Solar Wind Electrons and Langmuir Turbulence

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Abstract. Typical *in situ* spacecraft measurements made in the solar wind show that charged particle velocity distribution function (VDF) contains energetic component with quasi scale-free power-law velocity dependence, $f \sim v^{-\alpha}$. This paper proposes a theory for quiet-time solar-wind electrons that are in dynamical equilibrium with plasma turbulence. The theory predicts $f \sim v^{-6.5}$ for high electron velocities, while observations by WIND and STEREO spacecraft reveal $v^{-5.0}$ to $v^{-8.0}$ dependence. This shows that theory falls within the observed range.

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INTRODUCTION

In situ spacecraft measurements since the 1960s show that charged particles in solar-terrestrial environment deviate considerably from Maxwell-Boltzmann distribution in the high energy tail portion [1]. The observed velocity distribution function (VDF) can be empirically fitted with the kappa distribution [2], $f(v) \sim (1+v^2/\kappa v_{Te}^2)^{-(\kappa+1)}$, where $v_{Te} = (2k_BT_e/m_e)^{1/2}$ is the Maxwellian thermal speed, k_B is the Boltzmann constant, T_e and m_e are Maxwellian temperature and electron mass, respectively. The limit $\kappa \to \infty$ corresponds to the classic Maxwell-Boltzmann distribution, $f(v) \sim \exp(-v^2/v_{Te}^2)$. The kappa model is not only a convenient empirical tool but it may enjoy profound theoretical justifications for its use. It corresponds to the most probable state in the non-extensive thermo-statistics [3] or generalized Gibbsian thermodynamics [4]. It is also an equilibrium solution for generalized Boltzmann equation [5].

In this paper we propose yet another theoretical foundation in which a kappa-like electron VDF naturally emerges. Specifically, it may correspond to a time-asymptotic state of the electrons dynamically interacting with plasma turbulence. This follows from early one-dimensional (1D) studies of solar type-III electron beam and Langmuir turbulence problem [6, 7] in which it was found that the electron VDF evolves into a quasi time-asymptotic kappa-like state over a time scale much longer than quasi-linear relaxation time. Such a trend was confirmed in 1D particle-in-cell (PIC) simulation [8]. This naturally led to the question of whether a truly time-asymptotic dynamical equilibrium solution exists or not, especially in three dimensions (3D). The present paper addresses this issue. We shall apply the present finding to observation of heliospheric energetic electrons detected near 1 AU by WIND and STEREO spacecraft.

As noted already, solar wind electron VDFs contain high-energy tail [9] which is typically described as thermal core plus superthermal halo. Recently, the superhalo

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distribution was additionally identified by WIND spacecraft [10]. The solar wind is also characterized by pervasive quasi-thermal noise [11]. We assume that quiet-time solar-wind electrons are in dynamical equilibrium with quasi-thermal noise turbulence. Customary theories of superthermal electrons found in the literature rely on altitude-dependent collisional dynamics [12]. The present paper is concerned with local wave-particle (collective) dynamical processes, hence is complementary to the customary theories. As the solar wind expands there will be a constant competition between the time-of-flight beam reformation and wave-induced relaxation. We envision that the resulting dynamical steady-state will be that of kappa-like electron VDF and enhanced quasi-thermal Langmuir turbulence spectrum.

ASYMPTOTIC TURBULENT STATE

We now outline the actual theory starting from the kinetic equation for solar wind electrons,

$$\frac{\partial f_e}{\partial t} = \frac{\partial}{\partial v_i}\left(v_i\, G\, f_e + D\,\frac{v_i v_j}{v^2}\,\frac{\partial f_e}{\partial v_j}\right), \tag{1}$$

