GEOPHYSICAL RESEARCH LETTERS, VOL. 25, NO.13, PAGES 2365-2368, JULY 1, 1998

Current-Voltage relationship in the downward auroral current region

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Abstract. In the auroral zone of the earth currents flow along magnetic field lines. In the downward current region currents are mainly carried by upflowing electrons from the ionosphere. Because of the low plasma density along auroral field lines, substantial currents in the range of microamps per square meter require substantial potential drops parallel to the magnetic field in the range of a few hundred to a few thousand volts. The current-voltage relation along such magnetic field lines can be determined for simple profiles of the background ion density by invoking the condition of charge neutrality. For typical parameters, the current density is found to be a few times larger in the downward current region compared to currents in the upward current region for similar potential drops. Thus potential drops up to a few thousand volts and the consequent acceleration of ionospheric electrons up to keV energies, such as has been observed by the FAST satellite, are a necessary consequence of the observed current densities in the downward auroral current region.

Introduction

The auroral zone is characterized by currents that flow along the magnetic field into and out of the ionosphere. The general pattern of such currents was given by *Ijima* and *Potemra* [1976]. Potential drops along such magnetic field lines are important because they accelerate both ions and electrons. The current-voltage relation in the upward current region of the auroral zone was given by *Knight* [1973]. In the upward current region parallel electric fields produce inverted V's and presumably the discrete aurora. The basic idea of the Knight equation is that because of the low density of the plasma in the magnetosphere, a magnetic field-aligned potential drop is necessary to drive enough hot electrons into the ionosphere to produce the required current. Many such electrons cannot otherwise reach the ionosphere because of the magnetic mirror force. The current is presumably 'required' by boundary conditions near the equatorial plane. Thus the current-voltage relation of the upward field-aligned current depends on the temperature and density of the hot plasma (usually referred to as the 'plasmasheet plasma'). The downward current (sometimes referred to as the 'return current') is carried mostly by ionospheric electrons flowing up the field. Since the ionosphere is a plentiful source of electrons, it has often been assumed that little if any potential drop would be required in this region for such field lines to carry a current. It is purpose of this letter to show that this is not the case.

Recent FAST satellite data at altitudes near 4000 km in the auroral zone [*Carlson et al.*, 1998; *Ergun et al.*, 1998a] have shown, as previously suggested by, for example, *Burch et al.* [1982], *Gorney et al.* [1985], and *Marklund et al.* [1994], that significant potential drops occur in the return current region. FAST observations show that field-aligned potentials of up to several kilovolts can occur below 4000 km in the return current region with field-aligned potentials of a few hundred Volts being common. These observations have motivated this letter on the current-voltage relationship in the downward auroral current region.

The basic idea is that the current is limited by the ion density along the flux tube and the requirement that the plasma be charge neutral almost everywhere. The current density, j, is given by $j=env$ where e is the electron charge, n the electron density of the current carrying electron component, which (almost everywhere) cannot exceed the ion density, and v the average electron velocity. If the electrons start out as cold ionospheric electrons, v is related by energy conservation to the potential difference $\delta\phi$ as $v = (2e\delta\phi/m)^{1/2}$, where m is the electron mass. If the ion density is presumed to stay fixed on the electron transit time scale (this condition will be discussed further later), then $Ne(2e\delta\phi/m)^{1/2}$, where N is the ion density on the flux tube, is an upper limit on the field-aligned current density. As an example consider that the ion density is 1 cm^{-3}, then for a current of 10^9 el/cm^2-s (1.6 $\mu A/m^2$), the potential must be at least 283 V. In fact a more exact analysis shows that the potential should be substantially larger as is shown below.

Model

The current that a magnetic flux tube can carry is affected by the density along the flux tube. The density along auroral field lines is not in equilibrium but depends on the history of the field line. Auroral field lines carrying downward current may have convected to their present position from the polar cap or may have had a history of having carried upward field-aligned current. Typical densities in the polar cap are given by *Persoon* [1983]. Such densities are often less than 5 el/cm^3 at 10000 km altitude. Densities on field lines in the auroral zone carrying upward field-aligned current can be even smaller [*Calvert*, 1981]. Here the upward parallel electric fields can exclude nearly all ionospheric plasma except for the upward accelerated ion beam above a certain altitude which can be as low as 1270 km. (This is the lowest altitude at which an upward ion beam has been observed by the FAST satellite during its first year of observations. The energy of the upward ion beam was 2.5 keV.) Densities measured by FAST in upward ion beam regions, based on wave characteristics, are in the range of plasmasheet densities: 0.25-2.0 el/cm^3 [*Strangeway et al.*, 1997].

