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INFLUENCE OF EXTERNAL DENSITY FLUCTUATIONS ON PARAMETRIC 3–WAVE INTERACTION

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ABSTRACT

Recent satellite observations seem to invalidate the classical scenario of Langmuir turbulence. Among the possible candidates which may play an important role to be taken into account are external density fluctuations. Parametric three-wave interaction is studied in the presence of quasi-monochromatic large-scale and low-frequency density fluctuations.

INTRODUCTION

Langmuir turbulence plays an important role in the generation of type III radio bursts, in the relaxation of energetic electron beams in the solar wind, plasma-laser interaction and several planetary and astrophysical systems (see Robinson, 1997, for a review). Recent observations onboard of satellites WIND, POLAR, Ulysses and others provided new insights into this classical topic.

The traditional picture of Langmuir turbulence involves parametric instabilities, weakly turbulent homogeneous cascades and strongly nonlinear localized phenomena such as wave collapse (Galeev *et al.*, 1977, Robinson, 1997). However, this scenario is hardly supported by observations, where no clear evidence of Langmuir wave collapse has been found yet (Cairns and Robinson, 1995). Instead, quasi-monochromatic weakly nonlinear wave packets or small amplitude bursty electric fields are frequently observed (e.g. Bale *et al.*, 1996).

Among the phenomena which are thought to prevent the developement of the classical scenario and generate different behavior, are electron phase space holes (e.g. Omura *et al.*, 1996). It was also proposed that nonlinear phenomena play a neglectible role in the evolution beam–generated Langmuir waves in the solar wind, and that their fluctuations are just the effect of the random (Gaussian) fluctuations of the instability growth rate (Robinson, 1995). Another phenomenon which should not be overlooked is the presence of strong density fluctuations in the solar wind, which can reach a few percents of the average density (Neugebauer, 1975, Kellogg *et al.*, 1999). Strong Langmuir turbulence generates intense density fluctuations, which are weakly damped in a strongly non-isothermal plasma. Furthermore, strong density fluctuations can be generated by phenomena independant of Langmuir turbulence, such as compressible MHD turbulence. It can be expected that these fluctuations may achieve a level up to which they have a significant influence on parametric instabilities of Langmuir waves. Density fluctuations may also result in effects like Langmuir wave scattering, reflection or mode conversion, which we shall discard hereafter. Galeev *et al.* (1977) and Alterkop *et al.* (1976) have shown that the plasmon scattering on short wavelength can stabilize the modulational instability and prevent the formation of solitary waves in 1D. It was also recently shown that large scale density fluctuations can prevent the formation of Langmuir collapses (Robinson and de Oliveira, 1999). Concerning parametric 3–wave instability, Nicholson (1976) has shown that a sinusoidal density profile superposed to a linear gradient can transform the convective instability into an absolute one, and reduces the growth rate of the parametric instability. Hereafter we investigate the effects of external quasi-monochromatic low frequency density fluctuations on the dynamics of the classical parametric three-wave interaction.

MODIFIED 3–WAVE INTERACTION

Three wave interaction has not only its own interest from the theoretical side, but there exist some strong experimental indications that it is present in Langmuir turbulence in the solar wind (Bale *et al.*, 1996). Some early work considered the parametric three–wave interaction (3WI) in a plasma with weak, large scale and stationary inhomogeneities, which induce fluctuations in the local wavenumbers and a mismatch in resonance conditions $\kappa(x) = k_0(x) - k_1(x) - k_2(x)$ (Rosenbluth, 1972). Various cases were considered, for instance the case where one wave has constant amplitude (pump wave) in the presence of linear density gradient with fluctuations (Nicholson and Kaufman, 1974, Nicholson, 1976), or cases with all three amplitudes change in time (Tamoikin and Fainshtein, 1972, Reiman, 1979). Thus previous considerations were restricted to quenched (stationary) inhomogeneities, and also to the conservative case. Other extensions of the classical three-wave interaction are for example the study of finite bandwidth effects (e.g. Martins and Mendonça, 1988) or the inclusion of a fourth wave (e.g. Lefebvre and Krasnoselskikh 2001).