where $G = ne^4/(m_e v)^2 \int d\mathbf{k}\; k^{-2}\;\delta(\omega_{pe} - \mathbf{k}\cdot\mathbf{v})$ and $D = \pi e^2 \omega_{pe}^2/(m_e v)^2\; \int d\mathbf{k}\; k^{-2}$ $\delta(\omega_{pe} - \mathbf{k}\cdot\mathbf{v})\, I_L(\mathbf{k})$ are velocity space drag and diffusion coefficients, respectively; $\omega_{pe} = (4\pi n e^2/m_e)^{1/2}$ is the plasma frequency; e, n, and m_e being unit electric charge, density and electron mass, respectively; and $I_L(\mathbf{k}) = |E(\mathbf{k})|^2$ is the spectral Langmuir wave energy density. Note that Eq. (1) describes local wave-particle interaction processes between the electrons and Langmuir turbulence. The transport of particles and waves to the acceleration region requires the modification of the left-hand side of the particle and wave kinetic equations to include convective terms and inhomogeneity effects.

We seek an isotropic asymptotically steady-state isotropic solution to Eq. (1), $\partial f_e(v,t)/\partial t \to 0$. Even without specifying the wave spectrum, a formal solution can be obtained [13]

$$f_e(v) = C \exp\left(-\int dv\; \frac{vG}{D}\right). \tag{2}$$

However, the specific form of $I_L(\mathbf{k})$ must be provided to Eq. (2) through D. The Langmuir turbulence intensity must be a time-asymptotic solution, $\partial I_L(\mathbf{k},t)/\partial t \to 0$, of the wave kinetic equation, which is given by [14, 15]

$$\begin{aligned}
\frac{\partial I_L(\mathbf{k})}{\partial t} &= \left[\frac{\pi\omega_{pe}^2}{k^2}\int d\mathbf{v}\;\delta(\omega_{\mathbf{k}}^L - \mathbf{k}\cdot\mathbf{v})\left(\frac{ne^2}{\pi}\, f_e + \omega_{\mathbf{k}}^L\, I_L(\mathbf{k})\,\mathbf{k}\cdot\frac{\partial f_e}{\partial \mathbf{v}}\right)\right]_{\mathrm{a}} \\
&\quad + \left[\int d\mathbf{k}'\;\omega_{\mathbf{k}}^L V_{\mathbf{k},\mathbf{k}'}\;\delta(\omega_{\mathbf{k}}^L - \omega_{\mathbf{k}'}^L - \omega_{\mathbf{k}-\mathbf{k}'}^S)\left(\omega_{\mathbf{k}}^L\, I_L(\mathbf{k}')\,\frac{I_S(\mathbf{k}-\mathbf{k}')}{\mu_{\mathbf{k}-\mathbf{k}'}}\right.\right. \\
&\quad \left.\left. -\omega_{\mathbf{k}'}^L\,\frac{I_S(\mathbf{k}-\mathbf{k}')}{\mu_{\mathbf{k}-\mathbf{k}'}}\, I_L(\mathbf{k}) - \omega_{\mathbf{k}-\mathbf{k}'}^L\, I_L(\mathbf{k}')\, I_L(\mathbf{k})\right)\right]_{\mathrm{b}}
\end{aligned}$$

$$
-\left[\int d\mathbf{k}' \int d\mathbf{v}\, U_{\mathbf{k},\mathbf{k}'}\, \delta[\omega_{\mathbf{k}}^L - \omega_{\mathbf{k}'}^L - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}] \left(\frac{ne^2}{\pi\omega_{pe}^2}\,\omega_{\mathbf{k}}^L \right.\right. \tag{3}
$$

$$
\left.\left.\times \left[\omega_{\mathbf{k}'}^L I_L(\mathbf{k}) - \omega_{\mathbf{k}}^L I_L(\mathbf{k}')\right] f_i - \frac{m_e}{m_i}\,\omega_{\mathbf{k}}^L I_L(\mathbf{k}') I_L(\mathbf{k})\,(\mathbf{k}-\mathbf{k}')\cdot\frac{\partial f_i}{\partial \mathbf{v}}\right)\right]_{\mathrm{c}},
$$

