THE DYNAMICS OF MAGNETIC FLUX TUBES IN THE SOLAR CONVECTION ZONE

A Study of Active Region Formation

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1. Abstract

We describe some of our recent work on the dynamics of flux tubes in the solar convection zone. We focus on two topics, the first being the orientation ("tilt") of active regions, and how comparisons between observations of tilt and tilts computed from numerical simulations of flux tubes can be used to infer properties of magnetic fields deep in the solar interior. The second topic is an investigation of the kink instability of twisted flux tubes in the solar interior, and its possible relationship with flare-productive δ spot regions.

2. Introduction

Much of our research has focused on studying how magnetic flux emerges from the base of the solar convection zone, where the solar cycle dynamo is believed to operate, to the surface of the sun, where the effects of magnetic activity are observed. Our approach has been to develop, refine and study theoretical models for the motion of magnetic flux tubes. Such models are strongly motivated by detailed observations of magnetic flux emergence, which indicate that flux emerges in the form of discrete tubes surrounded by field-free gas. These models have been very successful in explaining many observed properties of active regions. Here, we review a few of our recent accomplishments.

3. Description of the Thin Flux Tube Numerical Model

A model for the dynamics of an isolated magnetic flux fiber, termed a *thin flux tube*, was proposed by Spruit (1981). The tube is "thin" in that all of its properties vary along its axis over scales much larger than the diameter of its cross section. This fact can be used to greatly simplify the equations of compressible magnetohydrodynamics (MHD). In the simplified equations, the tube itself is modeled as a three dimensional space-curve (its axis) moving within the interior of a star. Mass density, temperature, pressure, and magnetic field strength all vary along the tube's length and evolve according to dynamical equations. The axis itself moves under the actions of physical forces including: magnetic buoyancy, magnetic tension, aerodynamic drag (coupling the tube to the external plasma), and the Coriolis effect (the equations of motion are posed in a co-rotating reference frame). All of these forces are derived from the application of MHD equations to the case of a thin flux tube. There is also an effect of enhanced inertia due to the motion of the external medium around the moving tube. We have employed these principles to develop a sophisticated numerical model of flux tubes moving within the solar interior. The equations of motion used in our thin flux tube model are derived in Appendix A of Fan, Fisher and McClymont (1994, henceforth FFM), and the numerical techniques for their solution are described in the Appendix of Fan, Fisher and DeLuca (1993, henceforth FFD). Similar numerical models have been developed and are being used by several other research groups (see e.g. Choudhuri 1989; Caligari, Moreno-Insertis and Schüssler 1995).

4. Results of Model Calculations

4.1. JOY'S LAW

Perhaps the best known success of all of the numerical thin flux tube models has been their quantitative reproduction of "Joy's Law". Joy's Law is an empirical relation between the latitude of a bipolar magnetic region or a sunspot group and the observed angle (relative to E-W) made by its poles (Zirin 1988, Hale *et al.* 1919). This angle, termed its *tilt angle*, α, is defined to be positive when the leading pole is closer to the solar equator than the following pole. Binned active region data, from spot groups (Hale *et al.* 1919, Howard 1991b, Fisher, Fan and Howard 1995) and magnetogram measurements (Wang and Sheeley 1989), show a systematic increase of the tilt angles of active regions with the absolute value of latitude. For latitudes near 30° mean tilt angles are in the 8°–10° range. This effect may be extremely important; in several global models of the solar magnetic field (Wang and Sheeley 1991, Leighton 1964, 1969) the eleven year cycle is a direct consequence of this preferential orientation in newly emerged flux.

Tilt angles arise in a simple and compelling manner in thin flux tube models. As the apex of the loop rises, the plasma inside it expands. The Coriolis force on a parcel of expanding fluid will rotate it in the sense opposite to rotation (i.e. in an inertial frame, its rotation rate decreases). Furthermore, the amplitude of this rotation varies with latitude, θ, in the same manner as the Coriolis effect itself: $\sin(\theta)$. Thus the sense of the tilt angles and their dependence on latitude can be qualitatively explained in terms of a thin flux tube.

