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Anisotropy in Space Plasma Turbulence: Solar Wind Observations

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Abstract The local magnetic field induces many types of anisotropy in plasma turbulence, changing the rate of energy transfer and affecting the propagation of energetic particles. It is challenging to measure this anisotropy in the solar wind due to the limited number of sampling points and measurement difficulties and many aspects remain poorly understood. Nevertheless, in recent years considerable theoretical and experimental progress has been made in understanding the anisotropy of turbulence, the latter through new methods and multi-spacecraft data. A short review of recent work is presented, concentrating on observations rather than theory and discussing the principal limitations and restrictions of such measurements. Key results are discussed: the variation in observed power with angle of the magnetic field to the solar wind flow, and evidence for variations in the spectral index, on scales both above and below the ion gyroradius. A comparison of single and multi-spacecraft analysis methods applied to the same data intervals shows excellent agreement and provides measurements of anisotropy throughout the inertial range. Current outstanding issues are discussed along with their possible resolution.

Keywords Turbulence · Plasmas · Solar wind

1 Introduction

Turbulence is ubiquitous in space plasmas. In the solar wind, it is pervasive on a wide range of scales, from large scale streams to the electron gyroscale. The precise measurements of ion and electron distribution functions, as well as electric and magnetic fields, makes the solar wind a useful laboratory for the study of space plasma turbulence. Indeed, significant advances in the determination of the properties of plasma turbulence, on both fluid and kinetic

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scales, have been made using spacecraft data over the last 50 years. However, many aspects of plasma turbulence remain poorly known or understood. One of these is anisotropy: the breaking of isotropic symmetry in the properties of the turbulent fluctuations as a result of the presence of a local magnetic field. The presence of anisotropy is not surprising, given the central role of the magnetic field at both kinetic and magnetohydrodynamic (MHD) scales in determining plasma behaviour. However, its precise form and indeed its consequences for other phenomena such as heating rates, particle scattering and magnetic field topology are still unclear.

This paper is intended to provide an overview of recent advances in measuring the anisotropy of turbulence in the solar wind, aimed at a wider audience than the turbulence community itself. It is not a comprehensive introduction to space plasma turbulence: instead, we refer the reader to several recent reviews (Goldstein et al. 1995b; Tu and Marsch 1995; Horbury and Tsurutani 2001; Bruno and Carbone 2005; Horbury et al. 2005; Schekochihin et al. 2009; Matthaeus and Velli 2011). It is targeted at observations rather than theory, so while some recent theories are discussed, it is mainly to motivate the observations and allow us to discriminate between models. First, the underlying restrictions on measuring anisotropy with spacecraft are outlined: these limit what can be deduced from measurements. The various methods available to study anisotropy are then discussed along with their strengths and weaknesses, as well as the key results that have been found. Finally, the implications of anisotropic turbulence for other plasma processes are briefly discussed along with prospects for future advances. Before all of this, however, it is helpful to clarify precisely what we mean when we discuss anisotropy in plasma turbulence.

2 Types of Anisotropy

There are several manifestations of anisotropy with respect to the magnetic field, some of which are possible to measure directly while others are more difficult to quantify and some are currently only theoretical concepts.

Variance anisotropy It has long been known (e.g. Belcher and Davis 1971) that fluctuations are larger in components of the magnetic field perpendicular to the mean field than those in the parallel direction. This is consistent with the observation that field magnitude fluctuations are typically smaller than those in the components, as well as the tendency for fluctuations to be predominantly in the non-compressive Alfvénic mode. This so-called 'variance anisotropy' was the first strong evidence that the magnetic field affects the properties of solar wind turbulence. Variance anisotropy is scale dependent and can be different for the magnetic field and the velocity (Nicol et al. 2009).

Anisotropy of energy transfer rate In homogeneous hydrodynamic turbulence, the rate at which energy in turbulent fluctuations at a given scale is transferred down the cascade to smaller scales is the same in every direction: there is no anisotropy in the cascade depending on the direction of the wavevector of the fluctuations. Many theories of plasma turbulence, however, predict varying rates of energy transfer between scales depending on the direction of the wavevector to the magnetic field (e.g. Shebalin et al. 1983; Goldreich and Sridhar 1995). In particular, in these theories the energy in turbulent fluctuations can be transferred to smaller scales (higher wavenumbers) perpendicular to the field in wavevector space more quickly than that in field-parallel wavevectors. We discuss the physical background to these theories in Sect. 3.

Wavevector anisotropy If, at the large scale edge of the turbulent cascade (known as the "outer scale"), power is distributed isotropically with respect to the field, anisotropic energy transfer would result in the power being anisotropic in wavevector space at smaller scales, and indeed this anisotropy would be expected to be more pronounced at progressively smaller scales. This is what we term wavevector anisotropy.

We can think of wavevector anisotropy in terms of "eddies" in the turbulence. In a neutral fluid, eddies break up and form smaller daughters, which are statistically isotropic—this is, they are smaller than their parents in every direction. In a plasma, decay tends to increase the field-perpendicular wavevector more than the parallel and hence, daughter eddies tend to be shorter across the field than along it. This results in long, thin structures aligned along the magnetic field direction, with fine scale structure perpendicular to the field. In a very crude sense, we can think of this as being the result of the resistance of magnetic field lines to bending, but it being easier to braid them.

Wavevector anisotropy also results in anisotropy of the correlation function of the turbulent fluctuations (e.g. Matthaeus et al. 1990). Long, thin eddies aligned along the magnetic field will have a longer correlation length parallel to the field than perpendicular to it.

