Solar Wind Particle Distribution Function Fitted via the Generalized Kappa Distribution Function: Cluster Observations

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Abstract. One of the major issues in space plasma physics is that in spite of the inhomogeneity of interplanetary plasma and the complicated magnetic field topology we do not find strong deviations from Maxwellian distributions as it would be expected for a quasi-collisionless plasma. However, the presence of high energy tail and shoulders in the profile of distribution function stimulate to look for a better analytic representation of the observed distributions. Therefore, here we adopt a non-Maxwellian distribution function such as the Ellipsian distribution function, which is the generalized form of the Kappa distribution function. In this paper we have analysed the solar wind data recorded by Cluster s/c during early 2001 and 2002 when the s/c were repeatedly immersed in the solar wind, ahead of the Earth's bow shock. Data were modeled with the help of the Ellipsian distribution function and values of the best fit parameters were successively used to characterize the solar wind kinetics at different locations of one of the four Cluster s/c.

INTRODUCTION

In the natural space environment, e.g. planetary magnetospheres, the solar wind and astrophysical plasmas are generally observed to possess a particle distribution function with a non-Maxwellian tail (Chun-yu and Summers, 1998; Summers et. al., 1994) and heat flux shoulders (Marsch et. al., 1982). An appropriate particle distribution function for modelling such plasmas would be a generalized kappa distribution function such as "Ellipsian" distribution function defined in equation-1. This distribution function is characterized by spectral indices k and m, where k is the index of inverse power-law tail and m represents the shoulders of the distribution function, and can be reduced to Maxwellian for m=1 and $k \rightarrow \infty$. In this paper we use "Ellipsian" distribution function to model the solar wind ion data, because the fit can be best achieved in both the high energy tail as well as shoulders in the distribution function.In the present study we used plasma measurements recorded by the CLUSTER mission.

CLUSTER MISSION AND CLUSTER ION SPECTROMETRY (CIS)

The ESA Cluster mission consists of four identical spacecraft which allow to study in three dimensions the small-scale plasma structures in the near-Earth environment. The orbital parameters of the four spacecraft are slightly different to obtain a tetrahedral configuration in the regions of scientific interest. The size of this tetrahedron will be varied from 100 *km* to 18000 *km* during the course of the mission. The spacecraft cross the various near-Earth plasma regions through the year, for example

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the Earth magnetotail in the summer and the polar cusp and solar wind six months later. (Escoubet, C. P.; et al. , 1997).

The Cluster Ion Spectrometry (CIS), one of the 11 experiments on board the cluster spacecraft, is capable of measuring both the cold and hot ions from the solar wind, the magnetosheath, and the magnetosphere (including the ionosphere) with good angular, energy and mass resolution (Rème, H.; et al., 1997).

CIS experiment employs two sensors to obtain the full three-dimensional ion distribution of the major species. One sensor, ion Composition and Distribution Function analyzer (CODIF) is a top-hat electrostatic analyzer followed by a time of flight section which measures the distribution of the major ion species (H^+, He^+, He^{++}, and O^+) from 0 to 40 keV q^{-1} with an angular resolution of 22.5^o x 11.2^o and two different sensitivities. The high sensitivity side has a larger geometric factor (used for low fluxes), the low sensitivity side has a smaller geometric factor (used for high fluxes). The other sensor, the Hot Ion Analyzer (HIA), is also a top-hat electrostatic analyzer which measures the distribution of the ions without distinction of mass from 5 eV q^{-1} to 32 keV q^{-1} with a maximum angular resolution of 5.6^o x 5.6^o and two different sensitivities (Réme, H.; et al. , 1997).

ELLIPSIAN DISTRIBUTION FUNCTION

The distribution function which we shall use to model this data is called an "Ellipsian" distribution function, and has the form

$$f_E = A\left[1+\left\{\left(\frac{v_\|}{kv_{T\|}}\right)^2+\left(\frac{v_\perp}{kv_{T\perp}}\right)^2\right\}^m\right]^{-k} \quad (1)$$

where

$$A = \frac{1}{\pi^{3/2}\, v_{T\perp}^2\, v_{T\|}}$$

This distribution function is a generalized version of the Lorentzian (kappa) distribution function, where m=1 always (Summers et.al. 1991, 1992, 1994; Thorne et. al. 1991). Data fits of the magnetosheath electrons (Qureshi, M. N. S.; M.Phil thesis) showed that fits for the distribution functions are better achieved by our model distribution function than the Lorentzian (kappa) distribution function and bi-Maxwellian.

