes of anisotropic fluid theory and compare the results with kinetic theory. One class does not involve the magnetic field in the equations of state (simple polytropic model), whereas the second class does (the double polytropic model). Clearly, it would be extremely useful and desirable to have a fluid theory that can successfully predict the linear mode properties in a high β plasma. In that case, many problems associated with the existence and characteristics of discontinuities could be determined analytically. If, on the other hand, it turns out that the results show a large sensitivity to the free parameters (polytropic indices) involved, the fluid theories lose their predictive power.

Regardless of whether large-scale flows, discontinuities, or linear waves are considered, application of fluid theory requires closure of the set of moment equations. Two such relations are needed in the case of an anisotropic plasma. Recently, there has been much interest in describing the thermodynamic behavior of the magnetosheath. Both observational and simulation studies have been carried out that lead to empirical relations between the plasma anisotropy and the plasma β.[13,14] Such relations can be viewed as closure relations for magnetohydrodynamic theory. Typically, it has been found that the magnetosheath is neither adiabatic nor isothermal. These findings have led Hau *et al.*[15] to suggest a heuristical, generalized double-adiabatic closure of the form

$$\frac{d}{dt}\left(\frac{p_\perp}{\rho B^{\gamma_\perp - 1}}\right)=0,$$

$$\frac{d}{dt}\left(\frac{p_\parallel B^{\gamma_\parallel - 1}}{\rho^{\gamma_\parallel}}\right)=0, \qquad (1)$$

with polytropic indices $\gamma_\parallel$ and $\gamma_\perp$. Here, the limit $\gamma_\perp=2$ and $\gamma_\parallel=3$ corresponds to the CGL description, whereas $\gamma_\perp=\gamma_\parallel=1$ corresponds to the case of an isothermal plasma.

The properties of small-amplitude waves have been examined by several authors using the CGL[2–5] and double polytropic model.[6] In analogy with MHD, three modes referred to as slow, intermediate, and fast waves are identified. A fourth mode referred to as the entropy wave (mirror mode) is not of interest here and will not be discussed further. Although the properties of fast and intermediate modes remain qualitatively the same as in the isotropic case, the slow mode can, under certain conditions, have a different behavior. The three changes that can occur for the slow mode due to anisotropy are (1) its phase velocity can exceed the intermediate wave; (2) the density and magnetic field can be correlated; and (3) rarefraction waves rather than compression waves steepen. The phase velocities for the three low-frequency modes derived from (1) are given in Hau *et al.*[15]

One must keep in mind that the CGL limit is only expected to hold in very limited cases from the outset. When a finite heat flux exists and the magnetic moment is not conserved, one cannot necessarily expect that the form of the polytropic laws should in any way resemble the CGL equations of state. As an alternative approach, we consider theq simple polytropic model,

$$\frac{d}{dt}\left(\frac{p_\perp}{\rho^{\gamma_\perp}}\right)=0,$$

$$\frac{d}{dt}\left(\frac{p_\parallel}{\rho^{\gamma_\parallel}}\right)=0, \qquad (2)$$

which does not include the magnetic field perturbation. Of course, all such models are largely arbitrary and can find justification only through correct predictions. Here the idea is that the above two distinct approaches cover reasonable broad ground, such that meaningful and sufficiently general conclusions can be drawn when comparisons are made to the correct results from Vlasov theory.

It is straightforward to derive the phase velocities from Maxwell's equations, using the simple polytropic laws (2) in the anisotropic fluid equations (e.g., following Stringer's[16] prescription for the isotropic case). We assume that the electrons are isotropic and satisfy the polytropic law $(d/dt)\times(p_e/\rho^{\gamma_e}) = 0$. We introduce the sound velocities based on the ion mass,

$$C_k^2=\frac{\gamma_k T_k}{m_i}=\frac{\gamma_k}{2}\,v_A^2\beta_k, \qquad (3)$$

where the subscript k can assume the values $\perp$ and $\parallel$ for the ions, or e to indicate the electrons.

The intermediate mode phase speed then follows as

$$v_I^2=\cos^2\theta[v_A^2-(C_\parallel^2/\gamma_\parallel - C_\perp^2/\gamma_\perp)]. \qquad (4)$$

Note that this expression is similar to the known modification of the anisotropic intermediate mode speed, except that here only the ion β enters.

