

# A search for Langmuir solitons in the Earth's foreshock

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**Abstract.** The Earth's foreshock region, the region which is magnetically connected to the bow shock, is a region of intense Langmuir waves. Some of these waves have an envelope similar to that expected for one-dimensional Langmuir solitons. A computer search for such waveforms has identified a number sufficient for statistical investigation. It is concluded that there is no evidence for stable waveforms like one-dimensional Langmuir solitons.

## 1. Introduction

The time domain sampler (TDS) on Wind [Bougeret *et al.*, 1995] has returned a number of waveforms which resemble the expected forms of Langmuir solitons. Langmuir solitons are solitary waves in which dispersion is balanced by nonlinearity, so that the narrower the wave packet, or the larger the range in  $k$  space, the greater must be the nonlinearity and hence the amplitude. Various treatments predict an envelope whose shape is given by Mima and Nishikawa, 1984; Zakharov, 1984]

$$E_n = \frac{A}{\cosh\left(\frac{x}{W}\right)} \quad (1)$$

where  $A$  and  $W$  are inversely proportional though the relation of  $A$  and  $W$  to physical parameters varies among treatments.

The TDS samples the voltage difference between halves of two dipole antennas at sample rates up to 120,000 samples per second and stores 2048 sample "events" in a memory, for possible transmission to Earth. The two dipole antennas ( $E_x$  and  $E_y$ ) are perpendicular and lie in the ecliptic plane. When an "event" is requested by the data processing unit of the experiment, the TDS chooses either (fill events) the most recent event larger than a preset threshold, or (fast events) the event with largest amplitude on the X antenna since the last request and sends it to the telemetry stream. Almost all of the candidate solitons reported here were found among the TDS Fast events, as expected.

