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Ponderomotive Acceleration of Ions by Circularly Polarized Electromagnetic Waves

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Abstract. The ponderomotive acceleration of multi-species ions due to circularly polarized electromagnetic waves propagating parallel to the magnetic field is calculated. When applied to a low frequency left-hand circularly polarized wave, the results show differential accelerations among the different ion species. We demonstrate that differential acceleration of ion species by the ponderomotive force can occur even in the absence of a parallel electric field component. Our work extends the results previously obtained by *Li and Temerin* (1993) who considered the ponderomotive acceleration due to obliquely propagating Alfvén waves.

Introduction

It is becoming increasingly evident that the ponderomotive force resulting from large amplitude Alfvén waves is important for understanding solar wind and magnetospheric phenomena. For example, *Hollweg* (1971) has shown that Alfvén waves that are frequently observed in the solar wind can drive density fluctuations. More recently, *Allen et al.* (1990); *Allen* (1992); and *Guglielmi et al.* (1993; 1995) have shown that the geomagnetic pulsations in the Pc 1–5 frequencies observable from the ground (*Jacobs and Watanabe, 1964*; in the ionosphere (*Erlandson et al., 1990*) and the magnetosphere (*Bossen et al., 1976*) can significantly modify the plasma distributions in the magnetosphere.

Observations of large-amplitude Alfvén waves at rocket altitudes (*Bohm et al., 1990*) and of the escaping ionospheric ions that are frequently accompanied by low frequency, large-amplitude electric field fluctuations (*Lundin et al., 1990*) motivated *Li and Temerin (1993)* to calculate the ponderomotive force resulting from linearly polarized Alfvén waves. *Li and Temerin (1993)* show that this nonlinear force can produce a mass dependent ion acceleration which can effectively accelerate the ionospheric O^+ ions in the direction of the magnetosphere. This work provides an important first step toward understanding the long sought mechanism for energizing the ionospheric O^+ ions that are observed in the ring current and the geomagnetic tail (*Gloeckler et al., 1985*; *Lennartsson and Shelley, 1986*;).

Guglielmi et al., (1993; 1995) studied a static problem, and invoking a balance of force (no acceleration) calculated the ponderomotive effects on cold plasma resulting from the Pc waves assuming they are circularly polarized ion cyclotron waves. The main result they obtained is that the ponderomotive force will push the plasma toward the equator. On the other hand *Li and Temerin (1993)* dealt with a dynamical problem and studied the effects of ion acceleration on a cold, homogeneous and stationary fluid. The main purpose of this paper is to extend the theory of mass dependent ion acceleration by calculating the ponderomotive force of circularly polarized ion cyclotron waves on a warm, homogeneous and stationary two-fluid plasma. Our calculation ignores the pressure and gravitational effects.

The general expression of the ponderomotive force caused by arbitrary electromagnetic fields has not yet been calculated. Several authors have obtained expressions for special cases. For example, *Klima and Petrzilka* (1978) and *Karpman and Shagalov* (1982) have derived expressions that are based on the two-fluid description of a stationary cold plasma with no spatial dispersion. *Lee and Parks* (1983; 1988) calculated less restrictive ponderomotive force expressions for a warm, nonstationary, inhomogeneous two-fluid plasma.

The Ponderomotive Force of a Magnetized Plasma

For the special case of homogeneous and stationary plasma, the ponderomotive force density can be cast into a relatively simple form, (see for example *Lee and Parks,* 1983)

$$f_j = \frac{1}{16\pi}\frac{\partial}{\partial x_j}\{(\epsilon_{\beta\alpha} - \delta_{\beta\alpha})E_\alpha E^*_\beta\} + \frac{1}{16\pi}\frac{\partial}{\partial x_l}(\varepsilon_{jmp}\varepsilon_{klm}\Omega_p\frac{\partial\epsilon_{\beta\alpha}}{\partial\Omega_k}E_\alpha E^*_\beta) \qquad (1)$$

where $\epsilon_{\beta\alpha}$ is the dielectric tensor of a warm magnetized plasma

$$\epsilon_{\beta\alpha} = \delta_{\beta\alpha} - \frac{\omega_p^2}{\omega^2 - \Omega^2 - k^2s^2 + \frac{s^2}{\omega^2}(\mathbf{k}\cdot\mathbf{\Omega})^2}$$
$$\times[\delta_{\beta\alpha}(1 - k^2s^2/\omega^2) + k_\alpha k_\beta s^2/\omega^2$$