The slow (minus sign) and fast mode (plus sign) phase velocities are given by

$$v_{S,F}^2=\tfrac{1}{2}(A\pm\sqrt{A^2-4B}), \qquad (5)$$

with

$$A=v_A^2+C_\perp^2+C_e^2+\cos^2\theta\left[\left(\frac{\gamma_\parallel-1}{\gamma_\parallel}\right)C_\parallel^2-\left(\frac{\gamma_\perp-1}{\gamma_\perp}\right)C_\perp^2\right] \qquad (6)$$

and

$$B=v_A^2\cos^2\theta\{C_e^2+C_\parallel^2-(C_\parallel^2/\gamma_\parallel-C_\perp^2/\gamma_\perp)*[\cos^2\theta C_\parallel^2 - \sin^2\theta C_\perp^2+(\cos^2\theta-\sin^2\theta)C_e^2]\}. \qquad (7)$$

The last expression differs from the double-polytropic results[6] in the γ factors, multiplying the term $-\sin^2\theta\, C_\perp^2$. Otherwise (and except for our use of isotropic electrons) the results are identical.

We start with the low β case. For $\beta_i=0.2$, $\theta=60°$, and anisotropy $A\equiv T_\perp/T_\parallel=1.3$, the phase velocities for the kinetic S/SO, F/MS, and A/IC normalized to Alfvén speed are 0.506 07, 0.115 22, and 1.0826, respectively. Since the double-polytropic law of Hau and Sonnerup does not distinguish between ions and electrons, we assume $\beta_e=0$ in the kinetic theory for the remainder of this section. The phase

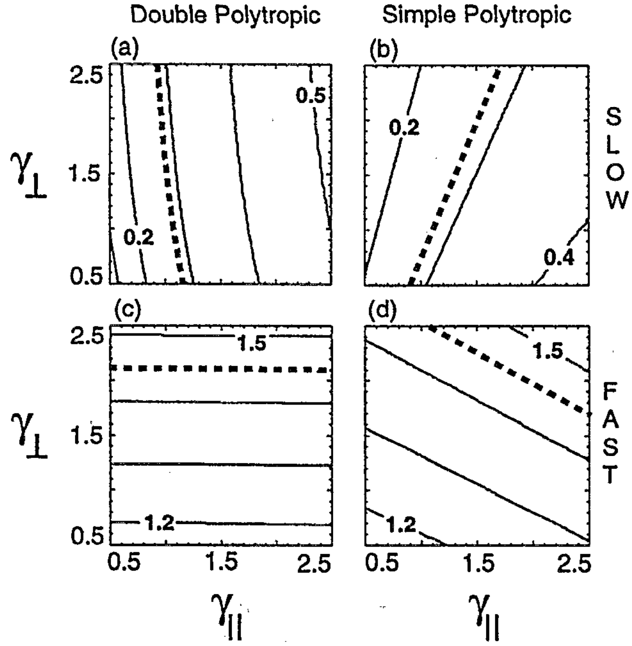

FIG. 2. Contours of the characteristic speeds for the slow and fast modes based on the double-polytropic and the simple-polytropic fluid models. The dashed contours correspond to the kinetic value of the phase velocity for each case. See the text for specific parameters.

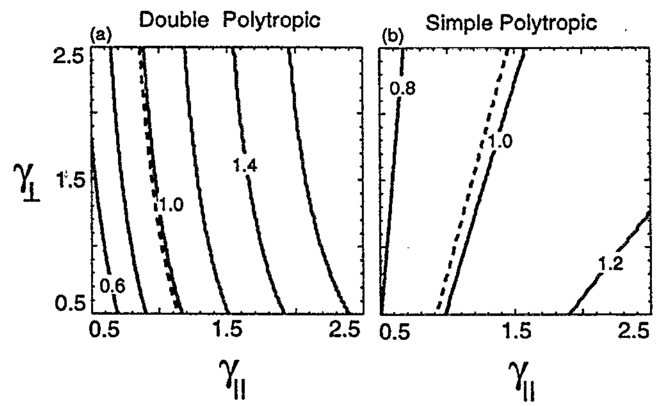

FIG. 3. Contours of the quantity V_S/V_I, where V_S and V_I are the characteristic speeds of the slow and intermediate modes, respectively. The dashed contours correspond to the kinetic value for each case.

velocity depends on $\gamma_\parallel$ and $\gamma_\perp$ in both of the fluid theories above; we consider changes in the phase velocity for γ's ranging from 0.5 to 2.5. The range in the resulting phase velocities of the fast mode is small and is 1.024–1.1 in the two fluid models. The combination of γ's that match the kinetic phase velocity of the F/MS in this case lies on a diagonal line (not shown) in the $\gamma_\parallel-\gamma_\perp$ plane, extending from $\gamma_\parallel\sim1.0$, $\gamma_\perp\sim2.5$ to $\gamma_\parallel\sim2.5$, $\gamma_\perp\sim1.83$ in the simple polytropic case, and it lies on a horizontal line with $\gamma_\perp\sim2.0$ (independent of $\gamma_\parallel$) in the double-polytropic case.

