Available online at www.sciencedirect.com

SCIENCE DIRECT®

Nuclear Instruments and Methods in Physics Research A 503 (2003) 562–566

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

www.elsevier.com/locate/nima

Polarization constraints on gamma-ray event circles in compton scatter instruments

S.E. Boggs[a,b,*]

[a] Department of Physics, University of California, Berkeley, CA 94720, USA
[b] Space Sciences Laboratory, University of California, Berkeley, CA 94720, USA

Received 11 July 2002; received in revised form 29 January 2003; accepted 7 February 2003

Abstract

A number of researchers are developing novel Compton scatter instruments, designed to track γ-ray interactions within their detector volume with high spatial and spectral resolution. Many of these instruments are designed as Compton imagers, where the initial photon direction can be determined through the Compton scatter formula to within an event circle on the sky for astronomy, or an event cone for near-field imaging. If the γ-ray scatters more than once before photoabsorption in the instrument, it is possible to use the polarization-dependent Klein–Nishina differential cross-section to constrain the probability density of the initial photon direction along the event circle. Here we derive this probability density, which will improve the imaging capabilities and sensitivities of Compton imagers.

PACS: 95.55.Ka; 29.30.Kv

Keywords: Gamma-ray; Compton telescopes; Gamma-ray tracking

1. Introduction

High-resolution Compton scatter instruments, designed to track soft γ-ray photons (0.2–10 MeV) as they Compton scatter through an active detector volume, are finding increased use in astronomy, nuclear physics, medical imaging, and industry. In astronomy, these instruments are known as Compton telescopes, and their development began in the 1970s [1–3]. This first generation of Compton telescopes culminated in the successful flight of COMPTEL on the Compton Gamma-Ray Observatory [4]. Currently, a number of research groups are working on much more powerful Compton telescopes, designed to track γ-ray scatters with very high spectral and spatial resolution, as well as high efficiency [5–9].

Compton telescopes work on a well-known principle (see Refs. [5,10] for overviews): by measuring the positions and energies of photon interactions in a detector volume, the initial photon direction can be reconstructed using the Compton scatter formula to within an annulus on the sky, known as an *event circle* (Fig. 1). The uncertainty, or width, of this event circle depends

*Corresponding author. Space Sciences Laboratory, University of California, Berkeley, CA 94720, USA. Fax: +1-510-643-8302.

E-mail address: boggs@ssl.berkeley.edu (S.E. Boggs).

0168-9002/03/$ - see front matter
doi:10.1016/S0168-9002(03)00989-6

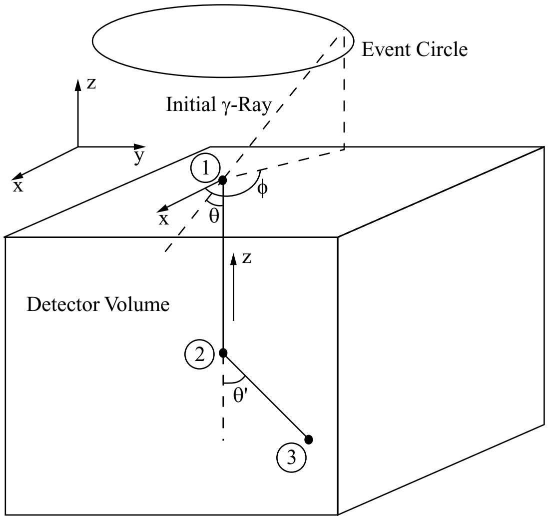

Fig. 1. Novel Compton scattering instruments track a γ-ray as it scatters (interactions 1 and 2) and eventually gets photo-absorbed (interaction 3) in an active detector volume—measuring the locations and energies with high resolution. Using the measured information, the initial photon direction can be determined to within an event circle. Here we define a coordinate system such that $\hat{z}$ is the direction from the second interaction site (2) toward the first interaction site (1), and the $\hat{y}$–$\hat{z}$ plane contains the first three photon interaction sites.

on the spectral and spatial resolution of the detectors, but also has a fundamental limit due to Compton scattering of bound electrons. These event circles form the starting point in a number of different techniques for image reconstruction [10,14,15]. In order to aid in the reconstruction, a number of researchers are developing instrumental techniques of constraining the photon direction to an arc on the event circle by measuring the recoil direction of the electron of which the initial photon scattered [7,8]. While this technique is promising for γ-ray energies above a few MeV, it remains difficult to measure the electron recoil direction at lower energies.