The plasma along auroral field lines may be considered to consist of four components: the relatively cold ionospheric ions and electrons and the much hotter plasmasheet ions and electrons. The ionospheric components are strongly affected by gravity: ionospheric ions are affected directly and the ionospheric electrons indirectly by the ambipolar electric field that maintains charge neutrality. The result is that the density of the ionospheric components decreases quickly with altitude at low altitudes where heavier ions dominate, and more slowly at higher altitudes where

Paper number 98GL01865.
0094-8534/98/98GL-01865$05.00

gravity is weaker and the ion composition is lighter [*Mozer et al. 1979*].

The plasmasheet component, whose kinetic energy is much larger than its gravitational potential energy, is essentially unaffected by gravity. In the absence of an electric potential along the field line and for an isotropic pitch angle distribution in the equatorial plane, the density of the plasmasheet component is constant along a magnetic field line. The temperature of the plasmasheet ions is typically much larger than of the plasmasheet electrons. Typical values are ~1 keV for the electrons and ~6 keV for the ions. If there is a potential along the field line, the plasmasheet electron density, however, will vary as $n_e \exp(-e\phi/(\kappa T))$ where n_e is the equatorial plasma density, T, the temperature of the plasmasheet electrons, and κ the Boltzmann constant. This is an exact solution for the electron density of this component provided the electron distribution is isotropic and Maxwellian and ϕ decreases monotonically from the equator. For simplicity we assume that plasmasheet ions are so hot that we can consider their density to be constant even in the presence of a field-aligned potential.

The density of the ionospheric electrons can be calculated since, assuming current continuity along the magnetic field, env/b is a constant along the magnetic flux tube. Here $b=|B|/|B_0|$ is the ratio of the magnetic field magnitude normalized to $|B_0|$, defined below, and thus $1/b$ is proportional to the flux tube cross-sectional area, e, the electron charge, n, the electron density of the ionospheric component and v, the average electron velocity parallel to the magnetic field line. Below a certain altitude an ambipolar electric field prevents most of the electrons from escaping. In the presence of a downward field-aligned current there exists an altitude at which the ambipolar field goes to zero (here we define $B_0=B$) and above which a parallel electric field accelerates electrons upwards. We consider the parallel potential of this latter electric field. At the location where the ambipolar field goes to zero, the ionospheric electrons have an average upward velocity of v_0. Above this altitude their velocity is given by $(v_0^2 + 2e\delta\phi/m_e)^{1/2}$, where $\delta\phi$ is the potential difference to the point where $B=B_0$. (Here we have considered the thermal energies of the ionospheric electrons small compared to $\delta\phi$ so that we can ignore the thermal spread of ionospheric electron velocities.) The potential along the field line can be calculated by assuming charge neutrality. Equating the electron and ion densities gives:

$$n_e \exp\frac{-e\phi}{\kappa T} + \frac{jb}{e(v_0^2 + 2e(\Phi - \phi)/m_e)^{1/2}} = N_i \qquad (1)$$

Here we set the potential along the field line, ϕ, to zero at the equator and thus the plasmasheet electron density along the field line is given by the first term. In the second term, which gives the ionospheric electron density (after solving for n in $j=env/b$ and using the expression discussed above for v), $\Phi-\phi$ is $\delta\phi$, the potential difference to the point where $B=B_0$, and N_i is the ion density. We wish to find Φ, the total potential difference along the magnetic field from the equator to the point where $B=B_0$.