The fluid theory is less successful for the slow mode where the range in the slow mode speed in the simple and double-polytropic theories are 0.089–0.218 and 0.091–0.224, respectively. The fact that the phase velocity can change by more than a factor of 2 by changing the γ's is problematical, since the proper values of γ's are not known *a priori*. The combination of γ's that match the kinetic phase velocity of the S/SO in this case lies on a line (not shown) extending from $\gamma_\parallel\sim0.68$, $\gamma_\perp\sim0.5$ to $\gamma_\parallel\sim0.8$, $\gamma_\perp\sim2.5$ in the simple-polytropic case, and the line $\gamma_\parallel\sim0.73$ (independent of $\gamma_\perp$) in the double-polytropic case. Thus, neither the isothermal ($\gamma_\parallel=\gamma_\perp=1$) nor the usual CGL ($\gamma_\parallel=3$, $\gamma_\perp=2$) in the double-polytropic case are valid. The phase velocity of the intermediate mode is independent of the choice of $\gamma_\parallel$ and $\gamma_\perp$ and matches that of the A/IC in kinetic theory.

We now consider the phase velocities in the high β regime. Figure 2 shows contours of phase speed in the long-wavelength limit for both slow and fast modes as a function of $\gamma_\parallel$ and $\gamma_\perp$. The plasma parameters are $\beta_i=1.2$, $\theta=60°$, the ion plasma to gyrofrequency ratio $\omega_p/\Omega=4000$, and $T_{\perp_i}/T_{\parallel_i}=1.3$. Again, we have chosen the electron beta to be zero. The phase speeds are calculated based on the double-polytropic model [Figs. 2(a) and 2(c)] and our simple-polytropic model [Figs. 2(b) and 2(d)]. The contour corresponding to the phase velocity in the linear kinetic theory is indicated with a dashed curve in each case. Note that there are no γ's in the kinetic theory and these dashed contours merely show what combination of γ's would yield the same phase velocity in fluid theory as that obtained from kinetic theory.

In the double-polytropic model, the phase speed of the slow mode is nearly independent of $\gamma_\perp$ [Fig. 2(a)], whereas the phase speed of the fast mode shows no significant variation with $\gamma_\parallel$ [Fig. 2(c)]. Both the slow and fast modes show a significant range in their phase velocities as a function of γ's. This also holds in the simple-polytropic model [Figs. 2(b) and 2(d)]. The combination of γ's that yield the correct phase velocities for the slow and fast modes is $\gamma_\parallel\sim1.0$ and $\gamma_\perp\sim2.0$ in the double-polytropic model and $\gamma_\parallel\sim1.6$ and $\gamma_\perp\sim2.2$ in the simple-polytropic model. As in the low beta case, neither CGL nor the isothermal case yield the correct phase velocity.

We have also carried out comparisons between kinetic theory and the two fluid models for some other parameter regimes. We have found no unique combination of $\gamma_\parallel$ and $\gamma_\perp$ that works for all cases. Thus, the polytropic approach is not very useful.

In anisotropic fluid theories of the types discussed above, the slow phase velocity can become larger than the intermediate speed (the so-called pseudo- or reverse MHD regime). Figure 3 shows contours of the ratio of the phase speed of slow mode to that of the intermediate mode in the long wavelength as a function of $\gamma_\parallel$ and $\gamma_\perp$. The plasma parameters are $\beta_i=2.0$, $\beta_e=0$, $\theta=60°$, and $T_\perp/T_\parallel=0.8$. The contour corresponding to the ratio of the phase velocity of the S/SO to the A/IC based on linear kinetic theory is indicated with a dashed curve in each case. For the present case, the S/SO speed is slower than the A/IC and the ratio is given by 0.9782. In both fluid models, there exists a large range in $\gamma_\parallel$ and $\gamma_\perp$, where the slow mode has a higher velocity than the intermediate mode. But there also exist regions where the slow mode is slower than the intermediate

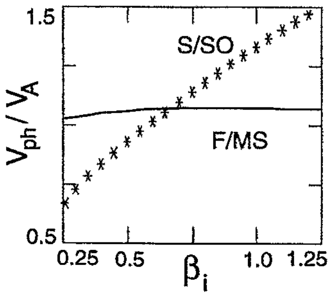

FIG. 4. Phase velocity of the S/SO and F/MS modes as a function of plasma beta.