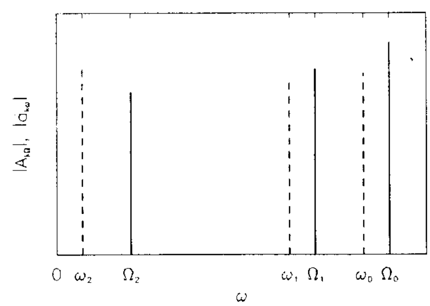

Figure 1: Assumed spectrum. Thick lines correspond to the original 3 waves $\{A_k\}$, and dashed lines are fluctuations $\{a_k\}$ induced by the forced low-frequency density fluctuations.

Hereafter we shall consider the parametric 3WI in the presence of quasi-monochromatic large-scale forced density fluctuations, in a plasma otherwise homogeneous. Weak fluctuations in the k–spectrum are assumed, so that we can concentrate on the temporal evolution. The classical system of 3WI including linear growth and damping rates, is extended by taking into account the modes induced by density fluctuations (see fig. 1). It reads

$$\begin{aligned}
(\partial_t - \Gamma_0 + i\delta\Omega_0)\, A_0 &= iUA_1A_2 + iVa_0a_2, & (1)\\
(\partial_t - \Gamma_1 + i\delta\Omega_1)\, A_1 &= iUA_0A_2^* + iVa_1a_2, & (2)\\
(\partial_t - \Gamma_2 + i\delta\Omega_2)\, A_2 &= iUA_0A_1^*, & (3)\\
(\partial_t - \gamma_0 + i\delta\omega_0)\, a_0 &= iVA_0a_2^*, & (4)\\
(\partial_t - \gamma_1 + i\delta\omega_1)\, a_1 &= iVA_1a_2^*, & (5)\\
(\partial_t - \gamma_2 + i\delta\omega_2)\, a_2 &= iV(A_0a_0^* + A_1a_1^*) + if(t), & (6)
\end{aligned}$$

Table 1: Parameters common to all simulations in section 2

Γ_0	Γ_1	Γ_2	γ_0	γ_1	γ_2
1	-0.6	-2.5	0.	-1.8	-5.1
$\delta\Omega_0$	$\delta\Omega_1$	$\delta\Omega_2$	$\delta\omega_0$	$\delta\omega_1$	$\delta\omega_2$
1.7	0.	0.	0.8	1.4	3.
F	ω_f	U			
50	$16\pi 10^{-3}$	1			

where Γ, γ are linear growth and damping rates, $\delta\Omega_k, \delta\omega_k$ are frequency shifts, U, V are matrix elements of the interaction and f is the forcing term for the low-frequency density fluctuation a_2.

Taking $V = 0$, one recovers the usual 3WI whose non-linear dynamics was extensively studied (Vyshkind and Rabinovich 1976, Wersinger *et al.*, 1980, Hugues and Proctor, 1994). They satisfy the resonance conditions

$$K_0 = K_1 + K_2,\ \Omega_0 = \Omega_2 + \Omega_2 - \delta\Omega,$$

with $\delta\Omega = \delta\Omega_0 - \delta\Omega_1 - \delta\Omega_2$, while the forced fluctuations satisfy

$$k_n = K_n + k_2,\ \omega_n = \Omega_n + \omega_2 + \delta\Omega_n + \delta\omega_2,$$

for $n = 0, 1$. If $V > 0$, forced fluctuations provide a nonlinear frequency shift to the primary Langmuir waves.

Eqs (1–6) can be normalized such that $\Gamma_0 = +1$ and $U = 1$. Moreover, we shall consider the case were in the absence of forcing ($f \to 0$) the forced modes disappear due to damping ($a_k \to 0$ for all k).

TRANSITION FROM CHAOTIC 3WI

In this section, the coupling coefficient V of the external fluctuations induced modes with the original ones is progressively increased. We start from a chaotic state of the pure 3WI (corresponding to $V = 0$), since these are the most structurally stable.

