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HF Chirps: Eigenmode trapping in density depletions

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Abstract. Short-duration, narrow-bandwidth, HF chirps occur at slowly drifting frequencies above f_{pe} when $f_{pe} > f_{ce}$ [*McAdams and LaBelle*, 1999]. We put forth a model of the HF chirp emissions as quasi-trapped eigenmodes in a density depletion perpendicular to the ambient magnetic field. The escaping waves retain the frequency structure of the eigenmodes and are observed as HF chirps. We show that this model is quantitatively consistent with observed characteristics of HF chirps, most notably the frequency spacing of the chirp emissions (0.1-4 kHz), the fact that they are equally spaced independent of mode number, and the number of modes predicted for a given density cavity. Several mechanisms for the escape of the waves from the cavity are suggested.

1. Introduction

High-frequency waves associated with electron beams have been observed in the auroral ionosphere by rockets and satellites for many years [*Kellogg et al.*, 1978; *Beghin et al.*, 1989; *Bonnell et al.*, 1997; *McFadden et al.*, 1986, and references therein]. Indications of frequency structure in waves near f_{pe} were observed by *Beghin et al.* [1989] although the time resolution was insufficient to determine the temporal structure of the waves. Recently, detailed observations of frequency and time structure in high-frequency waves have become available from the PHAZE II sounding rocket [*McAdams and LaBelle*, 1999; *McAdams et al.*, 1999]. One type of structured emission seen during the flight are short duration, narrow bandwidth, drifting frequency emissions termed HF chirps [*McAdams and LaBelle*, 1999]. In this paper, we put forth a mechanism for generation of HF chirps.

HF chirps have been observed in the topside auroral ionosphere at altitudes of 299-320 km. They have frequencies of 1.8-2.3 MHz which are above the local plasma frequency (f_{pe} ~1.7-2.2 MHz) and have been observed in the overdense plasma where the electron plasma frequency (f_{pe}) is higher than the electron cyclotron frequency (f_{ce}). The most striking feature of the HF chirps is their narrow bandwidth, $\Delta f \sim 300-600$ Hz. HF chirps often occur simultaneously at several equally spaced frequencies in "flocks". The spacing between the chirps in each "flock" is typically 1-2 kHz and remains constant for the duration of the emissions. The durations of the emissions range from 10 to 300 ms, but are generally less than 100 ms, corresponding to 30 m to 1 km assuming the structures are stationary and using the vehicle velocity of 3.4 km/s to convert duration to spatial size. Most HF chirps have decreasing frequency with time. HF chirps are often associated with density cavities or enhancements with $\delta n/n$ =1-5% and with spatial dimensions of approximately 50-1000 m. HF chirps are occasionally observed "tunnelling" into higher density regions from inside a density depletion. *McAdams and LaBelle* [1999] provide a more complete description and show observations of the HF chirps.

2. Trapping Theory

We put forth a model of the HF chirp emissions as quasi-trapped eigenmodes in a density depletion perpendicular to the ambient magnetic field (**B**). We model the density depletion as a shallow parabolic cavity: $n = n_o + (n''x^2)/2$, where $n_o \gg n''a^2$, a is the width of the cavity, x is the distance perpendicular to **B**, and z is parallel to **B**. From their frequency range ($f > f_{pe} > f_{ce} \neq 0$) we infer that the HF chirps are propagating on the Langmuir/upper hybrid wave surface (L/UH). The parallel wavevector k_z is constant, determined by the resonance with the beam electrons which are responsible for initially generating the L/UH waves ($k_z = \omega/v_{beam}$). We assume that these waves are primarily parallel to the background magnetic field $\mathbf{B} = B_z\hat{z}$ and $k_x << k_z$ allowing us to characterize the waves with the oblique Langmuir wave dispersion relation for the moderately magnetized case [*Ergun*, 1989; *Newman et al.*, 1994]:

$$\epsilon \cong \left(1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{3v_{th}^2k_z^2}{\omega^2}\right)\frac{k_z^2}{k^2} + \left(1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2}\right)\frac{k_x^2}{k^2} \tag{1}$$

Defining $k^2\epsilon = \gamma k_z^2 + \beta k_z^2$, and setting $\epsilon = 0$ gives $\beta k_x^2 = -\gamma k_z^2$. The assumption of $k_x << k_z$ is reasonable because when $\theta = \tan^{-1}(k_x/k_z)$ becomes large, the waves experience large cyclotron damping [*Newman*

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et al., 1994] given the plasma conditions during observations of the HF chirps.