Numerical thin flux tube calculations by D'Silva and Choudhuri (1993), Schüssler *et al.* (1994), and ourselves (FFM) have all reproduced Joy's Law to a convincing level of accuracy. Consistency with observation can be achieved with plausible choices for physical parameters such as the magnetic flux Φ in active region tubes, field strength B_0 at the base of the convection zone, and initial latitude θ_0 of the toroidal tube. In fact the latter two parameters are unobservable attributes of the solar dynamo. The application of thin flux tube models to match observations provides one of the few constraints available for these quantities.

4.2. ASYMMETRIC MOTIONS

The asymmetry in the shape of the emerging flux loop, clearly visible in Fig. 1, is also a consequence of the Coriolis effect, as described by Moreno-Insertis (1994). This shape could explain an observed asymmetry in the proper motion of spots during flux emergence (Moreno-Insertis 1994, Caligari *et al.* 1995): The shallower slope of the leading leg makes the leading spot appear to move faster than the following spot, in general agreement with observation. The Coriolis effect also generally results in a greater field strength in the leading leg of the flux loop, which we argue (FFD) is the origin for the more compact morphology of the leading side of active regions when compared to the following side.

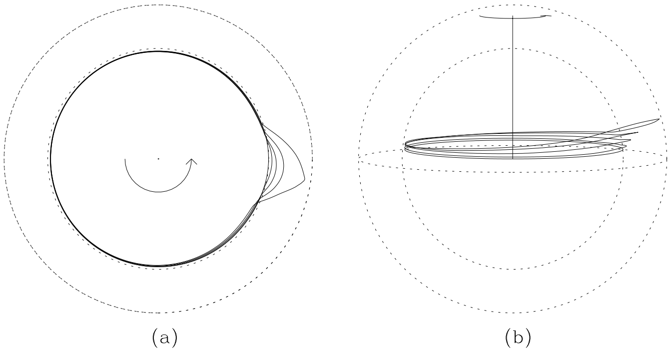
(a) (b)

Figure 1. The shape of an evolving flux ring with flux $\Phi = 10^{22}$ Mx and the initial field strength $B_0 = 3 \times 10^4$ G. The flux ring is viewed from the north pole and from 5 degrees above the equator in panels (a) and (b). 5 instants of the flux ring's evolution are shown as the solid curves; the inner and outer dotted circles mark the base of the convection zone and the photosphere.

4.3. A SCALING LAW FOR ACTIVE REGION TILTS

A more detailed analysis of the tilt angle variation was given in FFM. By considering the scaling of forces with Φ, B_0, and θ, we found that the tilt angles should follow

$$\alpha \ \propto \ \sin(\theta) \ \Phi^{1/4} \ B_0^{-5/4} \tag{1}$$

This simple relation agrees reasonably well with the results of our full numerical simulations provided $B_0 \gtrsim 2 \times 10^4$G (Figure 11, FFM). Thus the theoretical calculations not only reproduce the observed Joy's Law behavior, but also make testable predictions of how active-region tilts should vary with B_0 and Φ.

4.4. TESTS OF THE PREDICTED TILT-ANGLE SCALING LAW WITH SPOT-GROUP DATA

The tilt angle scaling law, Eq. (1), contains a dependence on latitude (Joy's Law) as well as a previously untested dependence on net flux, Φ. Motivated by this theoretical prediction, we sought to ascertain whether such a dependence exists in observed tilt angles. To do this we studied tilt angles in a dataset of 24,701 sunspot groups observed in white light between 1917 and 1985 at the Mt. Wilson Observatory (Howard, Gilman and Gilman 1984). Tilt angles were determined from the spot groups with an algorithm described in §2 of Fisher, Fan and Howard (1995, henceforth FFH). The size of this dataset permits statistically meaningful tests to be made on many subsets of the data. White light observations contain no direct magnetic information, but previously established proxy relationships allow this to be inferred. Specifically, Howard (1992) has established a proportionality between the net (unsigned) flux in an active region (from magnetograms) and the distance separating the centroids of its leading and following polarities. This enabled us to use the polarity separation d of a spot group as a proxy for the magnetic flux Φ.