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$$+\frac{i}{\omega}\varepsilon_{\beta\alpha\gamma}\{\Omega_\gamma - k_\gamma(\mathbf{k}\cdot\mathbf{\Omega})s^2/\omega^2\} - \frac{\Omega_\alpha\Omega_\beta}{\omega^2}] \quad (2)$$

$\mathbf{\Omega}$ and ω_p represent the cyclotron and plasma frequency of each plasma species, respectively, $s = \sqrt{k_B T/m}$ denotes the thermal velocity of the plasma (since the formal expressions for the electrons and ions are identical, symbols signifying the species of the plasma have been omitted), and ε_{jmp} and ε_{klm} represent the Levi-Civita symbols. Define a quantity Φ through the relation $\Phi = -\frac{1}{16\pi}\{(\epsilon_{\beta\alpha} - \delta_{\beta\alpha})E_\alpha E^*_\beta\}$ which, written explicitly using Eq. 2, is

$$\Phi = \frac{1}{16\pi}\frac{\omega_p^2}{\omega^2 - \Omega^2 - k^2 s^2 + \frac{s^2}{\omega^2}(\mathbf{k}\cdot\mathbf{\Omega})^2}$$

$$\times[(1 - k^2s^2/\omega^2)\mathbf{E}\cdot\mathbf{E}^* + \frac{s^2}{\omega^2}(\mathbf{E}\cdot\mathbf{k})(\mathbf{E}^*\cdot\mathbf{k})$$

$$+\frac{i}{\omega}\{\mathbf{\Omega} - \mathbf{k}(\mathbf{k}\cdot\Omega)s^2/\omega^2\}\cdot(\mathbf{E}^*\times\mathbf{E}) - \frac{1}{\omega^2}(\mathbf{E}\cdot\mathbf{\Omega})(\mathbf{E}^*\cdot\mathbf{\Omega})] \quad (3)$$

Eq. (1) can then be written in the vector form as $\mathbf{f} = -\nabla\Phi + (\nabla\times\mathbf{M})\times\mathbf{B}_0$. Here, $\mathbf{M} \equiv \nabla_{\mathbf{B_0}}\Phi$, where $\nabla_{\mathbf{B_0}}$ denotes that the derivatives are to be taken with respect to the ambient magnetic field $\mathbf{B_0}$. The first term represents the force which is derivable from the 'ponderomotive potential' Φ, and the second term may be interpreted as the Lorentz force $\frac{1}{c}\mathbf{J}\times\mathbf{B}_0$ acting on the magnetization current density $\mathbf{J} = c\nabla\times\mathbf{M}$. This is the general expression of the ponderomotive force for a warm homogeneous stationary plasma in which the effects of magnetization are taken into account. The dependence of magnetization manifests itself not only by $\mathbf{\Omega}$ (or $\mathbf{B}_0$) through Φ, but also by the existence of the 'Lorentz force' $(\nabla\times\mathbf{M})\times\mathbf{B}_0$.

Ponderomotive Force due to a Circularly Polarized Wave

Consider now the ponderomotive force for left-hand circularly polarized electromagnetic waves propagating along the magnetic field. No specific assumption will be made on the nature of the wave except of its polarization. The wave here can be for example a low frequency ($\omega < \Omega_i$) ion cyclotron wave. Assume that $\mathbf{B}_0$ is along the z–direction. We can thus write the electric field as $\mathbf{E} = \frac{1}{\sqrt{2}}E(\mathbf{e}_x - i\mathbf{e}_y)$ where $\mathbf{e}_x$ and $\mathbf{e}_y$ represent the unit vector in x– and y– directions, respectively. Using $\mathbf{E}^*\times\mathbf{E} = -i|E|^2\mathbf{e}_z$ and $\mathbf{\Omega}\cdot\mathbf{E} = 0$, we obtain

$$\Phi = \frac{1}{16\pi}\frac{\omega_p^2}{\omega(\omega - \Omega)}|E|^2 \quad (4)$$

$$\mathbf{M} = \frac{e}{mc}\frac{1}{16\pi}\frac{\omega_p^2}{\omega(\omega-\Omega)^2}|E|^2\mathbf{e}_z \quad (5)$$

where $\mathbf{e}_z$ is a unit vector in the z–direction.