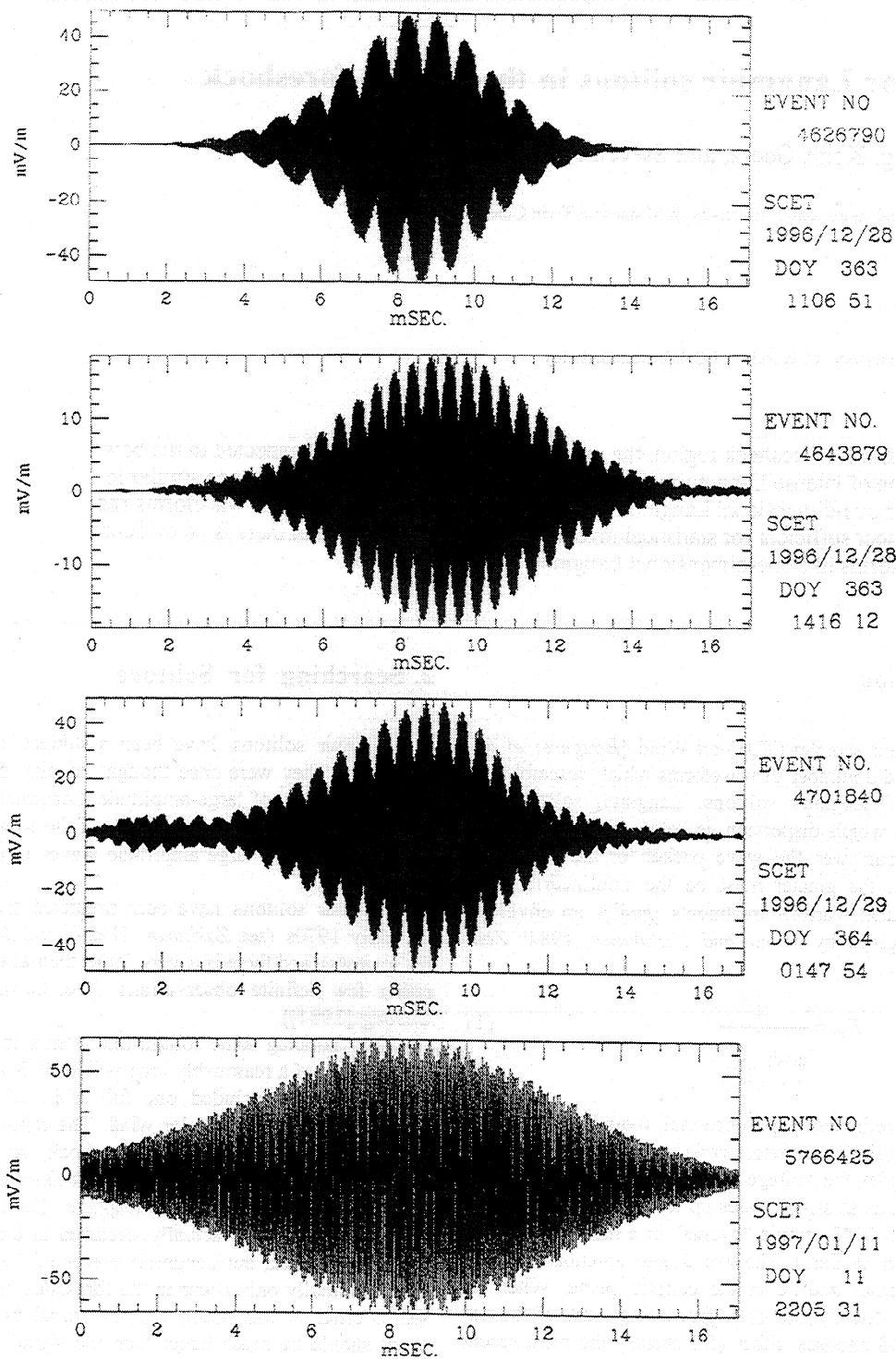
## 2. Searching for Solitons

Langmuir solitons have been a subject of interest since early days. They were once thought to play an important role in the behavior of large-amplitude Langmuir waves and as such play a part in the interaction of the solar wind with the Earth through the large amplitude waves of the Earth's foreshock.

Langmuir solitons have been predicted theoretically since the early 1970s (see Zakharov [1984] and Robinson [1997] for reviews), and there is a very large theoretical literature but rather few definite observations (see, however, Wong and Cheong [1984]).

After noticing some soliton-like events in the data, a computer search of a reasonably long period of data was made. This first data period included one full orbit, with excursions far upstream ( $87 R_E$ ) in the solar wind. The candidate solitons all occurred in the near Earth's foreshock, so further searches were confined to a period of 5 days before and 5 days after a traversal of the Earth's magnetosphere. No test was made to check that the events actually occurred in a region connected to the bow shock, but Langmuir waves of the observed amplitude essentially only occur in the foreshock region. The initial search criterion was simply that the signal in the center of the event should be much larger than the signal at the ends. This excludes wide solitons, though a few found by routine visual inspection of the data have been included. The solitons that satisfied this first criterion were all included in a preliminary list, but then a least squares fit to the expected description of solitons (equation (1)) was made, and only those candidates with fractional rms residual less than a certain value were included in the final list. This limiting value was determined by visual inspection of about 20% of the first list. Fifty-nine examples that passed these tests were found in  $\sim 160$  days of observation.

Figure 1 shows four candidate solitons which seem to have the right shape. The "scallop" of the envelope is due to beating of the sampling frequency, 120 kHz, with a harmonic



**Figure 1.** Examples of soliton-like waveforms captured by the time domain sampler on Wind. The scalloping of the envelope is due to beating of the sampling frequency with harmonics of the wave frequency. The top two examples are "good" solitons according to criteria discussed in the text, while the bottom two were rejected for different reasons.

of the wave frequency (which is 40.2 kHz for the first example), and the true envelope is the envelope of these "scallop." The top two solitons are "good" solitons, which have passed the selection tests including the shape test. The third, event 4710840, does not satisfy the least squares fit criterion be-

cause of the long tail to the left and some asymmetry and is rejected from the final list. One might consider that the tail is an unimportant part which does not justify its exclusion, but other candidates exist with similar residuals and much poorer resemblance to an ideal soliton, so computer-based criteria re-

quire its exclusion. The lowest one is too wide and would not have satisfied the initial choice criterion of the computer search but was found in routine data viewing.