The forcing term is simply taken in the form

$$f = F\exp(i\omega_f t) \tag{7}$$

with $\omega_f \ll \Omega_2$. And fixed parameters are shown in Table 1.

The 3WI dynamics is sketched on Fig. 2, which shows a projection of the attractor onto the $(|A_1|, |A_2|)$ plane. The parameters are shown on Fig. . The attractor is organised around the homoclinic orbit of the fixed point, which reads using previous notation

$$\begin{aligned} |A_0| &= \frac{\sqrt{\Gamma_1\Gamma_2}}{U\sin\Phi},\ |A_1| = \frac{\sqrt{\Gamma_0|\Gamma_2|}}{U\sin\Phi} \\ |A_2| &= \frac{\sqrt{\Gamma_0|\Gamma_2|}}{U\sin\Phi},\ \cot\Phi = \frac{\delta\Omega_1 + \delta\Omega_2 - \delta\Omega_0}{\Gamma_0 - \Gamma_1 - \Gamma_2}, \end{aligned}$$

with $\Phi = \arg(A_1) + \arg(A_2) - \arg(A_0)$.

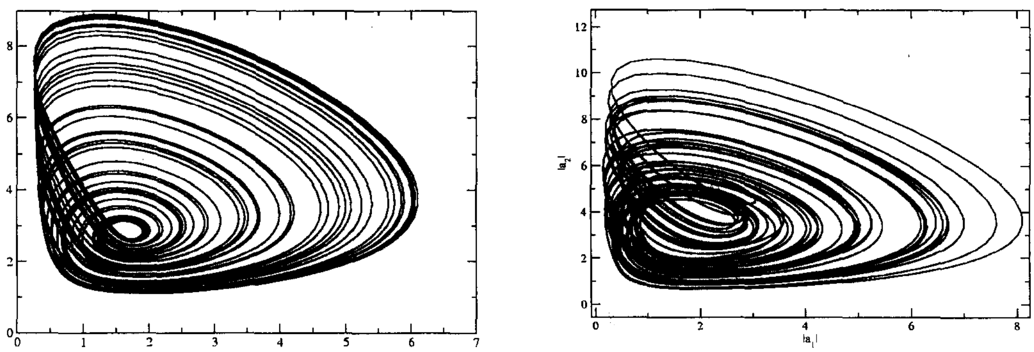

Figure 2: Projection of the attractor on the plane $(|A_0|, |A_1|)$, for $V = 0$ (pure 3WI, left) and $V = 4.5 \cdot 10^{-2}$ (right).

When the coupling coefficient V is increased, the attractor is slightly pertured in the vincinity of the fixed point (Fig. 3). Indeed, the pure 3WI fixed point is not a fixed point of the full system anymore. It is worth noting that increasing further V yields significant changes in attractor's shape and time series appearance. The orbit for short times and at random intervals get further and further away from the region where the 3WI attractor lies, which provide quite intermittent time series. In such a case, the electric field becomes bursty, and some preliminary investigations show that large electric field fluctuations may follow a power-law.

It was argued by Nicholson (1976) that external density fluctuations decrease the linear growth rate of the instability. Here we shall also investigate the non-linear instability, as given by Lyapunov exponents and entropy.

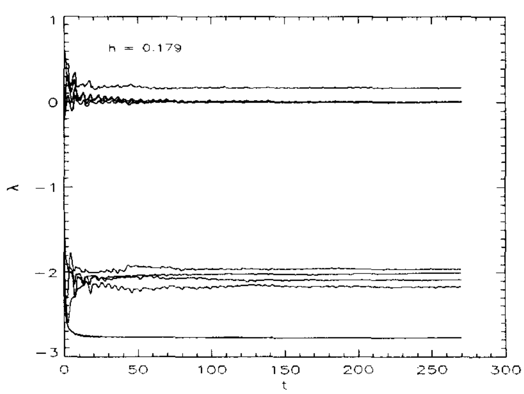

Figure 3: Convergence of Lyapunov spectrum for $V = 1 \cdot 10^{-3}$.