For concreteness let's consider a specific example and a specific idealized ion density profile. Assume that the plasmasheet component at the equator (i.e. idealized as the point where $b=0$) has a density of 1 cm^{-3}, an electron temperature of 1 keV, a large ion temperature that we take to be infinite, and that the current from the ionospheric electrons at 3000 km altitude is 2.0 $\mu A/m^2$ (i.e., 1.25×10^9 el/cm^2-s). (We ignore the small contributions to the current from the other components.) Assume further that the upward velocity of the ionospheric electrons at 3000 km, the point where the ambipolar electric field goes to zero, is equivalent to 0.25 eV or ~296 km/s (the density of the ionospheric component there is hence ~42 el/cm^3), and that the functional form of N_i as a function of b is given by

$$N_i = 1 + (C(2b-1)) \qquad \text{for } 0.5 < b \le 1.0 \qquad (2)$$

$$N_i = 1 \qquad \text{for } 0.0 < b \le 0.5 \qquad (3)$$

Thus in this model the ion density consists of a constant 'plasmasheet' component and an 'ionospheric' component. The ionospheric component $(C(2b-1))$, is a function of the relative magnetic field magnitude b and decreases from ~42 el/cm^3 to zero linearly as b goes from 1.0 to 0.5. The constant C must be equal to $\exp\left(\frac{-e\Phi}{\kappa T}\right) - \left(1 + \frac{j}{ev_0}\right)$ by (1) to have change neutrality at b=1. Thus the value of C is not known exactly until Φ is determined as described below. Under these assumptions the smallest potential for which charge neutrality can be achieved is 1.14 kV. The density and potential profiles (with the sign of the potential reversed) as a function of b are shown in Figure 1. Of the total potential drop ~0.67 kV occurs in the region where N_i=1, while the rest occurs where there are ionospheric ions.

These potentials are larger than estimated in the introduction because the presence of the plasmasheet electrons means that the potential, in addition to accelerating the ionospheric electrons, must reflect some of the plasmasheet electrons to allow 'room' for the ionospheric electrons. Thus the potential depends to a large extent on the plasmasheet temperature and density. Figure 2 shows the variation in the potential as a function of the density and temperature of the plasmasheet electrons for a current of 1.0 $\mu A/m^2$ at b=0.5 (the same current as in Figure 1). The curve labeled 'density at 1 keV' shows the variation of the potential drop as a function of density for a fixed temperature of 1 keV while the curve labeled 'temperature at 1 cm^{-3}' shows the variation of the potential drop as a function of temperature for a fixed density of 1 cm^{-3}. Higher plasmasheet electron temperatures or lower densities result in larger potentials.

Figure 3 shows the potential as a function of current density for different plasmasheet electron temperatures and densities. The current density is the current density at the point where the ionospheric density first disappears. That would be at b=0.5 for the

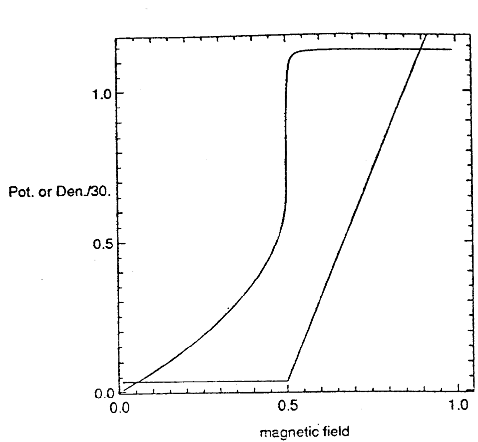

Figure 1. The density (cm^{-3}/30) and potential profiles (with the sign of the potential reversed in kV) as a function of b, the relative magnetic field strength along the magnetic field line normalized to the magnetic field strength at the point where the ambipolar parallel electric field goes to zero.

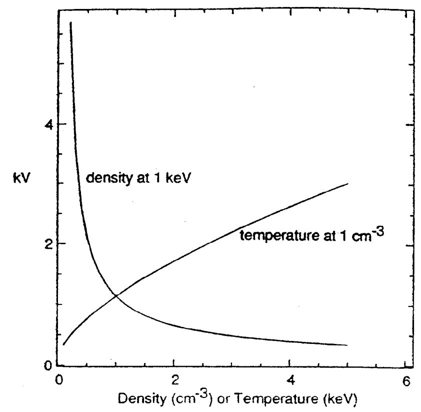

Figure 2. The variation in the potential as a function of the density and temperature of the plasmasheet electrons for a current of 1.0 μA/m^2 at b=0.5 (the same current as in Figure 1).

density model shown in Figure 1 but the relationship is not closely related to a particular ion density model. Different currents require slightly different density models since for larger currents the ambipolar field must go to zero at larger ionospheric densities to supply sufficient current-carrying electrons.