We solve the dispersion relation (1) in terms of k_x, assuming $k_z = \omega/v_{beam}$ remains constant as determined above. Using the above expression for n, we expand γ around $x = 0$ such that $\gamma \cong \gamma_o - \gamma'' x^2/2$. This gives

$$\gamma \cong \left[1 - \frac{\omega_{po}^2}{\omega^2} - \frac{3v_{th}^2 k_z^2}{\omega^2}\right] - \frac{x^2}{2}\left[\frac{n''}{n_o}\frac{\omega_{po}^2}{\omega^2}\right] \quad (2)$$

where ω_{po} is the plasma frequency when $n = n_o$. Furthermore, by assuming $\omega_{po} \simeq \omega_{pe}$ the expression for β simplifies to $\beta \simeq (-\omega_{ce}^2)/(\omega^2 - \omega_{ce}^2)$.

Figure 1 shows the dispersion relation (1) for the L/UH wave surface in an overdense plasma with $k_x \ll k_z$, $f_{pe} = 2.1$ MHz, and $f_{ce} = 1.4$ MHz. The vertical axis represents k_z and the horizontal axis represents k_x on a greatly expanded scale. Contours of constant ω are shown. The perpendicular wavenumber k_x varies with the varying electron density in the cavity. Traped modes have the property that the phase of the wave varies by an integer multiple of π between two reflection points at which $k_x = 0$. These trapped modes, as we demonstrate later, are equally spaced in ω and are represented by the contour lines in Fig. 1. The intersections between these lines and the $k_z = \omega/v_{beam}$ resonance condition (represented by a horizontal line in Fig. 1 for constant k_z) represent the trapped eigenmodes which achieve a high energy density within the cavity, essentially being pumped up by resonance with the particles until the rate of energy input from the electrons matches energy loss through escape from the cavity. The escaping waves retain the frequency structure of the eigenmodes and are observed as HF chirps. We shall show quantitatively that this model is consistent with observed characteristics of HF chirps, most notably the frequency spacing of the chirp emissions and the fact that these are equally spaced.

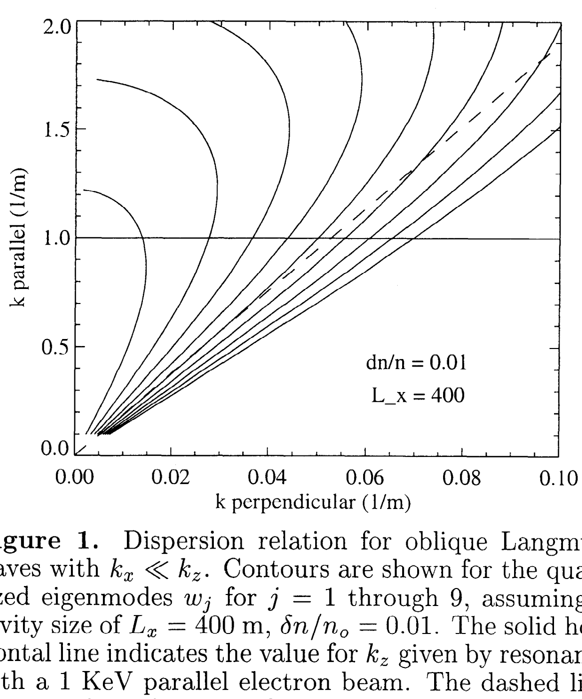

Figure 1. Dispersion relation for oblique Langmuir waves with $k_x \ll k_z$. Contours are shown for the quantized eigenmodes w_j for $j = 1$ through 9, assuming a cavity size of $L_x = 400$ m, $\delta n/n_o = 0.01$. The solid horizontal line indicates the value for k_z given by resonance with a 1 KeV parallel electron beam. The dashed line denotes where the angle of propagation equals 3 degrees; modes to the right of this line are subject to cyclotron damping [*Newman et al.*, 1994].