Power anisotropy Just as anisotropy of energy transfer rates leads naturally to wavevector anisotropy relative to the magnetic field in the plasma frame, so we might also expect to measure different power levels at a particular scale in the spacecraft frame relative to the field direction. This is indeed the case (e.g. Bieber et al. 1996), although the situation is complicated by measurement limitations: see Sect. 4. In this paper, we distinguish between spacecraft measurements of anisotropic power, which we call power anisotropy, and anisotropic power distributions in the plasma frame, which we call wavevector anisotropy. See Chen et al. (2010b) for the relationship between wavevector and power anisotropy.

Spectral index anisotropy The spectral index is a key diagnostic in studying turbulence: in the absence of other features in a broadband spectrum $P(k) \propto k^{-\alpha}$, the spectral index $-\alpha$ is often the only accessible parameter. Most theories of the turbulent cascade are in essence scaling theories: the spectral index is an important parameter in determining this scaling, and hence how energy is transferred from one scale to another (see Sect. 3). The celebrated spectral index of $-5/3$, observed in isotropic hydrodynamic turbulence, was derived by Kolmogorov (1941). A range of values have been predicted for MHD (e.g. Kraichnan 1965; Goldreich and Sridhar 1995; Boldyrev 2006) and kinetic-scale turbulence (e.g. Schekochihin et al. 2009). Crucially, just as the spectral index is a diagnostic of energy transfer, so the anisotropy of energy transfer rates will result in an anisotropy of the spectral index: the turbulent power will scale differently in different directions relative to the magnetic field. Such spectral index anisotropy is a natural outcome of all "critically balanced" theories of anisotropic turbulence (see Sect. 3) and it is therefore of interest to attempt to measure such anisotropy in the solar wind.

We can see therefore that wavevector, power and spectral index anisotropy are all closely related and it is these on which we concentrate in this paper. However, other types of anisotropy exist as we discuss below.

Imbalance Fluctuations can have wavevectors at a range of angles to the magnetic field, but these angles need not be acute. In general we consider the acute angle θ_{kB} between the wavevector and the field, but these may have positive or negative components along the field—for example, Alfvén waves can propagate parallel or anti-parallel to the field.

Assuming that all fluctuations are purely Alfvénic, we can study these two components separately using Elsasser variables (Elsasser 1950; Dobrowolny et al. 1980),

$$\mathbf{z}^{\pm} = \delta\mathbf{v} \pm \delta\mathbf{b}/\sqrt{\mu_0\rho} \tag{1}$$

Here, $\delta\mathbf{b}$ and $\delta\mathbf{v}$ are variations in the magnetic field and velocity respectively and ρ is the density. The normalisation of the magnetic field means that, for a pure Alfvén wave propagating parallel to the field, $\mathbf{z}^{+}$ will be zero and $\mathbf{z}^{-}$ twice the amplitude of the wave, and vice versa for an anti-parallel propagating wave. High speed solar wind streams are typically dominated by fluctuations propagating in an anti-sunward sense in the solar wind frame while slow wind streams are more mixed, typically with no overall dominant sense of propagation (Tu et al. 1990). Most theories of anisotropy have considered equipartition between the two senses (e.g. Goldreich and Sridhar 1995), the so-called "balanced" case. However, the nonlinear coupling between parallel- and anti-parallel modes (e.g. Sridhar 2010) means than any imbalance between them should have dramatic effects in the energy decay rates. Imbalanced turbulence has recently received more attention in the theoretical literature (e.g. Perez and Boldyrev 2010) and there is a growing interest in data analysis related to this topic (e.g. Wicks et al. 2011).

Alignment Wavevector anisotropy concerns the distribution of energy at a given scale between wavevectors at different angles to the local magnetic field. However, in order to describe a wavevector completely we need not just its magnitude and angle to the magnetic field, but in addition its clock angle about the field direction. One might naively assume that this clock angle is dynamically unimportant and so energy would be distributed at all angles equally. However, the nonlinear time of interactions between two Alfvénic fluctuations is dependent on the angle between their wavevectors, which includes their respective clock angles. In recent years there has been some theoretical work (Boldyrev 2006) arguing that this angle is scale dependent and therefore important in the turbulent cascade. Experimental support for this is rather limited to date but some recent results (Podesta 2009) suggest that it might be significant.

3 Theories of Anisotropic Turbulence

This section provides a very short introduction to the central points in anisotropic turbulence theories and is by no means comprehensive or rigorous; see Schekochihin et al. (2009) for an extensive recent discussion. In the simplest view of turbulence, energy is injected at a large ("outer") scale and is transferred nonlinearly to progressively smaller scales where it is eventually dissipated as heat (e.g. Frisch 1995): in between but far from the outer and dissipation scales, in the so-called "inertial range" where inertial forces dominate over viscous, we observe power law, scale-free behaviour. Several properties of inertial range turbulence in neutral fluids, such as a spectral index of $-5/3$, are universally observed. This spectral index was derived by Kolmogorov (1941), assuming isotropy, homogeneity, that there is a constant flow of energy from large to small scales and that the transfer takes place between neighbouring scales (that is, it is local in wavenumber space). The crucial factor in determining the spectral index is the time, $\tau(l)$, that is taken for energy to be transferred from scale l to smaller scales. In neutral fluids there is essentially only one possibility dimensionally, the nonlinear time, $\tau_{nl} = l/\delta u(l)$, where $\delta u(l)$ is the typical velocity shear across an eddy of size l. This leads directly to a spectral index of $-5/3$ (see Frisch 1995).

