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The relationship between electrostatic shocks and kinetic Alfvén waves

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Abstract. Auroral satellites and sounding rockets frequently observe large electric fields perpendicular to the magnetic field that have a narrow scale length perpendicular to the magnetic field if they are interpreted as spatial structures. These fields have been variously attributed to electrostatic shock structures or to kinetic Alfvén waves. These two models can be distinguished by considering the ratio of the magnetic field perturbation to the electric field. This ratio is calculated within the context of the electrostatic approximation, the fully kinetic Alfvén wave dispersion relation considered by *Lysak and Lotko* [1996], and the cold fluid model including ionospheric reflection presented by *Lysak* [1991, 1993]. Results for this model show that the ratio of the electric to magnetic field is not always equal to the Alfvén speed, especially for structures that are very narrow in the direction perpendicular to the magnetic field. These narrow structures have electric fields that are enhanced with respect to the Alfvénic value, and thus may appear as electrostatic.

1. Introduction

Large amplitude, spatially narrow electric fields perpendicular to the magnetic field have often been observed on magnetic field lines above auroral arcs since they were first observed by the S3-3 satellite [*Mozer et al.,* 1977]. These fields had magnitudes greater than 100 mV/m and spatial dimensions less than 10 km perpendicular to the magnetic field if they are interpreted as being spatial structures. These electric field structures were called "electrostatic shocks" by *Mozer et al.* [1977]. No corresponding magnetic field signatures were observed by S3-3; however, this satellite was incapable of measuring magnetic fields on the fine spatial scales of the electric field measurement. These structures were associated with upgoing ion beams, and the electrostatic nature of the structures was suggested by the fact that the ion beam energy was consistent with the electrostatic potential found by integrating the perpendicular electric field along the satellite trajectory, at least at higher altitudes [e.g., *Temerin et al.,* 1981].

Observations that were interpreted in terms of an Alfvén wave picture of electric fields were obtained by sounding rockets [e.g., *Boehm et al.,* 1990] as well as the Viking [*Marklund et al.,* 1990; *Block and Fälthammar,* 1990] and Freja [*Louarn et al.,* 1994; *Wahlund et al.,* 1994] satellites. The low-altitude measurements from the sounding rockets and Freja indicated strong magnetic perturbations with $E_x / B_y \sim 10$ (mV/m)/nT, consistent with the Alfvén wave relationship $E_x = V_A B_y$ for an Alfvén speed of 10,000 km/s. On the other hand, Viking observed waves in the 0.1-1.0 Hz band with electric fields the order of a few times 10 mV/m, but no corresponding magnetic signatures were found.

New and more detailed information on these structures is now becoming available from the Polar and FAST missions. It is the purpose of this letter to present calculations of the ratio E_x / B_y that can be compared with satellite observations. This ratio will be calculated in three ways: from an electrostatic model, from a model of freely propagating Alfvén waves, and from a model in which Alfvén waves are reflected from the ionosphere.

2. Electrostatic shock model

Although the large electric fields have been termed "electrostatic shocks," they have a finite magnetic component due to the fact that there are field-aligned currents flowing through the shock structure. At the ionosphere, the electric and magnetic fields are related by the expression $E_{xI} = B_y / \mu_0 \Sigma_P$ due to the fact that the electric field drives Pedersen currents in the ionosphere, which must be closed by a field-aligned current. This relationship must be modified in the presence of parallel electric fields. As was noted in *Lysak* [1985], integration of the electrostatic condition $\nabla\times\mathbf{E} = 0$ in a two dimensional geometry yields the relation $E_x(z) = E_{xI} + \partial\Phi_\parallel/\partial x$, where $\Phi_\parallel$ is the parallel potential drop between the ionosphere and the location z. In this expression a positive $\Phi_\parallel$ represents a potential that accelerates electrons into the ionosphere, and it is assumed that all quantities are mapped to ionospheric heights. It is convenient to assume a linear relationship between this potential drop and the field-aligned current density, $j_z = -K\Phi_\parallel$. We can also use Ampere's Law to write $j_z = (1/\mu_0)\partial B_y/\partial x$. Putting these expressions together with the expression above for the ionospheric electric field, we can write

$$E_x(z) = \frac{B_y}{\mu_0 \Sigma_P} - \frac{1}{\mu_0 K}\frac{\partial^2 B_y}{\partial x^2} = \frac{1}{\mu_0 \Sigma_P}\left(1 + k_\perp^2 L^2\right) B_y \qquad (1)$$

where we have assumed a plane wave variation and introduced the magnetosphere-ionosphere coupling scale length $L^2 = \Sigma_P/K$, which is typically in the 50-100 km range. Note that the mapped B_y is constant along field lines in this static model.