We found in FFH that the *mean* tilt behavior was consistent with the θ and Φ variation predicted in equation (1). (For reasons discussed in FFH, the quantity B_0 is not directly measurable, but its range of values appears to be quite restricted — see FFH for a more complete discussion of this issue). A more intriguing result of FFH was the large amplitude of the *fluctuations* $\Delta\alpha$ of individual tilts away from the mean behavior. These fluctuations are very significant, are much larger than estimated measurement errors, are not a function of latitude (unlike the "Joy's Law" behavior of the mean tilts), but are strongly decreasing functions of d. In FFH we argue that the fluctuations are solar in origin, and most likely from perturbations of the rising tubes by convective motions.

4.5. CONVECTION ZONE TURBULENCE IN MODELS OF RISING FLUX TUBES

In Longcope and Fisher (1995, henceforth LF) we show that turbulent motions in the convection zone do lead to tilt angle fluctuations $\Delta\alpha$ of the magnitude observed in FFH. We further find that a simple model explains the observed relation between tilt angle fluctuation and footpoint separation. To do this, we have developed a new technique for adding statistical perturbations to the thin flux tube model.

The first part of this technique defines an algorithm for generating realizations of the turbulent velocity field in the convection zone. A given turbulent realization is a function of space and time; the behavior of an ensemble of realizations gives statistical moments consistent with those assumed under mixing length theory. Although mixing-length convection models are highly idealized, they probably mimic the qualitative depth dependence of the velocity field that a rising flux tube encounters. For each individual realization of the velocity field, one can then compute the resultant perturbations to the motion of a rising flux tube.

To calculate the effect of the turbulent velocity field, we consider the limit of small deflections to a flux tube which has risen through a stationary atmosphere (i.e. we introduce the forces driven by convection zone turbulence as a first order perturbation to the general flux tube equation of motion). To further simplify our calculation, we use, for the unperturbed rise, a straight horizontal tube. After Fourier-transforming the perturbed equation of motion, we derive forced-damped harmonic oscillator equations for the evolution of each Fourier mode in the tube. This results in a spectrum of deflections which is linearly related to the spectrum of the turbulence. Using 250 numerically generated realizations of the turbulent velocity field, it is possible to calculate the fluctuations, $\Delta\alpha$, in the tilt angle of the emergent tube. Results of these Monte-Carlo calculations compare favorably to the observed values found in FFH (see Fig. 2).

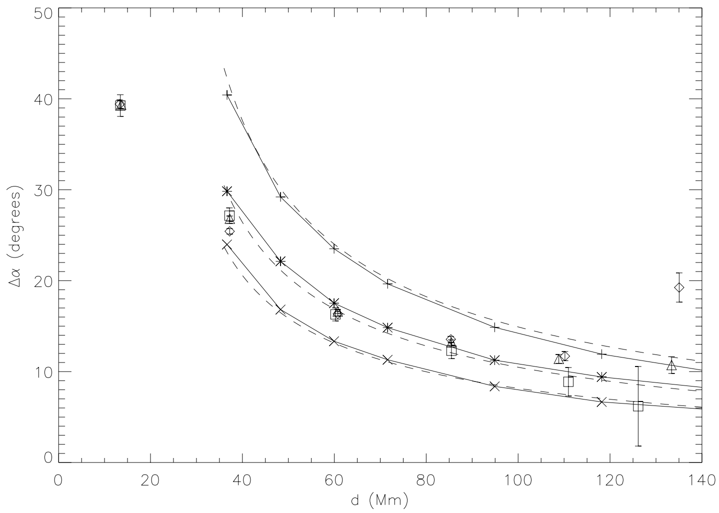

Figure 2. The RMS tilt angle, $\Delta\alpha$ as a function of d (Mm). The solid lines pass through the results of Monte-Carlo calculations with initial magnetic field strengths B_0, of 20 kG (+), 30 kG ($*$), 40 kG ($\times$). Observed values of $\Delta\alpha$ from FFH are shown for high ($\Box$), mid ($\triangle$) and low ($\diamond$) latitudes.