Perhaps rather surprisingly, spectral indexes of $-5/3$ are routinely observed in the solar wind, at least for the magnetic field (e.g. Bavassano et al. 1982), even though several aspects of the Kolmogorov (1941) derivation are not valid for MHD. In particular, there is more than one possible energy transfer timescale $\tau(l)$. Iroshnikov (1964) and Kraichnan (1965) pointed out that in MHD there is another important timescale, the Alfvén time $\tau_A(l) = l/v_A$. Unlike in a neutral fluid, Alfvénic fluctuations propagate and so oppositely moving wavepackets can pass each other before they decay. If this propagation time, which is τ_A, is shorter than the nonlinear decay time then the turbulent cascade will be slowed relative to the Kolmogorov (1941) case. Since τ_{nl} is inversely proportional to the amplitude of the fluctuations, this propagation condition, which is also scale dependent since δu varies with scale, is often called the "weak turbulence" assumption. Iroshnikov (1964) and Kraichnan (1965) showed that in this case, again assuming wavevector isotropy, the spectral index is $-3/2$. Interestingly, Podesta et al. (2007) showed that this is often the case for velocity fluctuations in the solar wind; however the magnetic field spectrum is well established as having a $-5/3$ spectrum.

The weak cascade is further complicated by anisotropy. Energy and momentum conservation in wave-wave interactions lead to power cascading to larger wavevectors (smaller scales) perpendicular to the magnetic field (Shebalin et al. 1983), but preserving the parallel wavenumber. Far down the cascade, this results in power being in wavevectors at very large angles to the field, so-called "2D" fluctuations. This led in the 1980's to a paradigm for solar wind fluctuations of a cascade in 2D fluctuations, combined with a "slab" population of field-parallel-propagating Alfvén waves (Matthaeus et al. 1990; Tu and Marsch 1993; Bieber et al. 1996).

As energy is cascaded to smaller scales in the anisotropic weak cascade, the nonlinear timescale decreases but the Alfvén timescale remains constant, because there is no parallel cascade (Galtier et al. 2000). Eventually these timescales will become equal and the turbulence is no longer weak. Goldreich and Sridhar (1995) argued that the cascade would remain in the "critically balanced" state between these two timescales. This naturally leads to anisotropy in wavevector space, which is itself dependent on scale, being more anisotropic further down the cascade: see, e.g. Schekochihin et al. (2009) for a more detailed discussion.

In a neutral fluid, turbulence is damped at small scales through viscosity. In a collisionless plasma, the situation is more complex since wave modes such as whistlers (Gary and Smith 2009) and kinetic Alfvén waves can exist below the ion gyroscale and indeed the range of possible wave modes, many of which are anisotropic, is large (Gary 2005). Theories based on cascades of kinetic Alfvén waves (e.g. Schekochihin et al. 2009) or whistlers (e.g. Cho and Lazarian 2004) include a nonlinear timescale which can again be balanced with a propagation timescale to produce a "critically balanced" cascade. Indeed, the critical balance paradigm appears to be one which can be applied to many cascade theories in order to predict wavevector anisotropy and its scale dependence. The precise cascade mechanisms between ion and electron scales are significant, since they potentially determine how energy is dissipated into plasma heating, although energy could also be dissipated directly near the ion gyroscale, for example by Landau damping. While this regime has been studied for many years (e.g. MacDowell and Kellogg 2001), the precise form of the cascade at scales below the ion gyroscale is, at this time, unclear. Eventually, some fraction of the energy will reach the electron gyroscale where the situation is even more uncertain.

Simulations have shown the presence of wavevector anisotropy (e.g. Shebalin et al. 1983) and results consistent with critical balance (e.g. Maron and Goldreich 2001) on magnetohydrodynamics (MHD) scales but the identification of such anisotropy in the solar wind requires detailed and rather complex analysis. In order to understand these problems, we need to consider in detail what is actually measured by a spacecraft in the solar wind.

4 Considerations in Measuring Anisotropy

There are fundamental limitations on the information regarding anisotropy that can be extracted from spacecraft measurements in the solar wind. In order to understand these limits, we need to consider how a time series of spacecraft data relates to variations in the plasma itself. For simplicity, in what follows we consider a component of the magnetic field, but the argument is the same for other parameters such as the velocity and density.

Supersonic flow: Taylor's hypothesis What does it mean to measure a time series at a spacecraft? The plasma flows past the spacecraft, but the waves and turbulent fluctuations are also in motion in the plasma frame and of course the spacecraft itself is also moving relative to the Sun. The solar wind flows nearly anti-sunward at several hundred km/s. Typical MHD wave speeds in the solar wind near the Earth are tens of km/s while spacecraft travel at a few km/s at most. The Mach number of the solar wind therefore, for any MHD wave mode, is large, typically around 5–10. This makes it possible to make a critical simplification, known as Taylor's hypothesis (Taylor 1938): that the time series measured by the spacecraft corresponds to a simple one-dimensional spatial sample. In essence, we can neglect any motion or dynamics of waves if the spacecraft passes through them much more quickly than the timescale over which they change (e.g. Perri and Balogh 2010). Measurements at the spacecraft at times t_i can then be considered as a straight line sample in the plasma frame, at a set of points $\mathbf{r}_i = \mathbf{r}_0 - \mathbf{V} \cdot (t_i - t_0)$ where $\mathbf{V}$ is the velocity of the plasma relative to the spacecraft; in practice it is nearly radially away from the Sun.

Taylor's hypothesis is typically reasonably well satisfied on MHD turbulent scales in the inertial range in the solar wind throughout the solar system (Perri and Balogh 2010) apart from close to the Sun, where the Mach number drops close to, and eventually under, unity: Solar Orbiter and Solar Probe Plus data will be complicated by this issue. In addition, in much of the magnetosheath the Mach number is below 1 and this approximation cannot be used. Further, at kinetic scales the plasma can support wave modes such as kinetic Alfvén waves and whistlers which propagate at higher speeds: if these velocities are comparable to the flow, again Taylor's hypothesis is not valid. In fact, anisotropy in the fluctuations can help to keep Taylor's hypothesis valid on kinetic scales. For the rest of this discussion, we will concentrate on situations where Taylor's hypothesis is well satisfied, and so our data can be treated as a spatial sample in the plasma.