Equation (1) can be used to compare electric and magnetic fields in the electrostatic model. If there are no parallel electric fields, $E_x / B_y = (800$ km/s$) / \Sigma_P$ (mho). On the other hand, if the second term in equation (1) is dominant, the field ratio is $E_x / B_y = (3.14\times10^7$ km/s$)/(\lambda_\perp^2 K)$, where $\lambda_\perp = 2\pi/k_\perp$ is the perpendicular wavelength in kilometers and K is measured in units of 10^{-9} mho/m^2. Thus, this expression shows that the ratio exceeds the speed of light for wavelengths less than 10 km for $K = 10^{-9}$ mho/m^2. It should also be noted that equation (1) indicates that the electric and magnetic fields from this model should be strictly in phase with each other.

3. Kinetic Alfvén wave model

The dispersion relation for kinetic Alfvén waves has been recently discussed by *Lysak and Lotko* [1996], and it was found that it could be written in the general form

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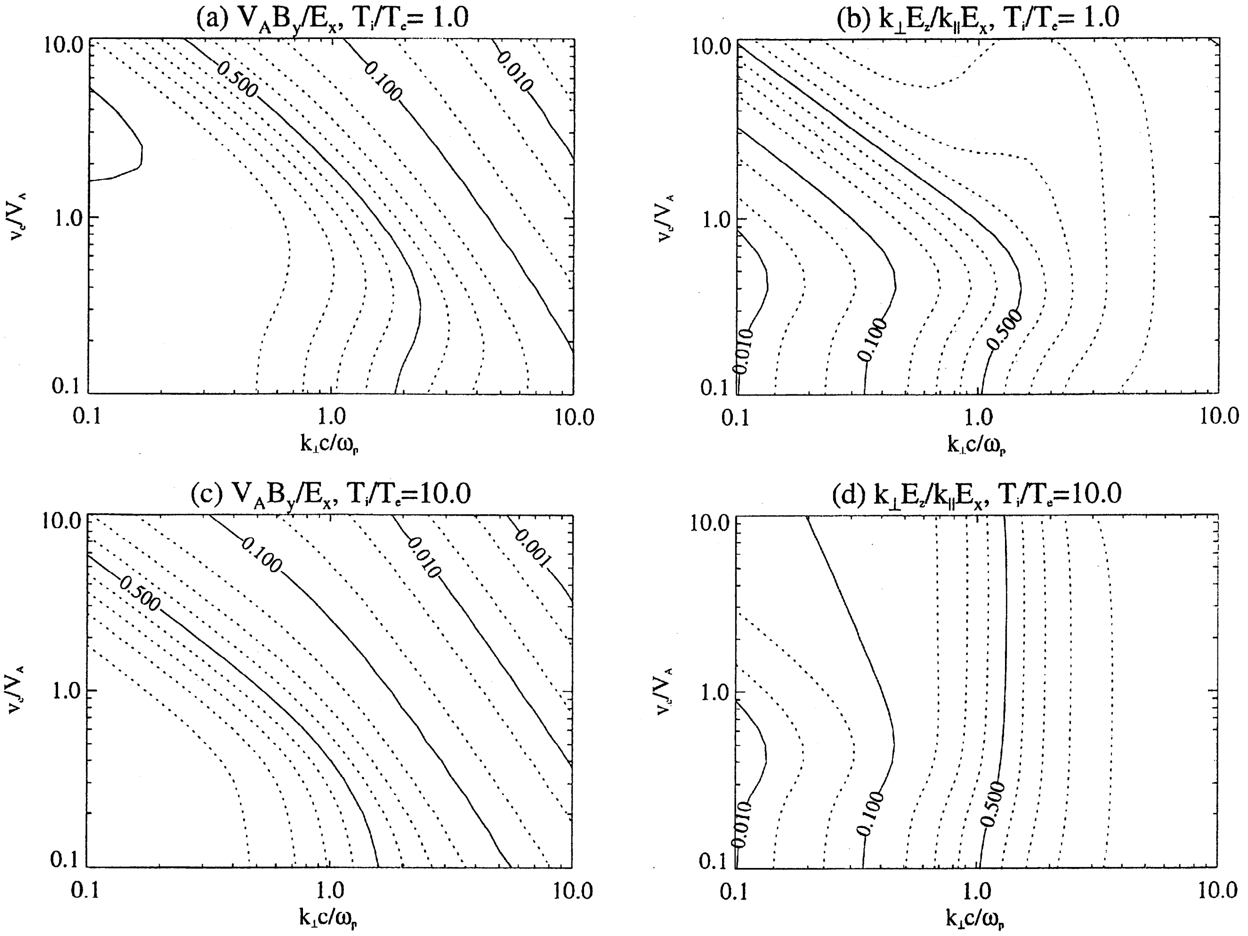