Lyapunov exponents indicate the average linear stability of a given trajectory (the attractor) in phase space with respect to infinitesimal perturbations and in different directions. They can be defined for almost all initial conditions by (Eckmann and Ruelle, 1985)

$$\lambda_n = \frac{1}{\|\delta x_n(0)\|} \lim_{T\to\infty} \frac{1}{T} \int_0^T \ln \|Df(x)\delta x_n(t)\| dt,$$

where δx_n is a suitably chosen perturbation, and Df is a linearization (Jacobi matrix) of Eqs. 1–6. Lyapunov exponents provide furthermore an upper bound on the entropy

$$h \leq \sum_{\lambda_n > 0} \lambda_n.$$

For the parameters chosen, the entropy (or to the least its upper bound) is typically

$$h \approx 1.8$$

and does not strongly change with V. However, the change in the time series is significant, and further caracterization of this change is left for futur study. Anyhow, an example where the finite V induces drastic changes in the nonlinear dynamics of the instability is shown in the following section.

STABILIZATION OF THE INSTABILITY BY FORCED DENSITY FLUCTUATIONS

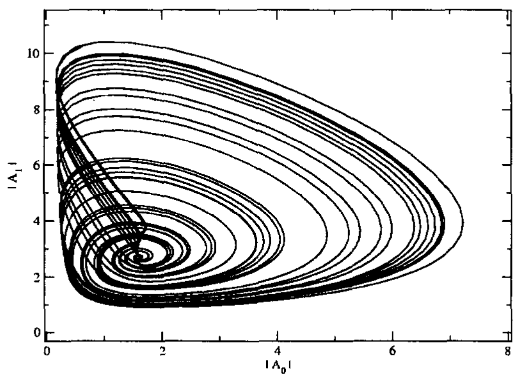

Figure 4: Projection of the attrator for a different set of parameters.

For certain parameters, the instability can even be stabilized by the external density fluctuations. Such an example is presented on Fig. 5, which shows that the strange attractor of the pure 3WI can turn into a limit cycle (stable periodic orbit), and thus into a non-chaotic attractor. In such case, the entropy vanishes and all Lyapunov exponents are negative.

CONCLUSION

Density fluctuations are experimentally known to have a quite high level in the solar wind. Hence plasma inhomogeneity and external density fluctuations may be some of the key elements to be considered in the theory of Langmuir turbulence in order to provide better understanding of the observations.

We have presented preliminary results on the study of the classical parametric interaction, which occurs in Langmuir turbulence as well as in many other contexts, in the presence of low-frequency quasi-monochromatic forced density fluctuations. It was shown how these fluctuations affect the dynamics. Although they do not always have spectacular effects on nonlinear instability indicators, such as entropy, it can lead to qualitative and quantitative changes in electric field time series. In particular, it makes time series more intermittent and bursty-like, in better agreement with quite typical observations in the solar wind. It was also shown that a rather small amount of forced fluctuations can turn the dynamics from chaotic (strictly positive entropy) to non-chaotic (null entropy).

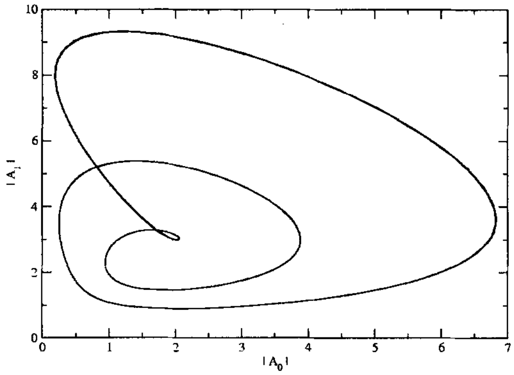

Figure 5: Example of the instability stabilization by external density fluctuations. The chaotic attractor of the pure 3WI (upper panel) is turned into a limit cycle for small V (here $5 \cdot 10^{-3}$), while other parameters are the same as in Fig. (4).

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