The reduced spectrum A one-dimensional cut through the plasma can only provide measurements of the plasma along that direction. How, then, can we learn about the properties of the fluctuations in any other direction? In fact, the time series contains information about fluctuations in all directions at the same time. In order to understand this concept, it is helpful to use a wavevector representation of the fluctuations, with a certain amount of power $P(\mathbf{k})$ at each wavevector. Anisotropy means that we must consider the vector $\mathbf{k}$ and not just the wavenumber. Consider a spacecraft measuring magnetic field fluctuations $b(t)$ in the plasma which flows at a speed $\mathbf{V}$. At a given frequency f in the spacecraft time series, this corresponds to a distance $\lambda = V/f$ in the flow direction and hence a flow-parallel wavenumber $2\pi f/V$. If there were only fluctuations with wavevectors parallel to the flow, our measured power spectrum would simply correspond to that in the plasma. However, we expect power in many directions in wavevector space and in general, at a spacecraft frequency f we are sensitive to power in all wavevectors with a component $2\pi f/V$ in the flow-parallel direction: see Fig. 1.

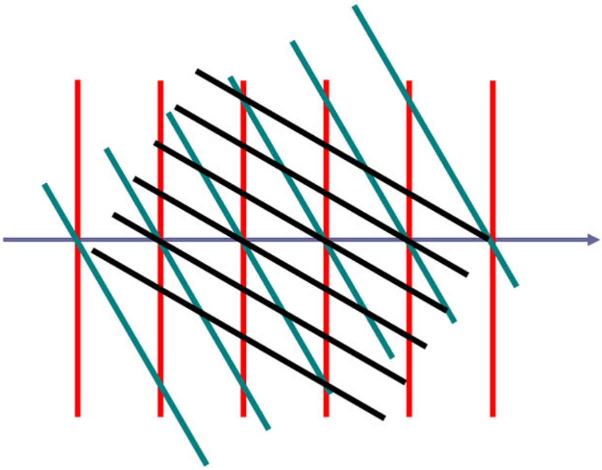

Fig. 1 Spacecraft measure a reduced spectrum: every fluctuation with a wavevector of $2\pi f/V$ in the flow direction contributes power at a spacecraft frequency f. Here, the spacecraft travels through the plasma (*blue horizontal line*) and passes through the crests of three planar waves (*red, green and black sets of lines*). The three waves have the same apparent wavelength along the spacecraft path and therefore appear to have the same frequency at the spacecraft, even though they have different orientations and wavelengths in the plasma

The general expression for the resulting, so-called "reduced spectrum" (Fredricks and Coroniti 1976) is

$$P(f) = \int P(\mathbf{k}) \cdot \delta(\mathbf{k} \cdot \mathbf{V} - 2\pi f)\, d^3\mathbf{k}. \tag{2}$$

It is the reduced spectrum that is measured by a spacecraft as the solar wind passes over it. The immediate consequence, when there are power law spectra in wavevector space, is that it is not possible uniquely to measure fluctuations in one particular direction of wavevector space with a single spacecraft, since power in other directions will contribute to the observed spectrum. In practice one must therefore make assumptions in order to make progress. The most common assumption is of axisymmetry of the fluctuations about the magnetic field direction. Even this, though, is not enough unambiguously to determine the spectral form. It is possible to determine the reduced spectrum for a few simple cases, such as all power lying only in wavevectors parallel (so-called "slab") or perpendicular ("2D") to the magnetic field (e.g. Bieber et al. 1996), where the spectrum is given by

$$P(f) = P_{slab}^{-\alpha} \cos(\theta_{BV})^{\alpha-1} + P_{2D}^{-\beta} \sin(\theta_{BV})^{\beta-1}. \tag{3}$$

Here, α and β are the spectral indexes of the slab and 2D components respectively and θ_{BV} is the angle between the magnetic field and the flow. As we can see, the power measured by a spacecraft will depend on θ_{BV} and indeed if α and β are different, then the measured spectral index will also vary; indeed the spectrum need not even be a precise power law.

No fully generalised analytical solutions have been published for more complex cases, although some special cases are known: for example, for the Goldreich and Sridhar (1995) model, $\alpha = 2$ when $\theta_{BV} = 0°$ and $\alpha = 5/3$ when $\theta_{BV} = 90°$. This makes it difficult to use spacecraft observations to distinguish between the various models of turbulence which predict different distributions of power with angle to the magnetic field in wavevector space (Forman et al. 2011).

Multi-point measurement methods The recent availability of multi-spacecraft data, particularly from the four spacecraft Cluster mission, has made it possible to analyse turbulence in more detail than with a single spacecraft. Crucially, with two or more spacecraft, one can compare between them and therefore recover information about how properties of the plasma vary across the flow rather than just along it, as is the case with one spacecraft. There are a number of ways to perform such an analysis, including detailed wavevector decompositions which are similar to those used when studying seismic data; these are known as k-filtering, or the wave telescope: see Pincon and Glassmeier (2008) for a review and Horbury and Osman (2008) for a review of multi-spacecraft methods of analysing solar wind turbulence, which also include cross-correlations and structure functions. All of these methods can provide information on the 3D structure of the turbulence. However, at least for the correlation and structure function methods, the limitations of the reduced spectrum still apply: only k-filtering can produce an estimate of the power in particular wavevectors, and it suffers from additional issues related to the number of individual wavevectors that can be identified at one time (Tjulin et al. 2007).