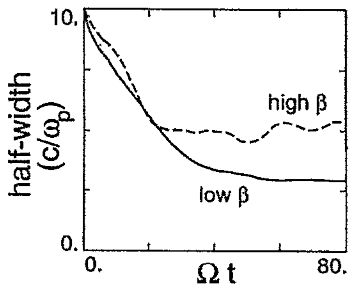

FIG. 5. Plot of shock thickness as a function of time for (a) $\beta_i=0.5$ and (b) $\beta_i=1.0$.

mode. Note also that the range in the ratio V_S/V_I is large for both below and above unity. Although fluid theory's prediction that the slow speed can be larger than the intermediate speed also occurs in the kinetic theory, the conditions for reversal to occur are not given correctly by fluid theory and are impossible to predict without prior knowledge of appropriate γ's. In addition, velocity reversals occur in the kinetic theory even in the isotropic case and in a more general sense than in fluid theory. For instance, in kinetic theory any combination of phase velocities for the three modes is possible, whereas in fluid theory the only reversal is for the slow mode to become faster than the intermediate mode, but it would still be slower than the fast mode.

III. IMPLICATIONS FOR SHOCKS

As we demonstrated in the previous section, the ordering of the kinetic phase velocities of the three modes can change both in the isotropic and anisotropic plasmas. An example, the S/SO mode can have a phase velocity that is larger than the F/MS mode. One of the important implications of this phase velocity reversal is in regards to structure of fast quasiparallel shocks. In the low β regime, the fast shock is associated with the F/MS mode and a steady (low Mach number) quasiparallel shock has an associated upstream phase standing whistler wave train. This is explained by the fact that a fast shock is formed as a result of the steepening of the F/MS mode that connects to the whistler branch. In the high beta regime, however, the S/SO can have a phase velocity larger than the F/MS mode (e.g., Table I). Since a fast shock is associated with the fastest mode, the fast shock would be associated with the S/SO in such cases. However, S/SO does not connect to the whistler branch and thus the fast shock cannot have a phase standing whistler wave train when S/SO is the fastest mode.

To test this theoretical expectation, we have performed hybrid simulations (fluid electron, kinetic ions) for the case where F/MS has a larger phase velocity than S/SO and for the case where S/SO is the faster mode. We have verified using linear theory that for $T_e=T_i$, and θ=20°, the F/MS has a larger phase velocity than the S/SO mode for $\beta_i \lesssim 0.7$, and a smaller phase velocity for $\beta_i > 0.7$. This is shown in Fig. 4, where we have plotted the phase velocity of the F/MS and S/SO modes as a function of ion β.

Using the same parameters as in Fig. 4, we have made two runs: one with $\beta_i=0.5$ and another with $\beta_i=1.0$. In order to keep the effect of backstreaming ions to a minimum, as well as to assure that the shock strength is the same in both cases, we have chosen the magnetosonic Mach number (M_F) to be low and the same in both cases, $M_F=1.24$. The Alfvénic Mach numbers are 1.38 and 1.7 for the two runs. In each case, the shock is initialized based on Rankine–Hugoniot relations and with a finite thickness. Given the difficulties associated with the fluid theory of simple linear waves outlined in the above section, our use of a fluid based magnetosonic Mach number as well as RH relations to set up the shocks may at first seem puzzling. But the RH relations, which are simple conservation laws, generally work much better than fluid theory of linear waves (see also the discussion in Karimabadi *et al.*). The reason for this can be seen as follows. Let us consider the high β case where the fluid theory of the three modes is clearly not valid. We have verified that the phase velocity of the (fluid) fast mode is almost identical to that of S/SO mode at $k=0$ (e.g., Fig. 1). Thus, one can interchangeably define the Mach number based on either the fast mode or the S/SO mode. Furthermore, the S/SO mode has a positive correlation between the magnetic field and density, which is what is required for a fast shock. Since the Mach number and the correlation of magnetic field and density are what is important in determining the jump conditions, the RH relations remain valid, although in fluid theory the shock is associated with the fast mode and in kinetic theory with the S/SO mode. Simply said, the importance of the association of the shock with S/SO rather than the fast mode comes into the structure of the shock and at finite wavelength, but it does not affect the jump conditions. It may, however, occur that the S/SO mode has an anticorrelated magnetic field and density oscillations. Whether a fast shock can be formed in such cases is not known and is beyond the scope of this paper.