For Figures 1, 2, and 3 the total potential drop (Φ) is found numerically by searching for the smallest potential drop that is consistent with charge neutrality (1) at the point where the ionospheric density first disappears (b=0.5). This gives a continuous monotonically decreasing potential (ϕ) as a function of b. In the numerical solution the total potential is first selected and then the percentage of the potential below and above the point b=0.5 is varied. For total potentials which are too small no solution can be found by varying ϕ at b=0.5. For larger total potentials (Φ) the solution for ϕ is discontinuous as a function of b. (See *Stern*, [1981] for a discussion of quasi-neutral equilibrium solutions in a dipole geometry.) The discontinuous solutions are not physical in the context of our model with prescribed ion densities since they do not satisfy, in the terminology of *Stern* [1981], the jump conditions for a double layer. The sufficient and necessary properties that an ion density profile needs to have for such a numerical solution method to work and for a continuous monotonically decreasing solution to exist have not been established. However, $N_i b$ is a monotonically increasing function of b for (2) and (3) and b/N_i (proportional to the average electron drift velocity) has a single maximum as a function of b. Such properties are probably necessary. For more complex density profiles, the potential profile is likely to take on more complex forms. We have used a particularly simple ion density profile for illustration. As discussed below the presence of waves and thus ion conics injects an ionospheric component which increases and smooths the density profile and will thus decrease the potential drop.

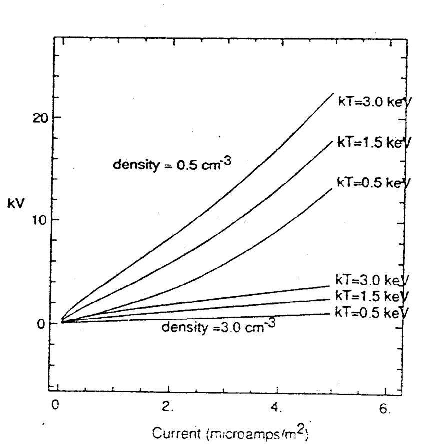

Figure 3. The potential drop along the magnetic field line as a function of current density for different magnetospheric electron temperatures and densities in the auroral downward current region.

Discussion

The above analysis suggests that a substantial potential is possible in the return current region even in the absence of wave turbulence. In our simple model ions should be accelerated downward by the parallel electric field. The downward acceleration of the ionospheric ions suggests that the ionospheric density profile might collapse into a double layer where charge neutrality does not hold. No such stable double layer appears possible. Instead the data show [*Carlson et al.*, 1998; *Ergun et al.*, 1998b] that intense waves and, occasionally fast propagating small-scale solitary waves occur in the acceleration region. This wave turbulence heats the ionospheric ions perpendicular to the magnetic field to produce ion conics and heats electrons parallel to the magnetic field to produce, in some cases, counterstreaming electrons [*Sharp et al.*, 1980]. The effect of the ion conics is to increase the density along the field line above what it would otherwise be and thus lower the potential that is required to carry a given current. The lack of a realistic, self-consistent, steady-state solution involving the ion dynamics, rather than being seen as a limitation of our model, should rather be seen as providing a strong hint as to the reason, as is observed, that the return current region is highly turbulent.

The consistent treatment of the ion density is the most problematic part of our analysis. A simple discussion of a current-voltage relation makes sense only in the context of a quasi-steady solution. Under our assumptions, because there is a parallel electric field, ionospheric ions will be accelerated downwards and the density and thus the potential will change with time. If one attempts to find a consistent steady-state solution where the density profile is in equilibrium with the current and potential without invoking waves, one does not find solutions that are consistent with the data. One finds solutions where the currents are too small. For example the solutions of *Stern* [1981], though based on idealized distribution functions, illustrate this point. However, because of wave turbulence, the ion distribution is heated. The heating combined with the magnetic mirror force produces a net upward flow of ions. Thus there is a possibility that the ion density may actually be, on the average, in quasi-static equilibrium. Given such a quasi-static equilibrium, our method of solution follows, though the exact value of the potential for a given current will be determined by the ion density profile which will also depend on the ion heating rate. The data show [*Carlson et al.*, 1998] that the ion densities in downward current regions can be in the range a few cm^{-3}, consistent with the assumptions here, and thus the current-voltage relations illustrated here may be close to a representation of reality. The self-consistent treatment of the waves generated in the downward current region is a major remaining problem.