One limitation common to all multi-spacecraft methods is that they are only sensitive to scales comparable to the inter-spacecraft distance and they cannot sense scales much larger or smaller. This makes it hard to use them to study the scaling of the fluctuations, which as we have seen is the most common prediction of turbulence theories. There are also lower limits on the number of data points that can be used: in general, while multi-spacecraft methods can provide useful additional information, they are by no means a panacea and care must be taken in their application.

Defining the magnetic field direction If plasma turbulence is anisotropic with respect to the magnetic field, we must study properties of the fluctuations relative to this direction. It seems straightforward at first sight to use the direction of the mean field, calculated over an interval of interest. In the presence of broadband fluctuations, however, the field direction is changing all the time. What, then, is the appropriate direction to use? The choice should be physically motivated and this raises a much more subtle question: to what field are turbulent fluctuations sensitive? In some sense, this must be the field local to the fluctuation (e.g. Schekochihin et al. 2009), but on what scale? Is it one much larger than the scale (or wavelength) of the fluctuation, or of the same order? Does this local scale vary with the scale of the fluctuations? Studies have traditionally used the "global" field direction averaged over a relatively long interval (say, an hour) but techniques have been developed to dynamically track the "local" magnetic field direction throughout an interval, in a scale-dependent way, using wavelets (Horbury et al. 2008), structure functions (Luo and Wu 2010) and multi-spacecraft data (Chen et al. 2010a). As we will see, there are indeed discrepancies between the results using the global and local field methods. Chen et al. (2011) suggested that since the anisotropy scales as $|\delta\mathbf{B}|/|\mathbf{B}|$ in a critical balance cascade, it is always necessary to use a local field tracking method, even at scales where the fluctuation amplitudes are low. As with most turbulence analysis techniques, all of these methods assume that the data are statistically stationary and that they are taken from a statistical ensemble. This requires careful selection of data intervals in practice.

5 Observations of Anisotropy on MHD Scales

Turbulence on fluid, or MHD, scales has been well studied for many years. This is partly because of its importance, for example in controlling energetic particle propagation, but also because it is relatively easy to measure. Anisotropy has been an increasingly dominant aspect of the analysis of MHD turbulence in the solar wind in recent years.

Power anisotropy The principal limitation in studying turbulent anisotropy in the solar wind is that measurements are only made along the flow direction in the plasma frame and hence at a particular angle relative to the ambient magnetic field. A crucial advance in our ability to measure turbulent anisotropy in the solar wind was the realisation, in the late 1980's, that one can take many intervals of data, when the magnetic field is at a range of angles to the flow direction, and then consider the properties of the observed fluctuations as a function of the field/flow angle θ_{BV} in order to probe the anisotropy of the turbulence itself. This is what we termed "power anisotropy" in Sect. 2.

Matthaeus et al. (1990) presented the first clear evidence of wavevector anisotropy, by considering how the correlation function of solar wind turbulent magnetic field fluctuations varied with θ_{BV}, the angle of the magnetic field to the flow. By averaging the correlation results from many solar wind intervals, Matthaeus et al. (1990) constructed an estimate of the correlation as a function of distance both parallel and perpendicular to the magnetic field, showing for the first time that the turbulence was not consistent with either "slab" waves (with wavevectors parallel to the field) or "2D" fluctuations (with wavevectors perpendicular to the field), but rather seemed to be a superposition of both. This was a vital step forward and demonstrated that anisotropic theories of plasma turbulence were required. Similar analysis has been performed on fast and slow wind separately (Dasso et al. 2005) and more recently using pairs of spacecraft (Weygand et al. 2009) to also study anisotropy of the correlation function, again revealing longer correlation lengths along the field than across it. Data from the four Cluster spacecraft have also been cross-correlated (Osman and Horbury 2007), using time lags to increase angular and spatial coverage, with similar results.

While the above methods used correlation functions, it is perhaps easier to relate measurements to theory using power spectra. Bieber et al. (1996) performed a detailed analysis of the variation in turbulence properties with θ_{BV} using power spectra and managed to quantify the anisotropy of turbulence, showing that around 20% of the power was in slab fluctuations and 80% in 2D. This result was pivotal in demonstrating the existence of power anisotropy in the solar wind, as predicted by some turbulence theories and discussed in Sect. 3. Horbury et al. (2008) used a wavelet method to extend the analysis of Bieber et al. (1996) to track the local magnetic field direction as it varies over time. They confirmed the existence of power anisotropy, producing more precise measurements of the anisotropy levels (around 5 in the high speed polar wind they studied); Podesta (2009) confirmed and extended these results to low latitude data. Luo and Wu (2010) used a similar field-tracking method but based on structure functions (which probe power levels using moments of increments and are well-established in the turbulence field (e.g. Marsch and Tu 1997) and also reached similar conclusions).

More recently, multi-point data have been used to measure power anisotropy. Time-lagged structure functions (Osman and Horbury 2009b) and k-filtering (Narita et al. 2006) both confirm the same sense of power anisotropy, with relative power anisotropies of a factor of a few between perpendicular and parallel.

Narita et al. (2010) recently presented evidence, using k-filtering of Cluster magnetic field data, that the properties of solar wind turbulence might not be symmetric about the mean field direction, as did Németh et al. (2010) using time-lagged cross-correlations. Loss of the assumption of rotational symmetry about the field invalidates many common theories of plasma turbulence and this result suggests that much more complex theories are required.

The anisotropy of power in wavevector space is, therefore, well established in the solar wind in the MHD inertial range. The precise levels of this anisotropy—the ratio of power perpendicular to the field to that parallel—are hard to measure and vary between studies and solar wind streams, but seem to be around 5–10.