The value of k_x for a given ω depends on the local density which varies only slightly spatially. To calculate the eigenmodes we solve $k_x^2 = -\gamma k_z^2/\beta$, ignoring contributions from $(\partial|k_x|/\partial x)/k$ and $(\partial n/\partial x)/n_o$ which is valid only for higher order modes. Thus we can approximate the phase of the wave as the spatial integral of k_x. We define two conditions which determine the quantization of the wave modes: (1) $k_x = 0$ at the turning points of the wave in the cavity, $x = \pm a_j$; and (2) the phase difference at the turning points must be $j\pi$ where j is an integer. Using the arguments above, we can express the latter condition approximately as:

$$\int_{-a_j}^{a_j} k_x(x)dx = \pi j \quad (3)$$

Substituting (2) and $\beta \simeq (-\omega_{ce}^2)/(\omega^2 - \omega_{ce}^2)$ into (1), we obtain an approximation for the dispersion relation in the inhomogeneous plasma which may be solved for k_x and substituted into (3). The solution of the resulting integral is :

$$\pi j = k_z \sqrt{\frac{\gamma''}{-2\beta}}\left[\xi^2 \sin^{-1}\left(\frac{a_j}{\xi}\right)\right] \quad (4)$$

where we define $\xi^2 = (2\gamma_o)/\gamma''$.

Combining (4) with the dispersion relation ($\beta k_x^2 = -\gamma k_z^2$), we set $k_x = 0$ at $x = \pm a_j$ which yields $\gamma = 0$. From the previous definition $\gamma = \gamma_o - \gamma'' x^2/2$ we find $a_j^2 = (2\gamma_o)/\gamma'' = \xi^2$. Requiring $j > 1$ due to the previous assumption that k_x is slowly varying and using (2) to approximate $\gamma'' \simeq n''/n_o$ gives the quantization rule:

$$j = \frac{k_z a_j^2}{2}\sqrt{\frac{(\omega^2 - \omega_{ce}^2)}{2\omega_{ce}^2}\frac{n''}{n_o}} \quad (5)$$

From the quantization rule, we estimate the frequency spacing of the trapped modes. The mode frequency (w_j) is determined by setting $k_x = 0$ when $x = \pm a_j$ and assuming $(3v_{th}^2 k_z)/(\omega^2) \ll 1$ (ie., neglecting the thermal correction).

$$\omega_j^2 = \omega_{po}^2(1 + \frac{a_j^2}{2}\frac{n''}{n_o}) \quad (6)$$

The quantization rule gives a_j in terms of j. Using the approximation $(\omega_j^2 - \omega_{j-1}^2) \simeq 2\omega_{po}(\omega_j - \omega_{j-1})$, valid when $\omega_j - \omega_{j-1} = \Delta\omega$ is small, and setting $\omega^2 \simeq \omega_{pe}^2$, we obtain an expression for the frequency spacing.

$$\Delta\omega_j = \omega_j - \omega_{j-1} = \frac{\omega_{po}^2}{2k_z}\sqrt{\frac{n''}{n_o}\frac{2\omega_{ce}^2}{\omega_{po}^2 - \omega_{ce}^2}} \quad (7)$$

The prediction from the eigenmode theory that the frequency separation of the waves does not depend on j is consistent with observations of HF chirps reported in *McAdams and LaBelle* [1999].