We transform this model distribution function into a form, which we find more convenient, from the point of view of numerical modeling. For this purpose, we define different parameters as:

$$b = \frac{v_{T\perp}^2}{v_{T\|}^2} \quad and \quad c = \frac{1}{v_{T\perp}^2} \quad (2)$$

And expressing $v_\|$ and $v_\perp$ through angle θ with respect to the ambient magnetic field B_o, as

$$v_\| = v\cos\theta \quad ; \quad v_\perp = v\sin\theta$$

and by taking the log on both sides of equation-1, we obtain

$$log\, f_E = a - k\, log\left[1+\left\{\frac{cv^2}{k}(b\cos^2\theta+\sin^2\theta)\right\}^m\right] \quad (3)$$

where

$$a = log\, A$$

Thus equation-3 is in the form, which we use to model the distribution function from the data given. We note that each experimental point is characterized by a value of v (the velocity of the particles) and θ (from the $v_\|$, $v_\perp$ planes) and we determine (numerically) the values of a, b, c, k and m and hence the values of log f_E.

In order to achieve the best possible fit of data with the model distribution function given by equation-1, we carry out the following steps.

1. For each measured point which is characterized by θ and v, we take the square of the differences between the observed data and model, in the following form:

$$dif_i = [log\, f_{oi} - log\, f_{mi}]^2 \quad (4)$$

Where f_o and f_m are the observed data and theoretical values of the distribution functions.

2. Secondly we take the sum of the differences dif_i .

$$sum = \sum dif_i$$

3. Now we minimize this sum to obtain the best possible values of a, b, c, m and k.

DATA SELECTION AND TREATMENT

In order to fit the data with our model distribution function we need the data in the form of log of distribution function, corresponding magnitude of the velocities of the particle and θ values on the $v_\|$ and $v_\perp$ plane in the plasma reference frame.

In order to select only protons, we used CODIF data. Unfortunately, we cannot use HIA data which would have a better angular resolution in phase space but cannot separate masses. Moreover, as fluxes are high in the solar wind, we used data only from the low sensitivity side.

Initially we have the data in the form of distribution function with thirty one energies and eighty eight angles measured every sixteen or eight seconds and correspondingly the number density, bulk speed and magnetic field data in the GSE coordinate system which are then transformed in the instrumen coordinate system.

In order to improve the statistics we average 30 successive distribution functions (sometimes 35) and normalized the averaged distribution function by number density. Then we transform the measured velocities from the instrument to the plasma frame of reference.

Each measured velocity (in the plasma reference frame) is then represented in the $v_{\parallel}$ and $v_{\perp}$ plane and is identified by v and θ.

To select the data, we chose time intervals in which the plasma parameters had little fluctuations and computed the time averages over such intervals, as described above. One such interval of 21st Feb. 2001 can be seen in the Fig.-2.The other time intervals are similar to this one in this respect but not in position. In that plot the selected time interval is from 77482 to 77947 seconds just before the shock. For this day we can also see the positions of the spacecraft in the solar wind in Fig.-1.

All the selected intervals which are considered in this paper are from the s/c4, because this came out to be the only spacecraft available in the solar wind mode in low sensitivity as explained above.

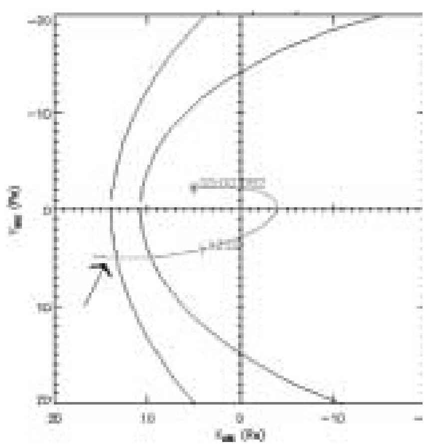

FIGURE 1. Position of the s/c4 on 21st Feb. 2001. The selected interval is indicated by the arrow in the solar wind.

FITTING RESULTS

We now present some fitting results and the corresponding values of the different parameters. In this procedure we fit all the points but here we are showing the results only in certain directions in the $v_{\parallel}$ and $v_{\perp}$ plane. The solar wind ion distributions are focused in a very narrow angular range and few energy steps in phase space,

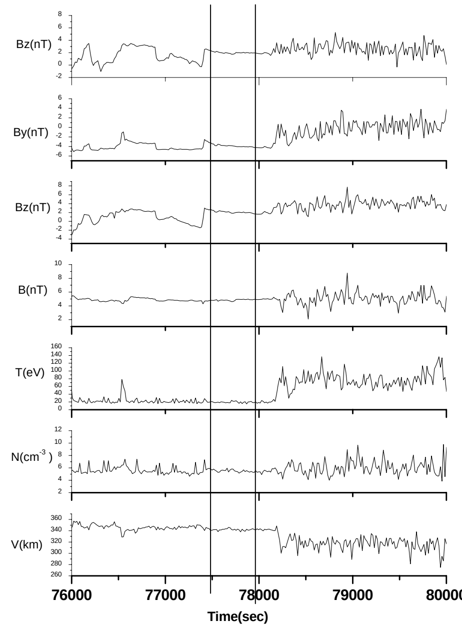

FIGURE 2. 16 sec. averages of magnetic field and Plasma parameters on 21st Feb. 2001 and the selected interval in between the vertical lines.

therefore we never have a large number of points in certain directions.