Figure 1. Normalized magnetic fields (panels a and c) and parallel electric fields (panels b and d) for the kinetic Alfvén wave dispersion relation given by equations (4) and (3), respectively. These are plotted as functions of the perpendicular wave number normalized to the electron inertial length, $k_\perp c/\omega_{pe}$ and the electron thermal speed normalized to the Alfvén speed, v_e / V_A. Equal electron and ion temperatures are assumed in panels a and b, while $T_i = 10\ T_e$ for panels c and d. A value of 1 in the magnetic field plots indicates an Alfvénic ratio, while 1 in the parallel electric field plots indicates an electrostatic structure.

$$\begin{pmatrix} \varepsilon_\perp - n_\parallel^2 & n_\parallel n_\perp \\ n_\parallel n_\perp & \varepsilon_\parallel - n_\perp^2 \end{pmatrix}\begin{pmatrix} E_x \\ E_z \end{pmatrix} = 0 \qquad (2)$$

where the wave vector has been assumed to lie in the xz plane, and $\varepsilon_\perp$ and $\varepsilon_\parallel$ give the perpendicular and parallel responses of the plasma, respectively, and are given by *Lysak and Lotko* [1996]. The wave polarization characteristics can be found from the first row of equation (2)

$$\frac{E_z}{E_x} = \frac{n_\parallel^2 - \varepsilon_\perp}{n_\parallel n_\perp} \qquad (3)$$

while the magnetic perturbation can be found from Faraday's law

$$cB_y = n_\parallel E_x - n_\perp E_z = \frac{\varepsilon_\perp}{n_\parallel} E_x \qquad (4)$$

The results from equations (3) and (4) are plotted in Figure 1 for temperature ratios of $T_i / T_e = 1$ and 10 as a function of $k_\perp c/\omega_{pe}$ and the ratio of the electron thermal speed $v_e = (T_e/m_e)^{1/2}$ to the Alfvén speed. It can be seen from this figure that the magnetic field ratio becomes Alfvénic and the parallel electric field is small for large perpendicular wavelength, $k_\perp c / \omega_{pe} \ll 1$. For small perpendicular wavelength, the wave becomes nearly electrostatic, with $B_y / E_x \ll 1 / V_A$ and $k_\perp E_z \approx k_\parallel E_x$. As noted in *Lysak and Lotko* [1996], a calculation using a model density profile shows that $k_\perp c / \omega_{pe} > 1$ up to about 7 R_E altitude for perpendicular wavelengths of 1 km mapped to ionospheric heights (corresponding to about 18 km at 7 R_E). Perpendicular wavelengths greater than 10 km mapped to the ionosphere have $k_\perp c / \omega_{pe} < 1$ over the whole field line in the model, and so these structures would be expected to have significant magnetic components. It can also be seen that an elevated ion temperature tends to suppress the magnetic field magnitude, especially for hotter plasmas. These purely propagating Alfvén waves also have the characteristic that the electric and magnetic fields are in phase.