### 3. Forms of Solitons

In the theory of Langmuir solitons, the amplitude  $A$  is inversely proportional to the width  $W$ . Their product, multiplied by the electron charge  $e$  is an energy and is a factor  $F$  times the electron thermal energy  $k_B T_e$ :

$$e A W = F k_B T_e \tag{2}$$

Mima and Nishikawa [1984] found that the factor  $F$  depends on the ratio of ion to electron temperature through the ion sound speed, but Robinson [1997] did not. For solitons at rest and equal temperatures, in both solutions the factor  $F$  is  $6\sqrt{2/3} = 4.9$ .

The soliton solution of the Zakharov equations depends also on the speed of the soliton relative to the plasma, a quantity which we cannot determine. More precisely, the solution of the Zakharov equations for the electric field which describes one-dimensional solitons is [e.g., Robinson, 1997]

$$\begin{aligned} \bar{E} &= E_0 \operatorname{sech} \left[ \frac{E_0 (x' - M t')}{\sqrt{2(1 - M^2)}} \right] \\ \exp \left\{ i \left[ \frac{1}{2} M x' - \left( \frac{M^2}{4} - \frac{E_0^2}{2(1 - M^2)} \right) t' \right] \right\} \end{aligned} \tag{3}$$

with

$$\begin{aligned} x' &= \frac{x}{\frac{3}{2} \sqrt{\frac{T_e m_i}{T_i m_e}} \lambda_D} \\ t' &= \frac{2 m_e}{3 m_i} \omega_p t \end{aligned} \tag{4}$$

and

$$E_0 = \frac{E_{\max}}{\left[ \frac{3 m_i}{4 m_e} \frac{\epsilon_0}{4 N_e k_B T_e} \right]^{1/2}} \tag{5}$$

Here  $E_{\max}$  is the electric field, in volts per meter, at the center of the soliton,  $m_e$  and  $m_i$  are the electron and ion masses,  $T_e$  and  $T_i$  are the electron and ion temperatures,  $k_B$  is the Boltzmann constant, and  $\lambda_D$  is the Debye length:

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{N_e e^2}} \tag{6}$$

This soliton moves through the plasma at a speed  $M C_s$ , where  $C_s$  is the ion sound speed. The factor  $F$  in (2) is

$$F = 6\sqrt{2/3} \sqrt{1 - M^2} \tag{7}$$

We have no way of measuring  $M$ , so take it to be zero below, so that solitons may have lower amplitude than the limit we take. In theory,  $M$  might be estimated from the ratio of the soliton frequency to the ambient plasma frequency, but the ambient plasma frequency is not known with sufficient accuracy and, in any case, is variable on a scale of a second or so.

### 4. Observation

The amplitude of the candidate solitons is easily measured. The width above is the width in the solar wind frame, while the measurements are of a traversal time in the spacecraft frame. The instability which gives rise to solitons must modulate the original monochromatic Langmuir wave with an envelope which travels at the group velocity, and this would be expected to propagate upstream in the solar wind in the same direction as the generating electron beam. One-dimensional solitons, however, only exist for speeds slower than the ion sound speed, so they must slow down to a speed which is therefore slow compared to the solar wind speed [Zakharov, 1984]. Hence the spatial width has been calculated by assuming that the structures are at rest in the solar wind.

Figure 2 shows histograms of the amplitudes, in mV/m, and of the observed widths, in ms, of the candidates which have satisfied the above tests. The initial test, that the ends should have much smaller fields than the middle, excludes wide structures. Note that the width is as defined in (1) and is a factor of