In principle, if a photon undergoes multiple Compton scatters in the detector volume before finally being photoabsorbed, we can use the polarization properties of Compton scattering to help further constrain the initial photon direction along the event circle [11,12]. Assuming the initial photon is unpolarized, the scattered photon will be partially polarized in a direction perpendicular to the scatter plane. When the photon undergoes a second scatter, the scatter direction is most likely to be perpendicular to this polarization direction, and hence in the same plane as the initial scatter. How well this helps constrain the initial direction of the photon depends on the photon energy, as well as the first and second Compton scatter angles. This technique has been demonstrated elsewhere through Monte Carlo simulations [11,12]. For image reconstruction, we would like to build upon this earlier work by deriving a simple analytical formula for determining the probability density of the initial photon direction along the event circle. Here we rederive the constraints that the polarization-dependent collisional cross-section places on the Compton event circles. Given a measured initial photon energy (by summing the individual interaction energies, or by using three-site Compton reconstruction [13]), and the Compton scatter angle of the first and second interactions, we derive the probability density for the initial photon direction along the event circle.

2. Scatter theory

Using polarization to constrain the Compton event circle is a two-step process. First, given the direction of the photon after its second scatter, we can determine the most-likely fractional polarization of the photon before this scatter (i.e. after the initial scatter). Second, given this fractional polarization and the initial Compton scatter angle implied by the interaction energies in the instrument, we can determine an azimuthal probability density for the initial photon direction along the event circle.

2.1. *Fractional polarization after the first scatter*

Here we treat the photon after its initial scatter as partially polarized, with the fractional intensity of each linear polarization proportional to the differential collisional cross-section for that polarization component to undergo the observed second scatter. The polarization of the photon after the second scatter is unimportant, so the Klein–Nishina differential collisional cross-section must be summed over all directions of polarization for this scatter. From Eq. (2.3) in Ref. [16],

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}[\beta + \beta^{-1} - 2\sin^2\theta\cos^2\eta] \tag{1}$$

where β is the ratio of the photon energy after the scatter to its energy before scattering (determined by the Compton scatter formula), θ is the Compton scatter angle, η is the angle between the photon polarization plane before scattering and the scattering plane, and r_0 is the classical electron radius.

We define a convenient coordinate system (Fig. 1), where $\hat{z}$ is the direction from the second interaction site toward the first interaction site, and the $\hat{y}$–$\hat{z}$ plane is defined as that containing the first three photon interaction sites. Using this coordinate system, we can define the two components of the scattered photon polarization as $\hat{\varepsilon}_1' = \hat{x}$ $(\eta = 90°)$ and $\hat{\varepsilon}_2' = \hat{y}$ $(\eta = 0°)$. Given these definitions, we derived the most-likely components of the linear polarization after the initial scatter using Eq. (1):

$$I_1' = \frac{[\beta' + \beta'^{-1}]}{2[\beta' + \beta'^{-1} - \sin^2\theta']} \tag{2}$$

$$I_2' = \frac{[\beta' + \beta'^{-1} - 2\sin^2\theta']}{2[\beta' + \beta'^{-1} - \sin^2\theta']} \tag{3}$$

where

$$I_1' + I_2' = 1 \tag{4}$$

and β' is the ratio of the photon energy after its second scatter to its energy before this second scatter, and θ' is the Compton scatter angle for this second scatter.