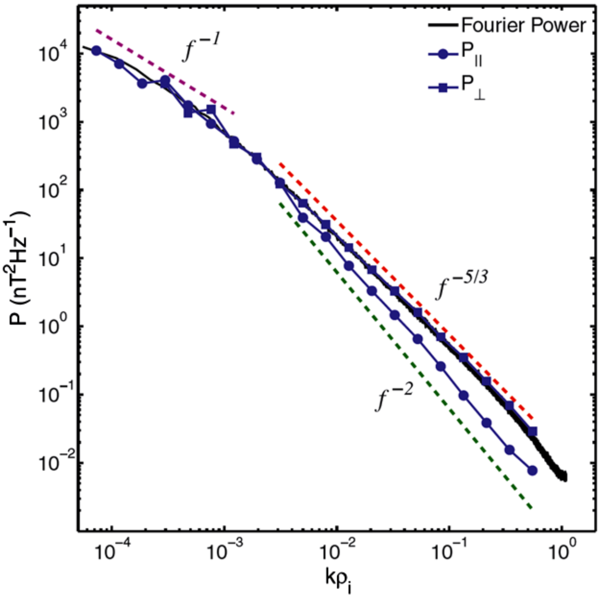

Fig. 2 Magnetic field power spectra (*black line*) and wavelet measures of the power when $\theta_{BV} = 0°$ (*filled dots*) and 90° (*filled squares*). Within the inertial range, the parallel spectrum is significantly steeper than the perpendicular, with a spectral index near −2. Power is isotropic at the large scale end of the inertial range and increases with decreasing scale. Figure from Wicks et al. (2010)

Spectral index anisotropy We have seen that there is anisotropy in power levels with respect to the local magnetic field, but what is its origin? One way to probe this is by considering how anisotropy varies with scale, or equivalently whether the spectral index of the fluctuations varies with respect to the magnetic field direction, since it is only by having different scalings in different directions that relative power levels can change. The existence of spectral index anisotropy in the solar wind is currently the subject of some controversy, in large part based on the definition of the relevant magnetic field direction and in particular on what scale the mean field should be measured. When the mean field is taken to be the length of an entire data interval (typically, of the order of an hour), then no anisotropy in seen in the spectral index (Tessein et al. 2009). However, methods that track the local magnetic field in a scale-dependent way, either using wavelets (Horbury et al. 2008; Podesta 2009) or structure functions (Luo and Wu 2010; Chen et al. 2011), do detect significant anisotropy. The sense is consistent: when $\theta_{BV} = 90°$, the spectral index is near $-5/3$ (or sometimes nearer $-3/2$), but when $\theta_{BV} = 0°$, it is nearer −2. This scaling is what we would expect from a critical balance cascade (see Sect. 3).

There is a close relationship between power and spectral index anisotropy: it is not possible to sustain equal power at all angles over a range of scales if the spectral index varies with θ_{BV}. Indeed, Wicks et al. (2010) have measured power and spectral index anisotropy over the entire inertial range in high speed wind and shown that power is isotropic at the large scale end of the inertial range, and increases to progressively smaller scales: see Fig. 2.

Together, these results are consistent with a critically-balanced cascade in the solar wind, but are not a proof and indeed other explanations, such as a population of discontinuities, could potentially give the same signature. A recent careful study by Forman et al. (2011) of the precise variation of power with θ_{BV} and scale showed that this is indeed largely consistent with a critical balance framework (at least if propagation anisotropy is taken into account) although it is remarkably difficult to eliminate other possibilities such as slab and 2D.

Multi-spacecraft data can be used to study spectral index anisotropy. Direct, k-filtering studies of the inertial range also seem to suggest that the parallel spectrum is steeper than the perpendicular (Narita et al. 2006); Osman and Horbury (2009a) and Chen et al. (2011) also found evidence of spectral index anisotropy using structure functions calculated from four-point Cluster data.

An obvious question is, if the scaling of the magnetic field spectrum is -2 along the field, why does one always observe a scaling of $-5/3$ when using a long interval of data (e.g. Tessein et al. 2009)? The answer is probably straightforward: as we can see from Fig. 2, the power when the field is at large angles to the flow (and the spectral index is $-5/3$) is much larger than when it is parallel to it. The fluctuations with a -2 spectral index are therefore swamped by the dominant $-5/3$ fluctuations: we need to look carefully to detect the scaling of the former, low power, component. Chen et al. (2011) recently showed, using simulation data, that the anisotropic scaling can only be measured when using the local magnetic field and not when using the global mean field.

6 Observations of Anisotropy on Ion Kinetic Scales

In the solar wind, below the proton gyroscale, fluctuations are still present and decay with scale as a power law with a steeper but variable slope (Smith et al. 2006; Sahraoui et al. 2009). The existence of plasma wave modes on these scales, such as kinetic Alfvén waves (Bale et al. 2005) and whistlers (Narita et al. 2011), suggests that an additional cascade might be operating. By analogy with hydrodynamic turbulence, these scales have often been termed the "dissipation range" but since a cascade, rather than damping alone, seems to be present it could also be termed the "dispersion" or simply "ion kinetic" range.

The precise scale of the transition between MHD and ion kinetic scales is somewhat unclear: is it the proton gyroradius, or the proton inertial length? A detailed study by Markovskii et al. (2008) found it difficult unambiguously to distinguish these two cases, although measurements of the rollover in the spectrum by Wicks et al. (2010) seemed to be better ordered by the proton gyroradius.

The presence of power anisotropy in the kinetic range was first demonstrated by Leamon et al. (1998) who used an analysis similar to Bieber et al. (1996) and found a lower level of power anisotropy than in the inertial range. Recent detailed analyses (Chen et al. 2010a; Sahraoui et al. 2010), using multi-spacecraft techniques, which are well suited to analysis of small scale turbulence, have shown higher levels of anisotropy comparable to or even larger than those on MHD scales. Power anisotropy is also present in magnetosheath turbulence (Alexandrova et al. 2008).