It is interesting to note that the effect of finite temperature in Φ and $\mathbf{M}$ disappears for the case of circularly polarized electromagnetic waves propagating along the magnetic field. The expression is the same as for the case of cold plasma since the temperature does not affect the dispersion relation of the waves propagating parallel to the magnetic field. Using Eqs. (4) and (5), the ponderomotive force density can be written as

$$\mathbf{f} = -\frac{1}{16\pi}\frac{\omega_p^2}{\omega(\omega-\Omega)}\nabla|E|^2 - \frac{1}{16\pi}\frac{\Omega\omega_p^2}{\omega(\omega-\Omega)^2}\nabla_\perp|E|^2 \quad (6)$$

$\nabla_\perp = \nabla - \nabla_\parallel = \nabla - \mathbf{e}_z(\mathbf{e}_z\cdot\nabla)$. Decomposing the first term into longitudinal and transverse components Eq. (6) becomes, for electrons and ions, respectively,

$$\mathbf{f}_e = -\frac{1}{16\pi}\frac{\omega_{pe}^2}{\omega(\omega+\Omega_e)}\nabla_\parallel|E|^2 - \frac{1}{16\pi}\frac{\omega_{pe}^2}{(\omega+\Omega_e)^2}\nabla_\perp|E|^2 \quad (7)$$

$$\mathbf{f}_i = -\frac{1}{16\pi}\frac{\omega_{pi}^2}{\omega(\omega-\Omega_i)}\nabla_\parallel|E|^2 - \frac{1}{16\pi}\frac{\omega_{pi}^2}{(\omega-\Omega_i)^2}\nabla_\perp|E|^2 \quad (8)$$

From Eq. (8), we can see that the ponderomotive force on ions can be very large as the wave frequency approaches the ion gyrofrequency. Note that this ion resonance occurs because the wave field used has left-hand polarization. If a right-hand circularly polarized wave were used, we would obtain the same expression as Eqs. 7 and 8, except the (+) and (-) signs in the denominator are reversed. Hence resonance with electrons is also possible for waves with frequency close to the electron gyrofrequency.

Let us now consider a plasma consisting of electrons and several species of singly charged ions. The charge neutrality condition requires $N_e = \Sigma_j N_j$ for the particle densities, where the subscript j denotes each ion species.

The equation of large scale motion for particles traveling along a field line is $ma_\parallel = qE_L + f_\parallel/N - ms^2\nabla_\parallel N/N$ where $\mathbf{E}_L$ is the ambipolar electric field created by the redistribution of charged particles in response to non-electrostatic forces. Because of their small mass, electrons reach an equilibrium in a short time scale. Thus, for electrons, $0 = -eE_L + f_{e\parallel}/N_e$ where we have ignored the pressure term $-ms^2\nabla_\parallel N/N$. Then the equation of motion for each ion species is

$$m_j a_{j\parallel} = f_{e\parallel}/N_e + f_{j\parallel}/N_j \quad (9)$$

Using Eqs. (7) and (8), Eqs. (9) reduces to

$$m_j a_{j\parallel} = -\frac{1}{16\pi}\{\frac{\omega_{pe}^2}{N_e\omega(\omega+\Omega_e)} + \frac{\omega_{pj}^2}{N_j\omega(\omega-\Omega_j)}\}\nabla_\parallel|E|^2 \quad (10)$$

Rewrite the terms in the curly brackets as

$$\frac{1}{\omega}(\frac{\omega_{pj}^2}{N_j\Omega_j} - \frac{\omega_{pe}^2}{N_e\Omega_e}) + \frac{\omega_{pe}^2}{N_e\Omega_e}\frac{1}{\Omega_e+\omega} + \frac{\omega_{pj}^2}{N_j\Omega_j}\frac{1}{\Omega_j-\omega} \quad (11)$$

Then, noting that $\frac{\omega_{pj}^2}{N_j\Omega_j} = \frac{\omega_{pe}^2}{N_e\Omega_e} = \frac{4\pi ec}{B_0}$ for each ion species, Eq. (10) can be simplified to

$$m_j a_{j\parallel} = \frac{1}{16\pi}\frac{\omega_{pj}^2}{N_j\Omega_j}(\frac{1}{\Omega_j-\omega} + \frac{1}{\Omega_e+\omega})\nabla_\parallel|E|^2 \quad (12)$$

So far, no approximation has been made, except the pressure term was neglected in the equation of motion. Thus, Eq. (12) is the general equation for large scale motions of ions acted upon by the ponderomotive force of left-hand circularly polarized electromagnetic waves propagating parallel to the background magnetic field.