2.2. Probability density along the event circle

From the measured interaction energies, we can reconstruct the initial Compton scatter angle θ from the Compton scatter formula. Given this angle and the fractional polarization derived above, we can find a probability density in ϕ along the Compton event circle for the initial photon direction. Here we assume that the initial photon is unpolarized. In our coordinate system defined in Fig. 1, for a given ϕ, we can define the initial photon direction, and two perpendicular directions for the polarization components,

$$r_0 = [\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta] \tag{5}$$

$$\hat{\varepsilon}_1 = [-\cos\theta\cos\phi, -\cos\theta\sin\phi, \sin\theta] \tag{6}$$

$$\hat{\varepsilon}_2 = [\sin\phi, -\cos\phi, 0]. \tag{7}$$

Assuming an initially unpolarized photon, we must sum over all possible orientations of the initial photon polarization. This is equivalent to taking $I_1 = I_2 = 0.5$. For each possible initial azimuthal angle, ϕ, we define its relative probability density as proportional to the sum of the four combinations of initial and scattered fractional polarizations multiplied by the differential collisional cross-section for scattering between the two polarization components,

$$p(\phi) \propto \sum_{i,j=1}^{2} I_i I_j' \, d\sigma(\hat{\varepsilon}_i, \hat{\varepsilon}_j') \tag{8}$$

where now we use the Klein–Nishina differential collisional cross-section formula that depends on both the initial and final polarization directions, from Eq. (2.1) of Ref. [16],

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}[\beta + \beta^{-1} - 2 + 4\hat{\varepsilon}_i \cdot \hat{\varepsilon}_j']. \tag{9}$$

Plugging in this cross-section and renormalizing $p(\phi)$ such that the integral over $d\phi$ is equal to unity, we find that,

$$p(\phi) = \frac{[\beta + \beta^{-1} - 2I_1'\sin^2\theta\sin^2\phi - 2I_2'\sin^2\theta\cos^2\phi]}{2\pi[\beta + \beta^{-1} - 2\sin^2\theta]} \tag{10}$$

where I_1' and I_2' are given by Eqs. (2) and (3).

2.3. Modulation of the event circle probability density

From the definition of $\hat{\varepsilon}_1'$ and $\hat{\varepsilon}_2'$ in our coordinate system, we always have $I_1' \geqslant I_2'$. Therefore, $p(\phi)$ is always maximum when $\phi = 0°, 180°$; and $p(\phi)$ is minimum when $\phi = 90°, 270°$. Therefore, we can define the modulation of the probability density along the event circle as