Chen et al. (2010a) considered separately the power and spectral index anisotropy of magnetic field fluctuations both parallel and perpendicular to the local magnetic field on scales between the ion and electron gyroscales. They showed that the field-perpendicular components scaled anisotropically, in a way that was consistent with both kinetic Alfvén wave and whistler turbulent cascades—assuming, in addition, critical balance. The parallel components, however, scaled significantly less steeply than expected for a kinetic Alfvén wave cascade, a result that does not as yet have a theoretical explanation.

Wave telescope measurements of this regime (Sahraoui et al. 2010) also show anisotropic scalings, with around $k^{-4.5}$ scaling of the field-perpendicular wavevectors and a steeper parallel slope. Sahraoui et al. (2010) suggested that their results were consistent with some Landau damping in addition to a turbulent cascade in the ion kinetic range, which is eventually damped at electron scales.

Our discussions have largely focused on magnetic field, and to a lesser extent, velocity, data. Recently, Malaspina et al. (2010) demonstrated that the electron density also shows spectral index anisotropy on scales between the ion and electron gyroradii. It is not clear what role compressive fluctuations play in this regime, but unexpectedly, Malaspina et al. (2010) found that the spectral index anisotropy of the electron density at these small scales was similar to that found for the magnetic field on MHD scales. There is no obvious interpretation of this result at this time.

Existing turbulence theories make no prediction as to how levels of anisotropy should change between the fluid into the ion kinetic cascade: would the shape of eddies be preserved, or completely changed? Chen et al. (2010b) showed how power and spectral index anisotropy should vary across these scales assuming that wavevector anisotropy is preserved across the transition. However, observationally, the anisotropy level seems to reduce around the proton gyroradius (Podesta 2009): this could be the result of the presence, at least some of the time, of pressure anisotropy instabilities which can inject power at ion scales (e.g. Kasper et al. 2002; Bale et al. 2009) and indeed there is evidence that such enhancements can directly affect anisotropy measurements (Wicks et al. 2010). Such fluctuations themselves appear to be anisotropic (He et al. 2011a; Podesta and Gary 2011). This raises the interesting possibility that not only is not all the turbulent energy damped at the ion scale, some additional power might actually be injected there and cascade to even smaller scales. Indeed, correlations between wave power and variance anisotropy near the proton gyroradius (Bourouaine et al. 2010, 2011) suggests a role for Alfvén-cyclotron instabilities in the solar wind. The relative partitioning of turbulent energy between protons, electrons and even heavier ions is unclear at this time but is of relevance to coronal heating (Marsch 2006; Cranmer et al. 2009) and wider astrophysical problems: see Schekochihin et al. (2009) and references therein.

Our overall picture of the fluctuations on ion kinetic scales is, clearly, incomplete. The observational challenges of measuring fluctuations on these small scales are significant and theories are also still evolving. Certainly further progress will be made, but with existing missions and instruments it is not clear that a satisfactory answer will be forthcoming as to the nature of the turbulence, if indeed a cascade is present, on ion kinetic scales, let alone around the electron gyroscale.

7 Comparison of Anisotropy Analysis Methods

The difficulty in measuring power and spectral index anisotropy in the solar wind, combined with discrepancies between results obtained from different methods and the importance of anisotropy to theories of plasma turbulence, make it of interest to compare results from more than one method applied to the same solar wind data. Figure 3 shows the results of just such a comparison. Here, magnetic field data (3.5 days starting on 11 January 2005) from the ACE spacecraft (Smith et al. 1998) have been analysed with the wavelet method and the powers when the field is parallel and perpendicular to the solar wind flow are plotted, showing clear power anisotropy. Spectral index anisotropy is also visible: power anisotropy is larger at smaller scales. The spectral indexes of the perpendicular and parallel powers are around $-5/3$ and -2, in agreement with Horbury et al. (2008), Podesta (2009), Luo and Wu (2010). A multi-spacecraft structure function analysis Chen et al. (2010a) has been performed on a 1 hour subset of the same solar wind stream measured by the four Cluster spacecraft. At this time, the spacecraft were around 10,000 km apart, making it possible to study scales above the proton thermal gyroradius. These data are also shown in Fig. 3;

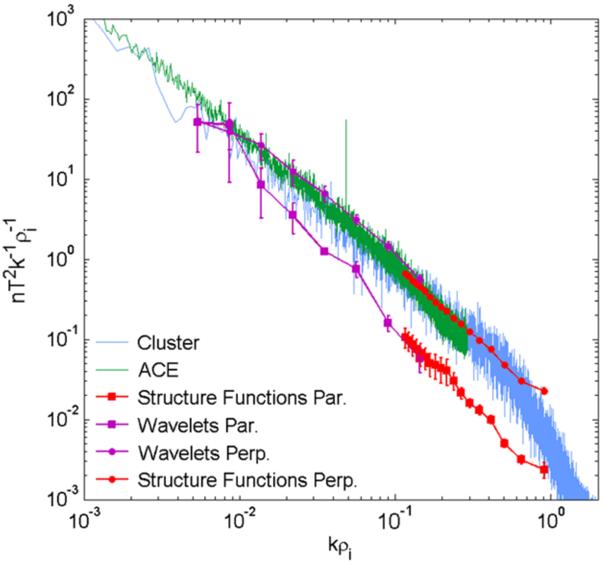

Fig. 3 Power spectra parallel and perpendicular to the local field, calculated from ACE data at large scales using the wavelet method and at small scales from Cluster data using the multi-spacecraft structure function method, with all data taken in the same solar wind stream. *Blue* and *green lines* show the total spacecraft power spectrum from Cluster and ACE respectively

note that the x-axis for both sets of results is normalised by the proton gyroradius (Wicks et al. 2010), which here was around 60 km. The relative normalisation of power spectra and structure functions is subtle (Essenwanger and Reiter 1969) and the details are not presented here. In addition, a correction of a factor of 8 has been made to allow for different overall power levels in the short Cluster sub-interval compared to the longer ACE interval, based on the relative power levels in the simple Fourier spectra.