where $V_{\mathbf{k},\mathbf{k}'} = \pi e^2/(2k_BT_e)^2\,\mu_{\mathbf{k}-\mathbf{k}'}\,(\mathbf{k}\cdot\mathbf{k}')^2/(k^2k'^2\,|\mathbf{k}-\mathbf{k}'|^2)$, $\mu_{\mathbf{k}} = (m_e/m_i)^{1/2}\,\omega_{pe}\,k\lambda_{De}\,(1+k^2\lambda_{De}^2)^{-1/2}(1+3T_i/T_e)^{1/2}$, $\lambda_{De} = \sqrt{T_e/(4\pi ne^2)}$ being the Debye length, and $U_{\mathbf{k},\mathbf{k}'} = [\pi e^2/(\pi\omega_{pe}^2)]\,(\mathbf{k}\cdot\mathbf{k}')^2/(kk')^2$. In Eq. (3) terms denoted with subscripts a, b, and c correspond to (a) spontaneous and induced emissions, (b) three-wave decay, and (c) spontaneous and induced scattering (or nonlinear wave-particle interaction) processes, respectively. In Eq. (3) the quantity $I_S(\mathbf{k})$ corresponds to the ion-acoustic turbulence spectrum, and $f_i(v)$ stands for Maxwellian ion VDF. A similar equation for the ion-sound turbulence exists but it is omitted here. In the above $\omega_{\mathbf{k}}^L$ and $\omega_{\mathbf{k}}^S$ stand for Langmuir and ion-acoustic dispersion relations, respectively. In the asymptotic state, $t \to \infty$, it can be shown that term (b) can be ignored when compared with term (a) or term (c). However, during the dynamical processes that lead to the asymptotic state term (b) must be retained.

According to previous studies [6, 7, 8], it was found that the formation of kappa-like energetic tail is suppressed in a purely collisionless Vlasov treatment where spontaneous effects are absent. This indicates it is reasonable to seek the steady-state solution by balancing spontaneous and induced terms in both linear and nonlinear terms. Let us first consider the linear term,

$$
0 = \frac{\pi\omega_{pe}^2}{k^2}\int d\mathbf{v}\,\delta(\omega_{\mathbf{k}}^L - \mathbf{k}\cdot\mathbf{v})\left(\frac{ne^2}{\pi}\,f_e + \omega_{pe} I_L(\mathbf{k})\,\mathbf{k}\cdot\frac{\partial f_e}{\partial \mathbf{v}}\right). \tag{4}
$$

Balancing the terms within the integrand, it can be shown that a self-consistent set of electron VDF and quasi-thermal noise spectrum emerges from Eqs. (2) and (4):

$$
\begin{aligned}
f_e(v) &= \frac{1}{\pi^{3/2}v_{Te}^3}\frac{\Gamma(\kappa)}{\kappa^{3/2}\,\Gamma(\kappa-3/2)}\frac{1}{(1+v^2/\kappa v_{Te}^2)^{\kappa}},\\
I(\mathbf{k}) &= \frac{k_BT_e}{4\pi^2}\left(1+\frac{1}{\kappa\,(kv_{Te}/\omega_{pe})^2}\right),
\end{aligned} \tag{5}
$$

where $\Gamma(x)$ represents the gamma function. The Maxwellian electron temperature T_e is related to effective kinetic temperature by $T_e^{\mathrm{eff}} = T_e\,\kappa/(\kappa-5/2)$. We should note that some authors recently made use of the kappa distribution to calculate quasi-thermal noise spectrum [17] without addressing the issue of self-consistency. On the basis of (5) one may derive the dispersion relation $\omega_{\mathbf{k}}^L$,

$$
\omega_{\mathbf{k}}^L = \omega_{pe}\left(1+\frac{3}{2}\frac{\kappa}{\kappa-5/2}\frac{k^2v_{Te}^2}{\omega_{pe}^2}\right). \tag{6}
$$

At this point, the value of κ is yet to be determined.