3. Discussion

The frequency splitting of the eigenmodes depends on four observable quantities: n''/n_o, k_z, ω_{ce}, and ω_{po}. The requirement that the waves must be in resonance with the electrons determines $k_z = \omega/v_{beam}$. Measurements of electron energy spectra on the PHAZE II payload at the time of the PHAZE II HF chirp observations reveal a beam energy of approximately 1 keV implying $k_z \simeq 1$ m^{-1} for $\omega \simeq \omega_{po} \simeq 2.1$ MHz as observed [*McAdams and LaBelle*, 1999]). The observed magnetic field implies $f_{ce} = 1.38$ MHz. Density fluctuations are observed with $\delta n/n_o$ ranging from 0.5 to 5.0% and with characteristic length scales (L_x) ranging from 50-800 m, where L_x is defined as the spatial size of an observed cavity from its center to the point where the density returns to the background level, and $\delta n/n_o$ describes the maximium depth at the center of the density cavity. The ratio n''/n_o can be estimated by $n''/n_o \simeq (2\delta n)/(L_x^2 n_o)$. Substituting these observed values into (7) leads to a frequency separation of 100 to 4000 Hz. Figure 2 shows the dependence of $\Delta\omega$ on $\delta n/n_o$ and L_x. The observed separation between the HF chirps is usually below 2 kHz, constraining the size and depth of the density cavities which can trap the HF chirp producing waves. For example, taking $\delta n/n_o =$ 1% and $\Delta\omega = 2000$ Hz requires $L_x \sim 100$ m; these numbers are consistent with density fluctuations observed within the HF chirp regions during the PHAZE II flight.

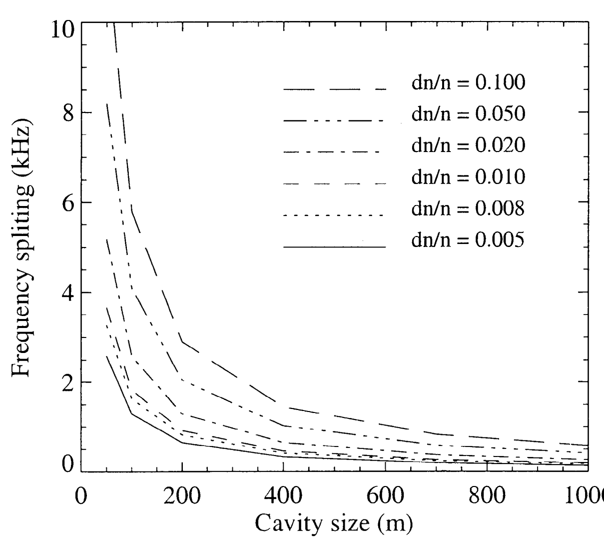

Figure 2. Relationship of $\Delta\omega$ to n''/n_o and L_x. Each line represents a different value of $\delta n/n_o$, the x axis represents L_x. The plot covers observed ranges of $\delta n/n_o$ and L_x for density fluctuations observed near HF chirps.

The number of modes in the cavity can be limited either by the depth of the density cavity $\delta n/n_o$ or by cyclotron damping. Under the conditions of Fig. 1 a 1% ($\delta n/n_o$) cavity of 400 m width could contain at most ~ 10 modes (with the constraint that the eigenfrequency is no more than 0.5% greater than f_{pe}). A shallower or narrower density well could not support as many modes. Cyclotron damping becomes significant if $k_x/k_z \sim \mathcal{O}(0.1)$ [*Newman et al.*, 1994] and also can limit the number of modes. In Fig. 1, the number of modes that can exist without severe damping is shown by the number of modes to the left of the dashed line representing $k_x/k_z = 0.05$. Cyclotron damping, under the conditions of Figure 1, would also restrict the number of modes to less than 5-10. The larger damping experienced by the higher j modes as k_x/k_z increases may help explain why relatively low numbers of HF chirps are usually observed outside of the density cavity. The predicted number of modes in the cavity (5-10 for $\delta n/n_o \sim 1\%$) as determined by either the finite dimensions of the cavity or cyclotron damping agrees with the observations of the HF chirps in the cavities.

Interpreting HF chirps as quasi-trapped eigenmodes in a parabolic density well predicts that the frequency spacing of the waves does not depend on the mode number j as observed. The range of frequency spacings predicted by the eigenmode theory (0.1-4 kHz), based on the observed $\delta n/n_o$ and L_x of density fluctuations observed near the same times as the HF chirps, spans the range of observed spacings. Conversely, the values of n''/n_o predicted by the theory for the observed HF chirp frequency spacings (0.3-2 kHz) are consistent with the characteristics of density cavities observed coincident with the most intense HF chirps.