The presence of relatively intense waves in the return current region suggests the possibility that so-called 'anomalous resistivity', that is, current resistivity due to plasma turbulence and waves, may play a role in influencing the current-voltage relation in this region. Whether this is the case can be determined by comparing measurements of the average ionospheric electron velocity (including any locally accelerated counterstreaming component) with measurements of the potential drop along the field line. If anomalous resistivity plays a important role then the average electron velocity should be less than that inferred from the potential drop. The average electron velocity can be determined directly from measurements of the electron distribution function while the potential drop can be inferred from measurements of the perpendicular potential drop and knowledge that perpendicular ionospheric electric fields are relatively small. Some examples of such comparisons [*Carlson et al.*, 1998] show that the average electron energy is comparable to that inferred from the potential drop. More exact comparisons are needed to establish the relative contribution of anomalous resistivity to the current-voltage relation in the return current region.

In contrast it is relatively clear that in the upward current region waves and turbulence have little effect on the current-voltage relation. For instance, *Lu et al.* [1991] found a good agreement with the Knight formula, which is based on the adiabatic motion of energetic electrons. Examination of electron distributions also shows, as would be required for the Knight formula to hold, that electron distributions are fairly consistent with adiabatic trajectories of plasma sheet electrons [*Croley et al.*, 1978]. The main exceptions are trapped electrons and backscattered and secondary electrons which carry little current in any case. Ions, which are not treated in the Knight relation, carry only a small portion of the current.

The current densities are somewhat larger in the return current region than in the upward current region for potential drops of the same magnitude. For the same parameters as in our example but with the sign of the potential reversed (plasmasheet electron temperature=1 keV, plasmasheet electron density=1 cm^{-3}, parallel potential drop=1.14 V at 3000 km altitude), the Knight formula gives a current density at the ionospheric level of 1.6 $\mu A\text{-}m^{-2}$, in contrast to 6.4 $\mu A\text{-}m^{-2}$ that the 2 $\mu A\text{-}m^{-2}$ at 3000 km altitude gives when projected to the ionosphere. Thus, for these parameters, the return current is a factor of four larger for the same magnitude of potential drop. *Weimer et al.* [1987] found that the magnitude of the current-voltage relation in the downward current region was comparable to that in the upward current region in one example. Given their limited number of events and the variation of the current-voltage relation with density and temperature, their result may be consistent with ours. In any case a casual examination of the FAST data shows that downward current densities are typically a factor of a few larger than adjacent upward currents for the same potential as estimated from the perpendicular electric fields (this provides an estimate of the potential below the satellite) though at FAST altitudes it is likely that often some of the potential drop is above the satellite.

The current discussed here refers only to the current carried by the upflowing ionospheric electrons. In addition there are current contributions from ions and precipitating magnetospheric electrons. One might suppose that the downward parallel electric field would result in a larger flux of magnetospheric ions into the ionosphere in analogy with the larger precipitation of magnetospheric electrons in the inverted V region as given by the Knight relationship. While this may be, as has been mentioned already the data show that the net current contribution from the ions is upward since upflowing ion conics typically dominate the ion contribution to the current. In any case the data show that the ion contribution is small compared to the electron contribution. The data also show that the contribution from the precipitating electrons is small for typical downward current regions containing field-aligned upflowing ionospheric electrons.

We have assumed a solution based on the idea that the plasma is on the average charge neutral. The justification for this is discussed well elsewhere [e.g., *Stern*, 1981]. In the opposite limit one gets a current-voltage relation similar to the Child-Langmuir solution for a charge limited diode. The current in that case would be orders of magnitude smaller for equivalent potential drops. Thus the background ion density allows much more current to flow but a substantial parallel electric field is still required.

Acknowledgments. This work was supported by NASA grant NAG5-3596.

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(Received March 6, 1998; revised May 13, 1998; accepted May 18, 1998.)