4. Alfvén waves including ionospheric reflection

The results presented above are for purely propagating Alfvén plane waves; however, observations of Alfvénic structures are often composed of the superposition of an incident wave and a wave reflected off the conducting ionosphere. A model to describe the resulting wave fields in this situation was presented by *Lysak* [1993]. In this model, the plasma is taken to be cold, which in this context means that the electron thermal speed is less than the Alfvén speed. This condition pertains below about 5 R_E altitude [*Lysak and Lotko,* 1996]. In this limit, the single wave results lead to the relation

$$\frac{E_x}{B_y} = \pm V_A \sqrt{1 + k_\perp^2 c^2 / \omega_{pe}^2} \qquad (5)$$

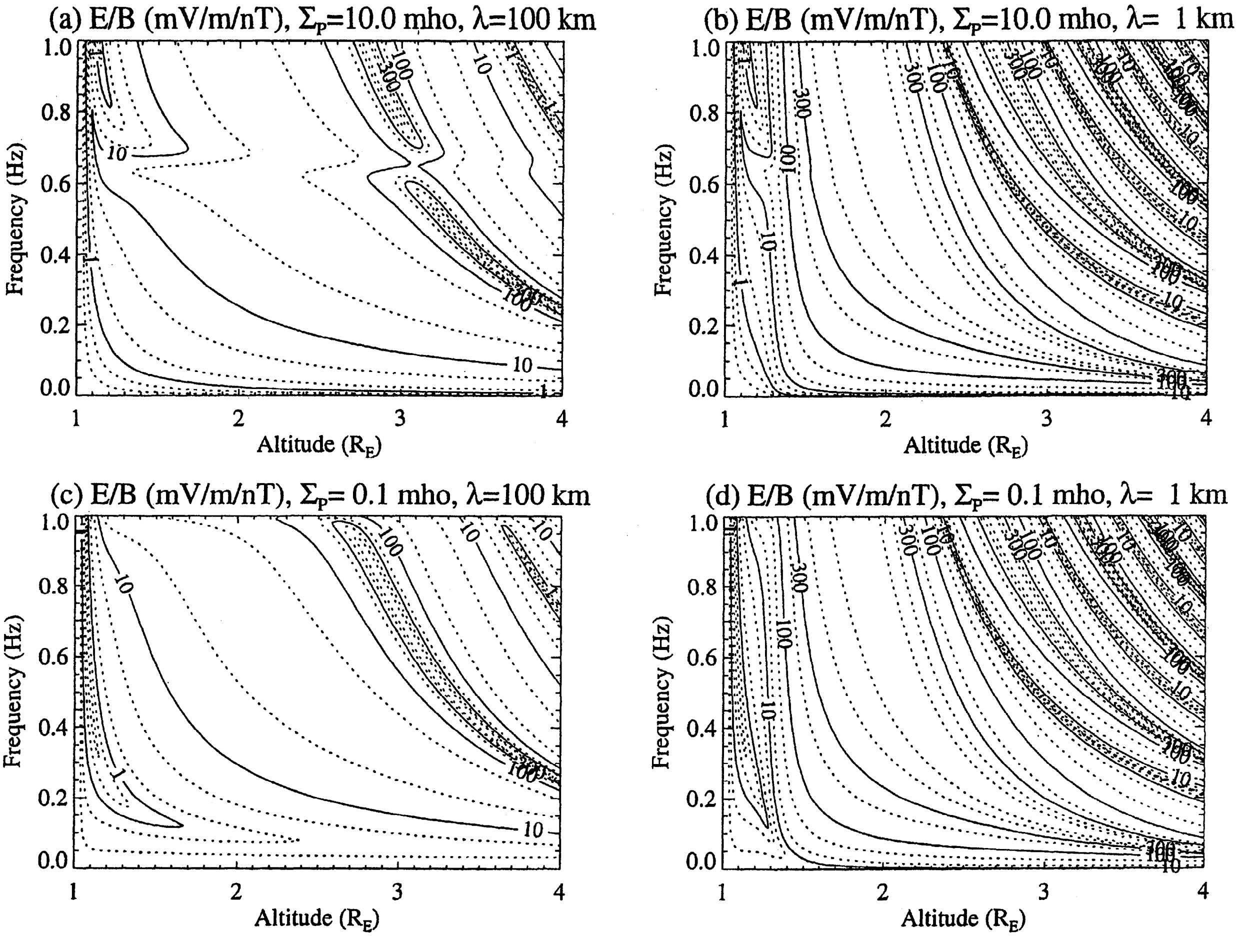

Figure 2. Contours of the E_x / B_y ratio for the ionospheric reflection model as a function of frequency and altitude, expressed in units of mV/m/nT, or equivalently, 1000 km/s. Panels a and b assume a Pedersen conductivity of 10 mho, while c and d assume 0.1 mho. Panels a and c are for a perpendicular wavelength of 100 km mapped to the ionosphere, while b and d are for 1 km. A value of 300 on these plots indicates a ratio of the speed of light.

where the sign depends on the direction of propagation. The observed wave fields are a superposition of the incident and reflected waves. This superposition is calculated by assuming a frequency and a perpendicular wavelength at the ionosphere as well as an ionospheric conductivity, as described in *Lysak* [1993]. A dipole field model and the density model from *Lysak* [1993] are assumed, giving a maximum Alfvén speed of about 140,000 km/s at 3000-km altitude that falls to about 10,000 km/s at a geocentric distance of 4 R_E.