Since k_z remains constant, the waves remain in resonance with the beam electrons, the waves grow continuously, and energy accumulates in the trapped wave modes. The dispersion relation indicates that for certain values of j the parallel group velocity, v_{gz}, remains close to zero, implying that modes near this j value would stay at approximately the same altitude along the field line as they grow.

A puzzling aspect of the HF chirp observations is their escape from the density cavity that creates them. HF chirps are often observed outside of density depletions [*McAdams and LaBelle*, 1999]. One possibility involves trapping inside a density cavity that does not provide 4π steradian containment, for example, a "box canyon" type of density cavity where one side is open but the other walls provide enough of a resonant cavity to trap the wave power. The waves can then leak out through the dip in the walls. A second possibility is tunnelling through a spatially thin density cavity boundary. For an HF chirp with $f = 2.106$ MHz propagating perpendicularly into a boundary where $f_{pe} = 2.126$ MHz, the skin depth is approximately 6.7 m. If the density barrier is thin enough to allow a significant fraction of the wave power through into a lower density region, the wave can then propagate freely until it is refracted enough that its angle of propagation becomes too oblique and it is cyclotron damped. Some evidence of this type of wave tunneling is seen in the HF chirp observations [*McAdams and LaBelle*, 1999]. A third possibility would be a non-linear wave process in the cavity leading to wave mode conversion.

The propagation characteristics of the HF chirp waves outside of the cavity are beyond the scope of this paper; however, Landau or cyclotron damping of oblique L/UH waves may account for the short duration of the HF chirps. If the escaping waves propagate more than a few degrees from parallel, they are quickly damped by electron cyclotron damping [*Newman et al.*, 1994]. Alternatively, the escaping waves may refract (change in k_z) so that they experience strong Landau damping.

Finally, it is worth addressing the unequivocable observation that the vast majority of the HF chirp emissions exhibit frequencies which decrease in time [*McAdams and LaBelle*, 1999]. If the electron density structure which imposes the eigenmodes results from precipitation induced ionization in the ionosphere which subsequently diffuses up the field lines, the movement of the density structure up the field lines means that the trapping conditions are produced at successively higher altitudes where the resonant frequencies are lower. Further treatment of this issue is beyond the scope of this paper and is left for more detailed modelling.

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References

Beghin, C., J. L. Rauch, and J. M. Bosqued, Electrostatic plasma waves and HF auroral hiss generated at low altitude, *J. Geophys. Res.*, *94*, 1359, 1989.

Bonnell, J., P. Kintner, J.-E. Wahlund, and J. A. Holtet, Modulated Langmuir waves: Observations from Freja and SCIFER, *J. Geophys. Res.*, *102*, 17233, 1997.

Ergun, R. E., Linear and nonlinear wave processes in the auroral ionosphere, Ph.D. thesis, Department of Physics and Space Sciences Laboratory, University of California at Berkeley, 1989.

Kellogg, P. J., S. J. Monson, and B. A. Whalen, Rocket observations of high frequency waves over a strong aurora, *Geophys. Res. Lett.*, *5*, 47, 1978.

McAdams, K. L., and J. LaBelle, Narrowband structure in HF waves above the electron plasma frequency in the auroral ionosphere, *Geophys. Res. Lett.*, *26*, 1825, 1999.

McAdams, K. L., J. LaBelle, M. L. Trimpi, P. M. Kintner, and R. A. Arnoldy, Rocket observations of banded structure in waves near the Langmuir frequency in the auroral ionosphere, *Accepted by J. Geophys. Res.*, 1999.

McFadden, J. P., C. W. Carlson, and M. H. Boehm, High-frequency waves generated by auroral electrons, *J. Geophys. Res.*, *91*, 12,079, 1986.

Newman, D. L., M. V. Goldman, R. E. Ergun, and M. H. Boehm, Langmuir turbulence in the auroral ionosphere: 1. Linear theory, *J. Geophys. Res.*, p. 6367, 1994.

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