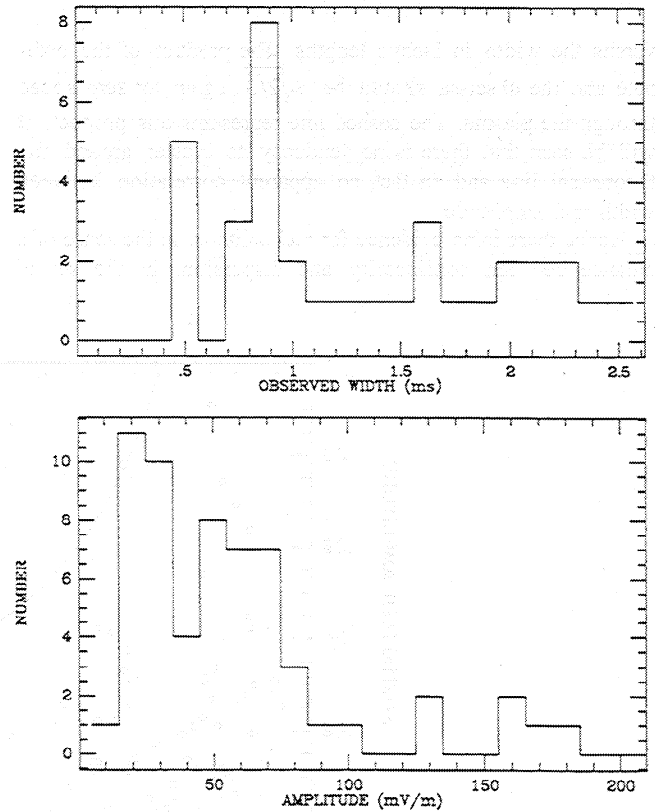


Figure 2. Histograms of the amplitudes, in mV/m, and of the estimated widths, in ms, of the soliton-like waveforms. The right edge on the width histogram represents an estimate of the largest width which would be caught by the computer search.

2.7 times smaller than the full width at half maximum. The right-hand edge of the graph is close to the estimate of the widest structure that would pass the first test, provided that this structure had an envelope of the cosh form.

It will be noted that the histogram of widths does not extend to zero widths but that the most narrow solitons have a width of about 0.5 ms. The translation to width in terms of Debye length will be done below.

If the soliton is nearly one-dimensional (i.e., very flat), the traversal time also depends on the angle between the solar wind velocity and the normal to the soliton plane. Since only two components of the field are measured, this angle cannot be established exactly. However, comparison of the projections of  $\mathbf{E}$  and  $\mathbf{B}$  shows that these projections are moderately well aligned, the median angle between them being  $9.5^\circ$ . This is as expected as the Langmuir waves are generated by an electron beam which is nearly along the magnetic field. Hence it is reasonable to take  $\mathbf{B}$  as a proxy for the direction of  $\mathbf{E}$ . If this is done, there are many packets with a large angle between the magnetic field and the solar wind. This is presumably because the orbit of Wind carries it far out to the sides of the bow shock. The width is then obtained from

$$W = t_{\text{trans}} \frac{v_{\text{sw}} \cdot \mathbf{B}}{|\mathbf{B}|} \quad (8)$$

Figure 3 plots the normalized amplitude, i.e., the electric field  $E_{\text{max}}$  at the center, divided by the quantity

$$\frac{1}{\lambda_D} \sqrt{\frac{k_B T_e}{e}} \sqrt{\frac{k_B T}{e}} \quad (9)$$

versus the width in Debye lengths. The product of the ordinate and the abscissa should be  $6\sqrt{2/3}$ , again for zero speed through the plasma. The dashed line represents this product. It will be seen that there is no tendency to cluster around the theoretical line and, in fact, no apparent correlation between width and amplitude.

Hence there is no evidence for real solitons, in the sense of a balance between nonlinearity and dispersion, in the set of

structures found here. "No evidence for" must be taken seriously, and we summarize the approximations made in Figure 3. First, the solitons have been taken at rest, i.e.,  $M = 0$  in equation (3). A finite value of  $M$  would reduce the product of amplitude and width, in accordance with what is observed. It is therefore possible that these candidate solitons have  $M$  near unity and are actual solitons.

However, if there were solitons being created, then we should expect that there were also wave packets with greater amplitudes than the equilibrium between dispersion and nonlinearity, as well as smaller. The packets with greater amplitudes would be in the process of collapse, and we would observe a continuum of widths down to zero. Robinson [1997] has commented that the process of collapse is so rapid, however, that one should expect just such a dearth of narrow packets. Hence the dearth of narrow packets is consistent with collapse, and we can draw no conclusion from it.

If the packets are nearly one-dimensional, i.e., are very flat disks with the electric field normal to the plane of the disk, then we should expect a correlation between their apparent width and the angle between  $\mathbf{B}$  and the solar wind. This is shown in Figure 4. The individual points are shown together with the result of a least squares fit to the form  $W = A + B \sin^2(\theta)$ . It will be seen that there is only a small correlation, though it should be noted that our selection criterion excludes widths wider than about 2.7 ms. Hence there is a suggestion that the structures are not flat but are nearly spherical.