Now consider waves with $\omega < \Omega_j$. Then, we can approximate $\frac{1}{\Omega_j-\omega}+\frac{1}{\Omega_e+\omega}\approx\frac{1}{\Omega_j-\omega}$. Using this approximation, Eq. (12) simplifies to

$$a_{j\parallel}=\frac{c^2}{4B_0^2}\frac{1}{1-\frac{\omega}{\Omega_j}}\nabla_\parallel|E|^2 \quad (13)$$

We can see that, since $\frac{\omega}{\Omega_j}=\frac{c\omega}{eB_0}m_j$, the acceleration depends on the mass of ions. We further note that the magnitude of the term $\frac{1}{1-\frac{\omega}{\Omega_j}}$ is a monotonically increasing function of both ω and m_j. Thus, ions with larger mass have larger acceleration into the region of higher wave fields and, for a given mass ratio between ions, waves with higher frequency are more effective in differentially accelerating ions than the ones with low frequency. For a wave with very low frequency, we can further approximate $\frac{1}{1-\frac{\omega}{\Omega_j}}\approx 1+\frac{\omega}{\Omega_j}$ and obtain

$$a_{j\parallel}=\frac{c^2}{4B_0^2}\frac{1}{1-\frac{\omega}{\Omega_j}}\nabla_\parallel|E|^2=\frac{c^2}{4B_0^2}(1+\frac{\omega}{\Omega_j})\nabla_\parallel|E|^2, \quad (14)$$

which is linear in mass of ions. For right-hand polarized low frequency waves, we obtain

$$a_{j\parallel}=\frac{c^2}{4B_0^2}(1-\frac{\omega}{\Omega_j})\nabla_\parallel|E|^2 \quad (15)$$

The acceleration for this case is smaller than for the left-hand circularly polarized waves. These results show the left-hand circularly polarized low frequency waves are more effective than the right-hand circularly polarized waves in accelerating ions.

The expressions for the ponderomotive force suggest that we can also have acceleration in the perpendicular directions, if the magnitude of the wave field has a gradient across the field lines. In fact, if the variations of the wave fields across the field line is comparable to that along the field line, the ratio of transverse force to the field aligned force on ions is $\frac{\omega}{\omega-\Omega_j}$. This can become quite large as the frequency approaches the ion gyrofrequency. This transverse force may play an important role in the transport of particles across the field line (see *Allan*, 1992). For low frequencies, we obtain the transverse acceleration to be

$$a_{j\perp}=\frac{c^2}{4B_0^2}(1+2\frac{\omega}{\Omega_j})\nabla_\perp|E|^2 \quad (16)$$

by applying a similar approach used to obtain Eq. (14). Eq. (16) indicates ions are accelerated differentially in the perpendicular direction as well.

Discussion

We note first that the structure of the ponderomotive force of circularly polarized electromagnetic waves given by Eqs. (7) and (8) is different from the equation used by *Li and Temerin* (1993). They used

$$\mathbf{F}=\frac{e^2}{4m}\nabla(\frac{1}{\Omega^2-\omega^2}E_\perp^2-\frac{1}{\omega^2}E_\parallel^2) \quad (17)$$

for the ponderomotive force of obliquely propagating Alfvén waves in a cold magnetized plasma. This equation can be obtained from Eq. (3) when $\mathbf{E}^*\times\mathbf{E}=0$ and $s^2=0$. (Note that $\mathbf{f}$ is the force density, while $\mathbf{F}$ of Eq. (17) is the force per particle). This can be realized if the polarization of the wave field is linear, which is the case for the obliquely propagating waves considered by *Li and Temerin* (1993). However, at higher frequencies, the Hall effect becomes important and the waves are generally elliptically polarized and the term $\mathbf{E}^*\times\mathbf{E}$ contributes to the ponderomotive force also. Eq. (17) is valid when the effect of the magnetization is small so that we can neglect the 'Lorentz force' $(\nabla\times\mathbf{M})\times\mathbf{B}_0$.

Eq. (17) can be written under the conditions to be discussed below as

$$a_{j\parallel}=\frac{e^2}{4m_j^2\Omega_j^2}\nabla|E_\perp|^2-\frac{e^2}{4m_em_j\omega^2}\nabla|E_\parallel|^2. \quad (18)$$

The first term in Eq. (18) is equal to $\frac{c^2}{4B_0^2}\nabla|E_\perp|^2$ and is denoted as a_p in *Li and Temerin* (1993). The second term is represented as $-|F_{pe}|/m_j$ and this term is responsible for the differential acceleration. Since F_{pe} is proportional to $E_\parallel^2$, the existence of $E_\parallel$, hence the obliquely propagating Alfvén wave, is essential for the mechanism of differential acceleration proposed by *Li and Temerin* (1993). *Li and Temerin* (1993) further indicated that (see their Eq. (6))

$$\frac{\omega}{\sqrt{\Omega_e^2-\omega^2}}<\frac{E_\parallel}{E_\perp}<\frac{\omega}{\sqrt{\Omega_i^2-\omega^2}}. \quad (19)$$