We have made several runs with different initial shock thicknesses. The initial transitory evolution of the shock consists of steepening or spreading of its thickness, depending on whether the initial thickness was set larger or smaller than its final asymptotic value. After this transitory stage, the shock thickness does not change significantly. Figure 5 shows the shock half-width as a function of time for the case where the low and high β shocks were initialized with a

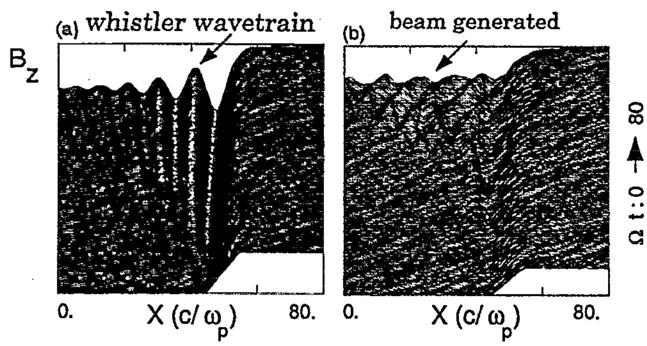

FIG. 6. Hybrid simulations of two quasiparallel fast shocks having the same parameters, except for the ion beta. (a) $\beta_i=0.5$ and (b) $\beta_i=1.0$.

half-thickness of $10c/\omega_p$. Both shocks evolve in time, steepening and reaching a fixed thickness after some time. As is evident in Fig. 5, the shock thickness is considerably larger in the high β case. In order to gain more insight into the change in the shock thickness as a function of β, we made new runs, starting with a shock thickness of $5c/\omega_p$. Stack plots (of B_z) are shown in Fig. 6. Here we started with an initial shock half-width of $5c/\omega_p$. In the low β case, the shock steepens until it reaches a thickness matching that in Fig. 5. There is no significant change in the shock thickness in the high β case, since it was initialized with a thickness close to its asymptotic value given in Fig. 5. The low β shock has an upstream phase standing whistler wave train [Fig. 6(a)], and the shock thickness is determined by dispersion. However, the dispersive whistler wave train is conspicuously absent as the ion beta is increased to 1.0 [Fig. 6(b)], and the shock thickness is determined by dissipation in this case. At late times in the simulation there are some whistlers generated upstream of the shock. These waves are generated due to the relative streaming between ions backstreaming from the shock and the incoming plasma. Artificial removal of the backstreaming ions eliminates these whistlers. The presence of whistlers, although beam generated, points to the fact that the absence of a whistler wave train is not simply due to damping effects. From linear theory it also follows that the damping is not strong enough to exclude a wave train.

Our explanation of the above results is that in the high beta case, the shock is associated with the S/SO mode, which does not connect to the whistler branch. The shock thickness is then given by dissipation rather than dispersion, yielding a different shock thickness. Another possible interpretation would be that the shock is still associated with the F/MS mode, but there may exist some scaling of shock thickness with β that also prevents a wave train from forming. We have ruled out the latter possibility by performing a third simulation for the same parameters as in Fig. 6(b), except that $\theta=60°$. The F/MS mode is then faster than S/SO mode, even for $\beta_i=1.0$. The shock showed a thickness similar to that in Fig. 6(a), and has a dispersive whistler wave train (not shown). Thus, we attribute the lack of a dispersive whistler wave train in Fig. 6(b) to the fact that the shock is indeed associated with the S/SO mode. Another fact supporting this conclusion comes from examination of the heating across the shock (not shown). In the low β case, the heating occurs almost equally in both directions with respect to the magnetic field, whereas in the high β case the heating is predominantly in the direction parallel to the field. Parallel heating with little perpendicular heating is consistent with a shock that is associated with the (highly electrostatic) S/SO mode.