We have not, therefore, provided any solid evidence for solitons which resemble one-dimensional solitons, except for their shape. This is as expected. Solitons in more than one dimension are subject to a transverse instability [Zakharov, 1984], which destroys them.

## 5. A Peculiar Event

Figure 5 shows the waveform of an event captured by the TDS which is too weak in amplitude to be considered a soliton candidate but which presents some features which incline

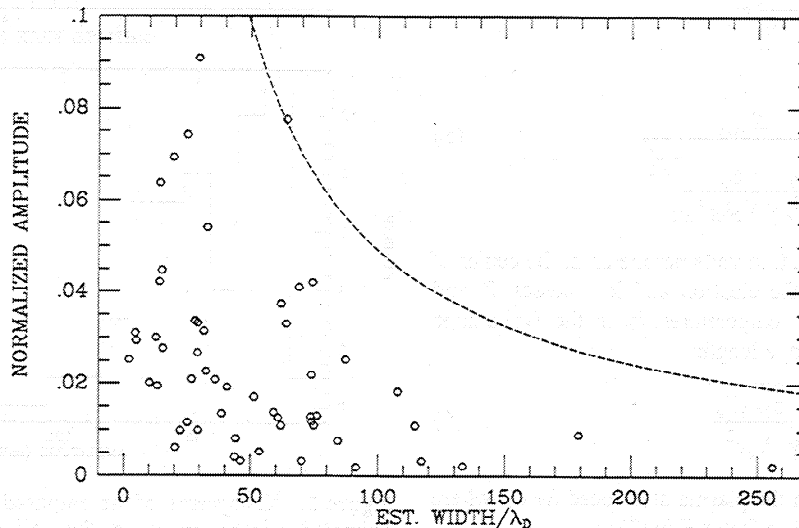
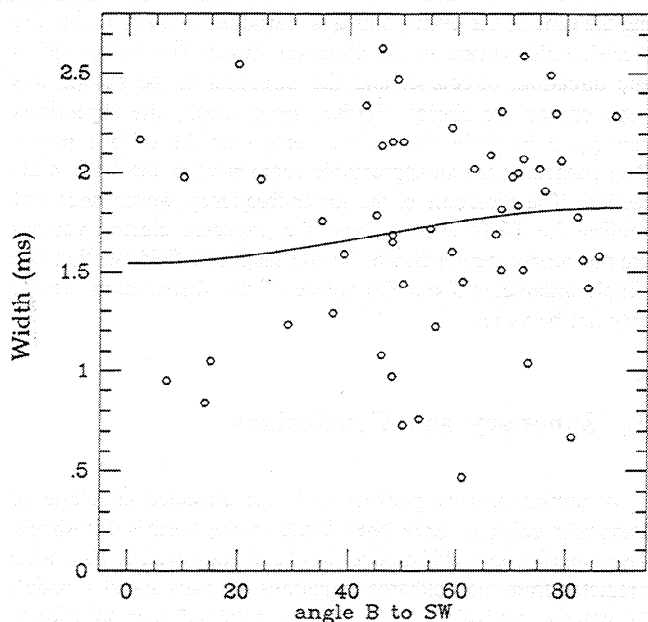


Figure 3. A scatter plot of amplitude normalized as in the text versus width in Debye lengths. The dashed line represents the expected relation for one-dimensional solitons at rest in the solar wind, as described in the text.



us to consider it in the context of solitons. It will be seen that there is a central packet at a higher frequency, 31.7 kHz, which appears to be radiating lower-frequency waves. The frequency of these is 4.04 kHz before the central packet and 4.80 kHz later. There are also short bursts of the higher-frequency waves, presumed to be Langmuir waves, mixed in with the low-frequency waves.

These lower frequencies fall into the range of what is usually identified in the solar wind as ion acoustic waves [Gurnett and Anderson, 1977; Gurnett and Frank, 1978]. For ion acoustic waves the Doppler shift due to the solar wind speed is larger than their frequency in the solar wind frame, an anomalous Doppler shift, so that the observed frequency  $\omega_{obs} = \omega - \mathbf{k} \cdot \mathbf{v}_{sw}$  is mainly just  $\mathbf{k} \cdot \mathbf{v}_{sw}$ .

Figure 4. A scatter plot and fitted curve to search for the dimensions of the packets parallel and perpendicular to the magnetic field.