$$\mu = \frac{p(0°) - p(90°)}{p(0°) + p(90°)}. \tag{11}$$

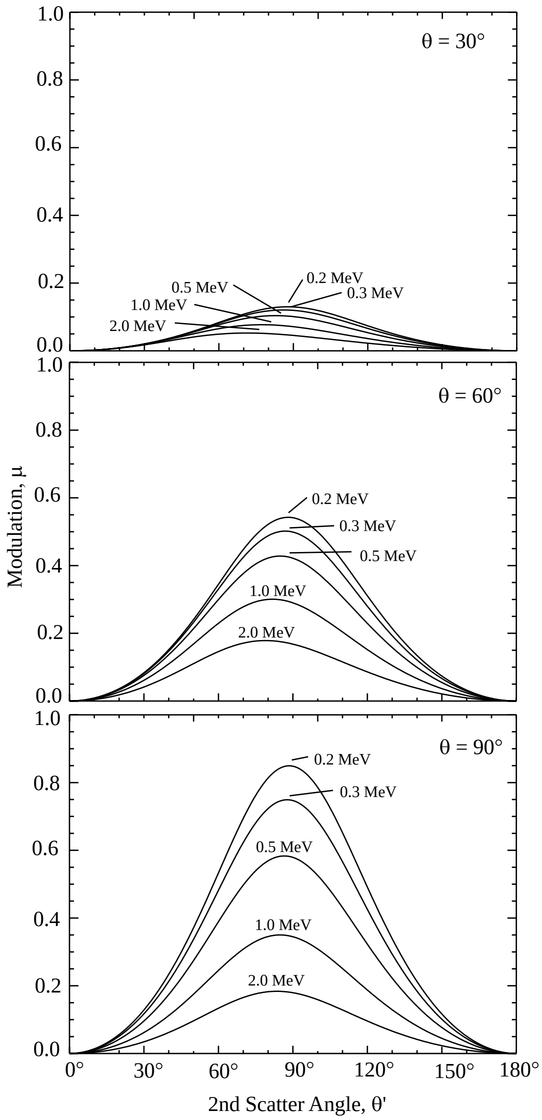

Fig. 2. Modulation, μ, of the event circle probability density as a function of the second Compton scatter angle, θ', for three values of the initial Compton scatter angle ($\theta = 30^\circ$, 60° and 90°), and several values of the initial photon energy between 0.2–2 MeV.

In Fig. 2, we plot this modulation as a function of the second Compton scatter angle, θ', for several soft γ-ray photon energies and initial Compton scatter angles, θ. As could be expected, the modulation is strongest at lower energies, but remains significant for photon energies above 1 MeV, especially when both scatters have Compton scatter angles near 90°.

3. Discussion

Image reconstruction using Compton telescopes often begins with the event circle probability distribution [10,14,15]. From our modulation curves, we can see that for photons below $\sim$0.5 MeV, taking the polarization effects into account, can dramatically improve the probability distribution for the initial photon direction along the Compton event circle. Even at higher energies, this probability distribution should significantly improve image reconstruction techniques. This will be especially true for very compact Compton scatter instruments, which are dominated by large-angle scatters, as opposed to traditional Compton telescopes which predominantly measure shallow-angle scatters. Even a modulation of 0.33, readily achieved at 1 MeV, corresponds to a density variation of a factor of 2 around the event circle. While this provides marginal help in localizing any individual photon, it provides a very significant improvement for image reconstruction techniques designed to maximize a likelihood statistic for many events.

Acknowledgements

This research was supported by NASA grant NAG5-5350. S.E. Boggs would like to thank the California Space Institute for support, as well as R. Diehl and A. Zoglauer for useful discussions on Compton imaging. The author would also like to thank an anonymous reviewer for bringing previous work on this subject to his attention.

References

[1] V. Schönfelder, A. Hirner, K. Schneider, Nucl. Instr. and Meth. 107 (1973) 385.
[2] D. Herzo, et al., Nucl. Instr. and Meth. 123 (1975) 583.
[3] J.A. Lockwood, L. Hsieh, L. Friling, C. Chen, D. Swartz, J. Geophys. Res. 84 (1979) 1402.
[4] V. Schönfelder, et al., Astrophys. J. Suppl. 86 (1993) 657.

[5] S.E. Boggs, P. Jean, Astron. Astronom. Suppl. 145 (2000) 311.
[6] E. Aprile, et al., AIP Conf. Proc. 510 (2000) 799.
[7] F. Schopper, et al., 2000 IEEE Nuclear Science Symposium Conference Proceedings, Lyon, France, Vol. 3, 122, 2000.
[8] T.J. O'Neill, et al., AIP Conf. Proc. 510 (2000) 804.
[9] R.A. Kroeger, et al., 2000 IEEE Nuclear Science Symposium Conference Proceedings, Lyon, France, Vol. 8, 7, 2000.
[10] P. von Ballmoos, R. Diehl, V. Schönfelder, Astron. Astrophys. 221 (1989) 396.
[11] N. Dogan, et al., IEEE Trans. Nucl. Sci. NS-39 (1992) 1427.
[12] N. Dogan, Ph.D. Dissertation, University of Michigan, 1993.
[13] R.A. Kroeger, et al., 2001 IEEE Nuclear Science, Symposium Conference Proceedings, San Diego, CA, Vol. 1, 68, 2001.
[14] R. Diehl, Exp. Astronom. 6 (1995) 103.
[15] S.J. Wilderman, N.H. Clinthorne, J.A. Fessler, W.L. Rogers, 1998 IEEE Nuclear Science Symposium Conference Proceedings, Toronto, Canada, Vol. 1176, 1998.
[16] R.D. Evans, The Atomic Nucleus, McGraw-Hill, New York, 1955 (Chapter 23).