5. The Helical Kink Instability of Twisted Flux Tubes in the Interior

There is a small but important class of active regions which appear to be twisted and possibly kinked or knotted when they emerge. The structure and dynamics of the magnetic field in these active regions is especially important because they are strongly linked to the occurrence of large solar flares. The link between solar flares and emerging flux loops which are kinked can be inferred from the fact that the "island δ" spot configuration is typically the site of the largest solar flares (see *e.g.* Leka *et al* 1995; Kurokawa 1991; Tanaka 1991; Zirin 1988, p. 337). Here, two spot umbrae of opposite magnetic polarity emerge within one spot penumbra; furthermore, the orientation of the two polarities is frequently reversed from the usual Hale configuration. One interpretation of the δ spot configuration is that a rising loop of a twisted magnetic flux tube has kinked into a braided structure, similar to that which results when twist is applied to the loop of a rubber band. Such an interpretation would account qualitatively for the reversed polarity configuration, the close proximity of the opposite polarity spots, and the occurrence of flares as the oppositely directed magnetic fields in the intertwined loop legs undergo reconnection.

Motivated by a desire to understand the dynamics of twisted active region flux tubes below the solar photosphere, we have investigated the kink stability of these tubes (Linton *et al* 1995). Following previous studies of the kink mode, we apply linearized equations of MHD to a cylindrical magnetic equilibrium (screw pinch), but with significant differences from the earlier work. In our case, the magnetic field vanishes outside some radius, $r = R$, where it is confined by the higher pressure of the unmagnetized plasma. This outside boundary of the tube is free to move, displacing the unmagnetized plasma as it does so. We consider equilibria in which all field lines have the same helical pitch: $B_\theta / r B_z = q = \text{const}$. Our main results are as follows:

(1) These equilibria are stable to kinking, provided that the field line pitch does not exceed a threshold; $q \leq q_{\text{cr}}$ for stability. The threshold is $q_{\text{cr}} = \sqrt{\alpha}$, where α is the r^2 coefficient in the Taylor series expansion of the equilibrium axial magnetic field (B_z) about the tube axis ($r = 0$): $B_z(r) = B_0(1 - \alpha r^2 + \ldots)$. When this criterion is violated, there are unstable eigenmodes, $\xi \propto e^{\imath(\theta + kz)}$. The most unstable of these have a helical pitch k which is near (but not equal to) the field line pitch.

(2) For weakly twisted tubes ($qR \ll 1$) we are able to derive growth rates and unstable eigenfunctions analytically. For strongly twisted tubes ($qR \gtrsim 1$) we find growth rates and unstable eigenfunctions numerically.

(3) The maximum growth rate and range of unstable wavenumbers for a strongly twisted tube can be predicted qualitatively by using the analytical results from the weakly twisted case. The maximum growth rate in that case is given by $\omega_{max} = v_A R(q^2 - q_{\text{cr}}^2)/3.83$, where v_A is the axial Alfvén speed. The range of unstable wavenumbers is given by $(-q - \Delta k/2) < k < (-q + \Delta k/2)$, where $\Delta k = 4qR\sqrt{q^2 - q_{\text{cr}}^2}/3.83$.

(4) The kink instability we find corresponds primarily to internal motion. Helical translations of the entire tube are found to be stable.

(5) We argue that an emerging, twisted magnetic flux loop will tend to have a uniform q along its length. The increase in the tube radius R as it rises results in a decreasing value of q_{cr}. This means that the apex of the flux loop will become kink unstable before the rest of the tube.

(6) Our results lead us to believe that most twisted flux tubes rising through the convection zone will be stable to kinking. Those few tubes which are kink unstable, and which presumably become knotted or kinked

active regions upon emergence, only become kink unstable some time after they have begun rising through the convection zone.

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