From the foregoing we conclude that not only can fluid theory not be used to explain the types of waves that phase or group stand at shocks, it also does not provide an accurate framework for classification of shocks. Although we have chosen large beta for our comparison, it is quite relevant. For instance, the observed slow shocks in the magnetotail[17–20] and the newly discovered slow shock at the magnetopause[21] have small ion beta upstream but high ion beta downstream. Given the above reversal of phase velocities, it may become possible for a RD to stand downstream of a slow shock. Such a situation was observed at the magnetopause.[21]

IV. DISCUSSION AND SUMMARY

The linear properties of low-frequency waves are widely used in studies and identification of various discontinuities in space physics. In the low beta regime, fluid theory provides a good approximation to kinetic theory and has been used extensively in theory of shocks. Low beta plasmas admit two simplifications. One is that the modes can be ordered according to their phase velocities; e.g., the fast mode is faster than the slow and intermediate modes at all angles and independent of T_e/T_i as long as the total plasma beta is small. The other is that mode properties such as correlation between density and magnetic field remain intact as a function of propagation angle, etc. This facilitates a simple classification scheme of discontinuities and jumps across discontinuities can be related to linear modes. For instance, the fact that density and magnetic field always increase across a fast shock can be explained by the positive correlation of the density and magnetic field for the fast mode. The above scheme has worked so well that its applicability in high beta plasmas if often not questioned.

Here, we reconsider the relation between linear modes and shocks in high beta plasmas using both fluid and kinetic theory. Since fluid theory has the advantage that it yields simple analytical expressions for linear modes we first considered its validity in high beta ($\gtrsim 0.5-1.0$) and isotropic limit. We found that fluid theories of linear modes (both MHD and Hall MHD) do not yield the correct mode properties, even in the long-wavelength limit. Using linear kinetic theory, we demonstrated two complications that are not present in the low beta regime.

First, modes can no longer be ordered according to their phase velocities. We studied in detail the example that at small and moderate propagation angles, the S/SO mode can have the fastest phase velocity. In such cases, a (quasiparallel) fast shock should then be associated with the S/SO mode rather than the usual F/MS mode. This has important implications for fast shocks. Since it is the F/MS rather than the S/SO mode that connects to the whistler branch, low Mach number quasiparallel shocks associated with S/SO would not be expected to have a phase standing whistler wave train upstream and their thickness is determined by dissipation rather than dispersion. Moreover, the shock is not expected to have a S/SO wave train because the S/SO mode does not have the proper phase and group velocities and also it is heavily damped. We verified this expectation through kinetic (hybrid) simulations. Further, S/SO can become heavily damped at high beta, and thus the existence and structure of quasiparallel shocks in high beta plasmas warrants further study. At large propagation angles, F/MS remains the fastest mode and thus the above difficulty does not apply to quasiperpendicular shocks.

Another complication in a high beta plasma is that the linear properties of a given mode can change with propagation angle and ratio T_e/T_i. For instance, the S/SO mode can possess both a positive or negative correlation of density and magnetic field, depending on the parameters (see Table I). The implication of this behavior for the relation of linear properties to jumps across the shock remains to be explored.

We also examined the validity of two general class of fluid theories in anisotropic plasmas. As in the isotropic case, fluid theory becomes inaccurate in the high beta regime and questions regarding the steepening of the mode, and relation of mode properties to shocks, have to be explored within the context of kinetic theory. In regards to the breakdown of fluid theory, the ion beta matters more than electron beta since for low-frequency waves, electrons act approximately as a fluid, whereas ion kinetic effects play a significant role. Anisotropic fluid theory can, however, be useful in the calculation of characteristic speeds with the issue of its applicability dictated mainly by the level of accuracy required in a given application. In general, fluid theory provides accurate predictions for the intermediate speed, less accurate result for the fast speed and yet less accurate result for the slow mode (e.g., see Fig. 2). Since we have found no unique combination of $\gamma_\parallel$ and $\gamma_\perp$ that yields the correct value of slow and fast characteristic speeds for all parameters, whether one can use the fluid speeds depends on the size of error or uncertainty one can tolerate in a given application. For instance, if one is interested in identifying a certain discontinuity in the data and a factor of 2 uncertainty in the slow speed does not change the identification of the discontinuity, fluid theory can be used. As demonstrated in Fig. 2, the kinetic values of slow and fast speeds usually fall within the range of speeds predicted by fluid theory when varying $\gamma_\parallel$ and $\gamma_\perp$ between 0.5 and 2.5. On the other hand, if one is interested in knowing whether the slow speed is larger or smaller than the intermediate speed and this ratio changes from a value smaller to a value larger than unity for a reasonable range in $\gamma_\parallel$ and $\gamma_\perp$ (see Fig. 3), then fluid theory obviously cannot be used.

In short, our results show a strong need to continue studies of discontinuities in high beta plasma in general, and their classification as well as relation of linear waves to shocks in particular. The present work has raised and addressed some of the important issues involved. We hope that this work will inspire more research in this area.

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