The agreement of the two methods—and in particular the level of power anisotropy and the spectral indexes—over the range of scales where they overlap is excellent. This is strong evidence that both methods are reliable measures of anisotropy and also makes it possible to cover a wide range of scales. In the future, application of this 'dual-technique' method when the Cluster spacecraft were at smaller separations should make it possible to cover the MHD turbulent cascade from the outer scale through the proton gyroradius and into the ion kinetic cascade, revealing how anisotropy changes between these two regimes.

8 Effects of Anisotropy

We have seen that wavevector anisotropy is present in solar wind turbulence from the outer scale all the way down to the electron gyroscale and that the majority of analysis methods also report spectral index anisotropy. While these results are of interest in diagnosing the physical processes within the turbulent cascade, they are also important in a number of other areas where turbulence can affect the plasma. Arguably most important is the effect on energetic particle propagation, both parallel and across the magnetic field. Energetic particles resonate with and scatter from fluctuations with a field-parallel wavevector component, but we have seen that most power is at large angles to the field and so this dominant power does not contribute to field-parallel diffusion: using just the field-parallel power, one recovers field-parallel diffusion coefficients closer to those measured directly from solar energetic particles (Bieber et al. 1996). However, it has been pointed out by Chandran (2000) that under a critical balance framework, there is so little field-parallel power that one would expect very little scattering—potentially much less than is observed. The anisotropy of power in MHD turbulence also leads to the "braiding" or shredding of magnetic flux tubes (e.g. Bieber et al. 1996): this has the effect of increasing the diffusion of particles perpendicular to the mean field (e.g. Zimbardo 2005). The quantitative effects of anisotropy on particle propagation are still poorly understood and indeed there is still considerable uncertainty whether discontinuities in the heliospheric magnetic field are the edges of flux tubes (e.g. Borovsky 2008) or caused by the turbulence itself (e.g. Seripienlert et al. 2010).

The turbulent cascade of energy from large to small scales heats the solar wind plasma (Cranmer et al. 2009) but cascade rates can be significantly affected by both the power anisotropy and imbalance (inward vs outward propagating fluctuations) of the cascade: this effect has not been studied in detail to date.

One effect of power anisotropy is that the apparent power level of the fluctuations as measured by a spacecraft can be rather different to that in the plasma itself. The trailing edges behind high speed solar wind streams, when the plasma velocity declines steadily over several days, often contain near-radial magnetic fields with very low power levels (e.g. Gosling and Skoug 2002). The origin of these regions is debated but the near-radial magnetic field means that the plasma frame power levels are probably rather higher than those measured directly by spacecraft; again, this has consequences for the interpretation of particle propagation through these regions.

We also note one more unusual consequence of wavevector anisotropy. The interstellar medium (ISM) is filled with turbulence which is likely to be anisotropic just like that in the solar wind. Measurements of the ISM spectrum are made using radio scintillation generated by density fluctuations, which are expected to be carried as a "passive scalar" and hence have the same properties as the bulk turbulence. If this is the case, with power in wavevectors at large angles to the magnetic field and that field direction randomly distributed at large scales, we would expect to measure a power level only around 84% of the actual power. This is a relatively small effect but can be much larger if the magnetic field tends to be aligned with the line of sight.

9 Discussion

The existence of wavevector anisotropy in the solar wind, on both MHD and ion kinetic scales, seems clear. Anisotropy of scaling is not universally accepted but the evidence is growing. Analysis of solar wind data, combined with new data sets, is proving one of the driving forces in the development of our understanding of plasma turbulence. However, much remains unclear.

The question of whether the turbulence cascade is critically balanced remains largely unanswered given the remarkable difficulty in distinguishing this case from other examples such as slab and 2D. The different effects that these two configurations have on energetic particles, however, means that this remains an important issue. Indeed, the attractiveness of critical balance as a concept has led to its widespread adoption in turbulence theory on both MHD and kinetic scales so it is important to determine if this paradigm is indeed applicable to plasma turbulence. Most work to date has concentrated on the magnetic field components and to a lesser extent the velocity. Little progress has been made in studying compressive fluctuations in the density and field magnitude: although there is evidence for power anisotropy (He et al. 2011b) the nature of their cascade is not well understood. We know that turbulence is often "imbalanced" in the sense of supporting Alfvénic fluctuations with a dominant sense of propagation and there appears to be anisotropic—and different—scaling in the two senses of propagation (Wicks et al. 2011), but no existing theories seem to explain the observations adequately. The details of alignment in solar wind turbulence, and its interactions with wavevector anisotropy, are only just beginning to be investigated. Finally, the transition from fluid to ion kinetic scales, and the effect of anisotropy in how energy flows across the boundary, has yet to be explored in any depth. The prospects of progress in many of these areas are excellent, with the availability of good data sets of many parameters (velocity, density, etc.) from both single and multi-spacecraft missions. Perhaps the biggest measurement limitation at the moment is the lack of very high quality data on ion, and even electron, kinetic scales. Upcoming missions such as MMS, Solar Orbiter and Solar Probe Plus will greatly improve this situation, but in order to finally characterise the very challenging electron cascade in particular, a dedicated "turbulence explorer" mission may be required.

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