To see where this comes from, use Eq. (17) and write in terms of $\mathbf{f}/N$ and rewrite the ion equation of motion Eq. (9) similar in form to Eq. (12). We then obtain,

$$m_ja_{j\parallel}=\frac{e^2}{4m_j}\nabla(\frac{1}{\Omega_j^2-\omega^2}|E_\perp|^2-\frac{1}{\omega^2}|E_\parallel|^2)+\frac{e^2}{4m_e}\nabla(\frac{1}{\Omega_e^2-\omega^2}|E_\perp|^2-\frac{1}{\omega^2}|E_\parallel|^2) \quad (20)$$

Eq. (19) is obtained by requiring the first term of Eq. (20) (which originates from the ponderomotive force on ions) to be positive, and the second term (the ponderomotive force on electrons), to be negative. However, to arrive at Eq. (18) from Eq. (20), the first term dominates the second term in the first line while the second term is much larger than the first in the second line. Thus, the inequality sign in Eq. (19) should be ($\ll$).

Second, as $E_{\parallel}$ and $E_{\perp}$ of the obliquely propagating Alfvén waves are given by

$$E_{\parallel}\ /E_{\perp} = \frac{\omega}{\omega_{pe}}\epsilon^{1/2}(1+\frac{\omega_{pe}^2}{k_{\perp}^2 c^2})^{-1/2}$$

where

$$\epsilon = 1 + \frac{\omega_{pe}^2}{\Omega_e^2-\omega^2} + \Sigma_j \frac{\omega_{pj}^2}{\Omega_j^2-\omega^2},$$

we can thus rewrite Eq. (18) in terms of $E_{\perp}^2$. If the condition $1 \ll \omega_{pe}/\Omega_e \ll \omega_{pj}/\Omega_j$ is satisfied, we can approximate $\epsilon = \omega_p M^2/\Omega_M^2(1+\alpha)$, where the subscript M refers to the ion species with the largest mass. α is defined by $\alpha = \sum_j' N_j m_j/N_M m_M$ where the summation is over all ion species except for the heaviest species. The acceleration given by Eq. (18) then becomes

$$a_{j\parallel} = \frac{e^2}{4m_j^2\Omega_j^2}[1-(1+\alpha)\frac{N_M m_M}{N_e m_j}\frac{1}{1+\omega_{pe}^2/k_{\perp}^2 c^2}]\nabla|E_{\perp}|^2$$

Thus, for $a_{j\parallel}$ to be positive, that is for ions to be accelerated in the direction of the gradient of the wave field, the condition

$$1+\omega_{pe}^2/k_{\perp}^2 c^2 > (1+\alpha)\frac{N_M m_M}{N_e m_j} \qquad (21)$$

is required. For a plasma consisting of O^+ and H^+ with approximately equal concentrations, α is nearly equal to 1/16. Thus, O^+ is always accelerated in the upward direction since Eq. (21) is trivially satisfied for $m_j = m_M$. However, for H^+ to be accelerated in the upward direction, the condition $\omega_{pe}^2/k_{\perp}^2 c^2 > 7.5$ must be satisfied.

In conclusion, note that ponderomotive calculation of *Li and Temerin* (1993) requires a small ($E_{\parallel} \ll E$) but finite $E_{\parallel}(E_{\parallel} \neq 0)$. This can be realized in the case of obliquely propagating Alfvén waves. These requirements are not needed for the circularly polarized waves propagating parallel to the magnetic field. Differential acceleration can occur even if $E_{\parallel} = 0$ when applied to low frequency circularly polarized waves. Finally, note the difference between Eq. (13) and Eq. (18). $\frac{c^2}{4B_0^2}\nabla_{\parallel}|E|^2$ is the lower-bound of the ponderomotive acceleration in Eq. (13), while it is the maximum acceleration predicted by Eq. (18). Thus, the ponderomotive force of circularly polarized ion cyclotron waves can give larger acceleration than the obliquely propagating Alfvén waves when the electric fields are of the same magnitude and the frequency is very low. The theory in this paper may also be applied to the solar wind. We are studying if our theory can account for the positive correlation that has been observed between the Helium density and the solar wind velocity (*Hirshberg et al.*, 1972).

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