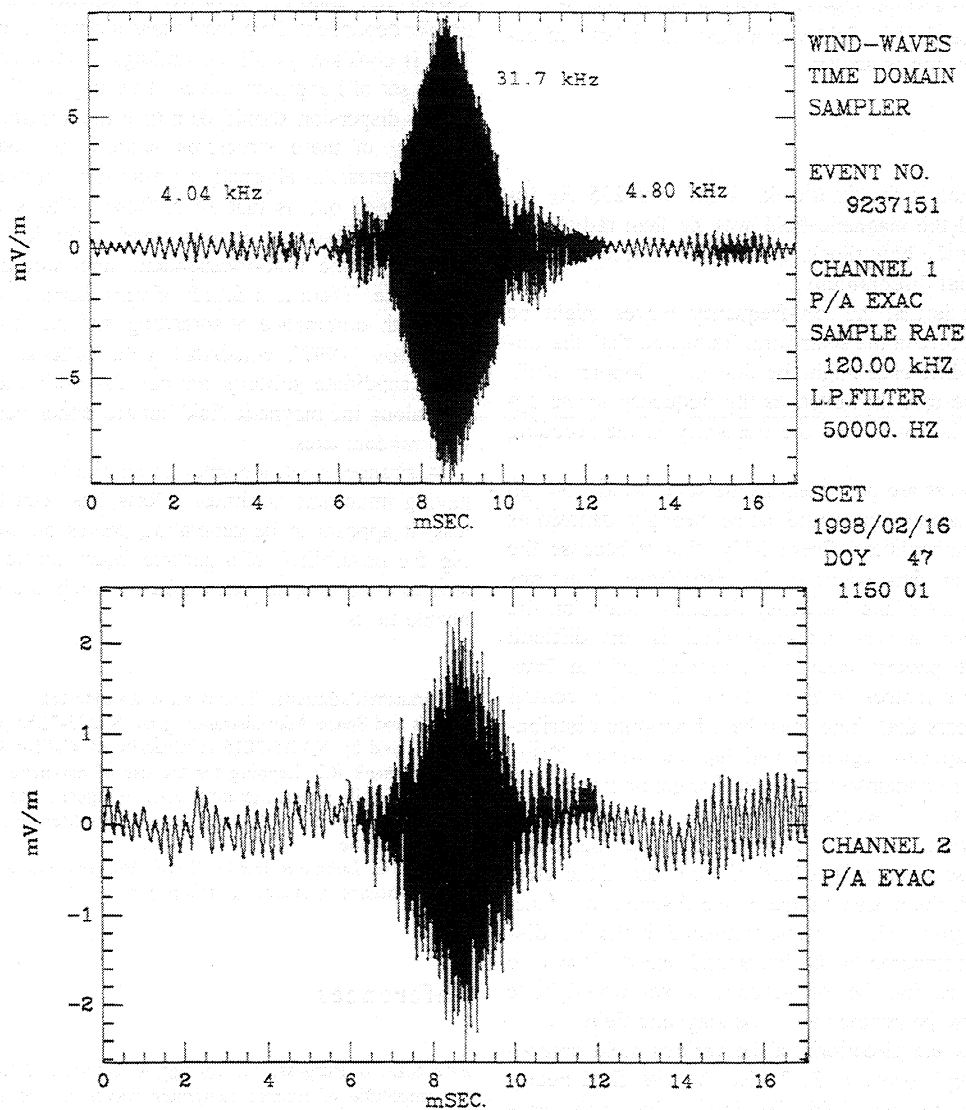
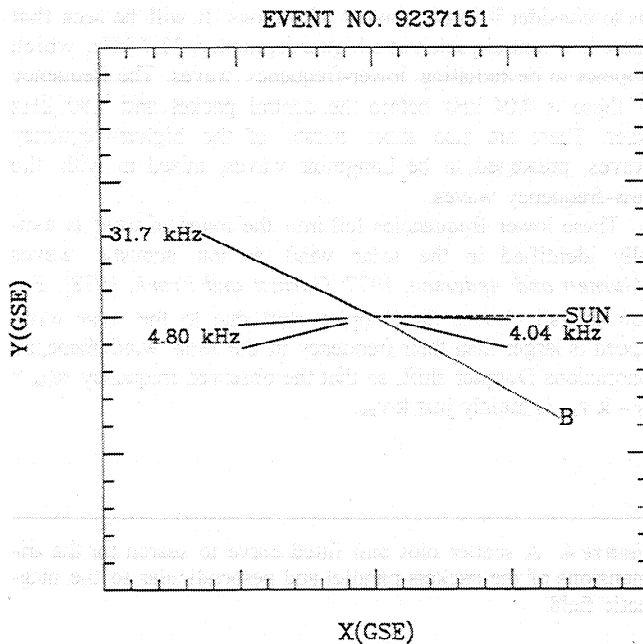