The resulting profiles of the E_x / B_y ratio are plotted in Figure 2 for two conductivities, Σ_P = 10 mho and 0.1 mho, and for two perpendicular wavelengths, 100 km and 1 km. These values encompass a range of typical parameters that might be encountered in the auroral zone. These plots show the field ratio in millivolts per meter per nanotesla. These units are equivalent to an effective speed of 1000 km/s. Thus, the contours labeled 300 on these plots correspond to a field ratio equal to the speed of light; values greater than this are often considered to be indicative of electrostatic waves. Solid lines correspond to ratios of 1, 10, 100, and 300, while the dotted lines give ratios that are a factor of 2 and 5 above the previous solid contours.

From these plots, it can be seen that the electric-to-magnetic field ratio is strongly dependent on the perpendicular wavelength of the wave. For the larger perpendicular wavelength of 100 km, the field ratio is in the range of 1 to 10 mV/m/nT at FAST altitudes. For 1 km perpendicular wavelength, the electric fields are enhanced, and ratios at or greater than the speed of light can be observed above 1.5 R_E. It should also be noted that for these waves the phase shift between electric and magnetic fields (not plotted) is generally near ±90°, except at very low frequencies, where the structure becomes static, and at the resonant frequencies of the system [*Lysak,* 1991].

5. Discussion

The results presented here support the suggestion made by *Goertz* [1984] and others that the so-called "electrostatic shocks" first observed by S3-3 and repeatedly observed by other spacecraft can simply be the signatures of narrow scale inertial Alfvén waves. For the density model assumed by *Lysak* [1993], the electrostatic nature of the waves becomes apparent for wavelengths less than a few kilometers, mapped to the ionosphere. On these scales, the E_x / B_y ratio can exceed the speed of light for a wide range of frequencies and altitudes.

This model is in general agreement with the observations made by sounding rockets and polar-orbiting satellites. At lower altitudes, observations from rockets [*Boehm et al.,* 1990; *Knudsen et al.,* 1992] and from Freja [*Louarn et al.,* 1994; *Wahlund et al.,* 1994] showed clear signatures of Alfvén waves. It can be seen from Figure 2 that at altitudes below 2000 km, the predicted field ratios are 1-10 (mV/m)/nT, consistent with these measurements. At higher altitudes, small-scale waves become more electrostatic, and ratios of $E_x / B_y > c$ are possible over a large range; thus, higher altitude satellites such as Viking and Polar would be more apt to see the field structures as electrostatic.

These results indicate that "electrostatic shocks" that are comparable in scale size to individual auroral arcs (< 1 km) can be explained by the structure of inertial Alfvén waves of small perpendicular wavelength. Larger scale structures with $E_x / B_y > c$ would be more easily explained in terms of the electrostatic picture, including the effects of parallel electric fields. It was noted by *Lysak and Dum* [1983] that electrostatic fields could arise on larger scales from an Alfvén wave picture with anomalous resistivity included. It is also possible that strong double layers could form driven by the field-aligned currents inside the Alfvén waves. Such structures could enhance the local value of the ratio between the parallel and perpendicular fields.

One important observation would be a measurement of the phase difference between the electric and magnetic fields. For an electrostatic structure or for an Alfvén wave propagating without reflection, E_x and B_y should be in phase. When reflection from the ionosphere is included, however, there will be frequency-dependent phase differences between the fields, as was noted by *Knudsen et al.* [1990, 1992] and *Lysak* [1991].

In summary, it is shown that Alfvén waves with scales smaller than a few kilometers mapped to the ionosphere have an electrostatic wave structure in which the magnetic field perturbation becomes small. Such wave structures would be consistent with an "electrostatic shock" field structure in that the electric-to-magnetic field ratio can become greater than the speed of light. Phase shifts between the electric and magnetic field components would provide a means of distinguishing such structures.

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