In Fig.-3 we can see the fits in certain directions for the selected intervals from 21st Feb. 2001, 28th March 2001 and 11th Feb. 2002 and the corresponding parameters in the Table-1. For sake of brevity, we are not showing fit for 22nd Feb. 2001, although the parameters are given for this day in the Table-1, because of the similar type of fit in the similar conditions of slow solar wind as day 21st Feb. 2001.

SUMMARY AND DISCUSSION

In this paper we fitted the solar wind ion data with "Ellipsian" distribution function. All the selected intervals are from the slow solar wind except the interval of 28TH Mar. 2001 in which solar wind is intermediate. This interval is also different from the other intervals in the respect that it is in a magnetic cloud. Data fits in Fig.-3 show that the Ellipsian distribution function fits quite well for high energy tails as well as shoulders in the distribution function.

The parameters m and k in the Table-1 are the spectral indices and in general represent the flat part and the high energy tail of the distribution function respectively.

TABLE 1. Distribution function's parameters for the Solar Wind.

Day	a	b	c	k	m
21^{st} Feb. 2001	-15.1582	0.6925	0.000628	0.8456	2.8958
22^{nd} Feb. 2001	-15.0088	0.7511	0.001186	0.9798	2.2758
28^{th} Mar. 2001	-15.8503	0.9936	0.000268	0.8348	2.6785
11^{th} Feb. 2002	-14.9241	0.6278	0.0004619	2.8642	1.2156

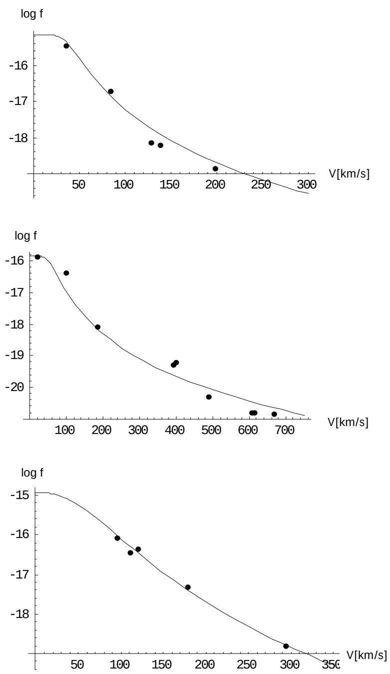

FIGURE 3. Data fits for 21^{st} Feb. 2001, 28^{th} March 2001 and 11^{th} Feb. 2002 are shown in the upper, middle, and lower panels, respectively. The dots indicate the observed data points and lines indicate the model distribution functions. Each plot refers to only one direction in $v_{\|}$ and $v_{\perp}$ plane.

If value of k becomes higher it tends to shift towards Maxwellian (Leubner and Schupfer 2000; Summers et al. 1991). Typical values of k in space plasmas lies in the range $1 < k \leq 5$ (Summers and Thorne, 1992, and references therein whereas in our case it lies in the range $0 < k \leq 5$). The other parameters like a which contains log of number density and thermal speeds is almost the same for all the intervals and b, which is the anisotropy ratio (see equation.-2), is not much different in all three intervals except 28th March, where the value of b is almost one. This particular time interval is in the magnetic cloud and due to the compression and thermal heating of the plasma, anisotropy is much reduced. In the last interval of 11th Feb. 2002, the value of m is lowest and $k$is highest than the other intervals indicating the presence of less high energy tail in the distribution function. Also the value of b is less than from the other intervals, indicating high anisotropy with parallel temperature greater than the perpendicular temperature.

We conclude that data fits of the magnetosheath electrons (Qureshi M. N. S.; M.Phi Thesis) and solar wind ions distribution functions can be best achieved by our "Ellipsian" distribution function (with $m > 1$) than by Generalized Lorentzian (Kappa) distribution function where m=1 always.

This is a preliminary study and only one CLUSTER spacecraft is available up to now for this type of data analysis. But in the future, we want to carry out the same analysis with the data from at least two spacecraft at the same time, located at different positions to see the spatial variation in plasma.

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