Figure 5. A peculiar event captured by the time domain sampler. This event is not in the foreshock but in the undisturbed solar wind.



**Figure 6.** Directions of the electric fields of the waves of Figure 5. Note that the electric field vectors extend in both directions, though only one is shown.

This event is not in the foreshock. Wind was  $235 R_E$  toward the Sun, and the magnetic field was far from radial and could not intersect the bow shock. The event occurred near the edge of a tangential discontinuity.

A first reaction is that the low-frequency waves might be radiated symmetrically in the solar wind frame and that the observed frequency difference might be due to a Doppler shift. This cannot be the case, however, as the frequency in the approaching region is lower than the frequency in the receding region.

Ion acoustic waves are prominent in the solar wind, though in a Maxwellian plasma they would be so strongly damped as to be essentially nonexistent. Presumably, this is because the damping depends on derivatives of the distribution functions (whereas the real part of the frequency depends mainly on the moments), and these can vary on a scale which is very difficult to measure. In the present case, the amplitude of the low-frequency waves increases slightly away from the central packet, and it appears that there must be a beam-like distribution in the low frequency region to maintain the waves. If this beam is to generate or maintain both the Langmuir waves and the ion acoustic waves, it seems likely that it is a beam in the electron distribution. At any rate, to generate Langmuir waves, whose phase velocity is at least of the order of the electron thermal speed, the positive slope of the distribution function must fall at a point which is fast compared to the ion distribution and fast compared to the ion sound speed. It would therefore be expected that the ion acoustic waves would have wave vectors nearly perpendicular to the magnetic field.

Figure 6 shows the directions of the various field components. The "primary" wave, at 31.7 kHz, has its field nearly aligned along the magnetic field. This is not always the case for Langmuir waves in the foreshock [Bale *et al.*, 1998]. The directions of the lower-frequency waves vary somewhat dur-

ing the time of the event, and this variation is shown. All the electric fields extend in the direction shown and in the opposite direction, of course, and the direction to be shown has been chosen for clarity. Further, once again, the directions shown are the projections of the vectors on the ecliptic plane. *B* in particular has an appreciable component in the GSE *Z* direction. The direction of the lower-frequency waves does not confirm the above expectations: the projected electric vectors are not nearly perpendicular to the magnetic field, so that the simple arguments about the nature of the distribution above may not be valid.

## 6. Summary and Conclusions

A number of wave packets with the expected envelope of Langmuir solitons have been found in the Earth's foreshock. Their amplitude-width product has been compared to the value predicted from the Zakharov equations for zero speed through the plasma, equivalent to the nonlinear Schrodinger solutions. For the candidate solitons which have been found, Figure 3 shows that there are more with a smaller amplitude than the time-independent form than there are with a larger amplitude. This is consistent with the findings of Bale *et al.* [1997] on a wider set of Langmuir waves. This implies that the spreading due to dispersion should dominate the contraction so that the majority of these structures should not be collapsing to smaller ones. An alternative possibility, however, which cannot be ruled out, is that all of these solitons are moving relative to the plasma at speeds close to the ion sound speed, in which case the time-independent form would have a smaller amplitude. There is a dearth of very narrow solitons, consistent with dominance of spreading and but also, according to Robinson [1997], consistent with collapse. It appears that these candidate solitons are not flat with their short dimension along the magnetic field but are either nearly spherical or with random axes.

A strange event, too small in amplitude to be considered as having important nonlinear effects, has been found. Nevertheless, it appears to be generating waves on each side, indicating the possibility of a particle beam emitted by the central structure. This event and others which are similar remain a puzzle to us.

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