To determine κ, we now balance nonlinear spontaneous and induced scattering terms,

$$
\begin{aligned}
0 &= \int d\mathbf{k}' \int d\mathbf{v}\, \frac{(\mathbf{k}\cdot\mathbf{k}')^2}{k^2 k'^2}\, \delta[\omega^L_{\mathbf{k}} - \omega^L_{\mathbf{k}'} - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}] \\
&\quad \times \left(\frac{k_B T_i}{4\pi^2} [\omega^L_{\mathbf{k}'} I_L(\mathbf{k}) - \omega^L_{\mathbf{k}} I_L(\mathbf{k}')] + I_L(\mathbf{k}') I_L(\mathbf{k}) (\omega^L_{\mathbf{k}} - \omega^L_{\mathbf{k}'}) \right) f_i, \qquad (7)
\end{aligned}
$$

where T_i is the ion temperature. Since the resonance $\omega^L_{\mathbf{k}} - \omega^L_{\mathbf{k}'} - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v} = 0$ is satisfied only for $\mathbf{k} \approx \mathbf{k}'$, we may assume $\mathbf{k}' = \mathbf{k} + \delta\mathbf{k}$, where $|\delta\mathbf{k}| \ll 1$, and expand the integrand,

$$
\begin{aligned}
0 &= \int d(\delta\mathbf{k}) \int d\mathbf{v}\, \frac{(\mathbf{k}\cdot\mathbf{k}')^2}{k^2 k'^2}\, \delta[\omega^L_{\mathbf{k}} - \omega^L_{\mathbf{k}'} - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}] \\
&\quad \times \delta\mathbf{k} \cdot \left(\omega^L_{\mathbf{k}} \frac{dI_L(\mathbf{k})}{d\mathbf{k}} + \frac{4\pi^2}{k_B T_i} \frac{d\omega^L_{\mathbf{k}}}{d\mathbf{k}} [I_L(\mathbf{k})]^2 - \frac{d\omega^L_{\mathbf{k}}}{dk} I_L(\mathbf{k}) \right) f_i. \qquad (8)
\end{aligned}
$$

The solution to Eq. (8) is given by

$$
I(k) = \frac{k_B T_i}{4\pi^2} \left(1 + \frac{(4/3)(\kappa - 5/2)}{\kappa (k v_{Te}/\omega_{pe})^2} \right). \qquad (9)
$$

Upon comparing Eqs. (5) and (9), we find that the simplest solution is when $T_i = T_e$ and $(4/3)(\kappa - 5/2) = 1$. From the latter requirement we easily obtain the value of κ as $\kappa = 13/4 = 3.25$. The assumption of $T_i = T_e$ is not too inconsistent with observed ratio of electron to proton temperatures T_e/T_p at 1 AU. For instance, according to Ref. [16] the mean value of T_e/T_p is ~ 1.1 for high-speed solar wind.

With $\kappa = 3.25$ the asymptotic VDF is given by $f_e(v) \sim v^{-6.5}$, for $v \gg v_{Te}$. Let us compare this against the quiet-time solar wind electron VDF. We have made a preliminary survey of $\sim$2 to 100 keV electron observations from the SupraThermal Electron (STE) instrument on the STEREO A & B spacecraft, during quiet times in the interplanetary medium in 2007–2008. In general, the quiet-time VDFs of superhalo electrons fit well to a single power-law ($f \sim v^{-b}$), ranging from v^{-5} to v^{-8}. Figure 1 shows that on 9 January 2007 when WIND and two STEREO spacecrafts were close together ($< 140\ R_E$ or $\sim$0.06 AU), the observed superhalo electron VDFs show similar power-laws, $\sim v^{-7.3}$. About 10 months later when the STEREO spacecrafts were separated by $\sim 42^\circ$ ecliptic longitude ($\sim$0.7 AU), the superhalo electrons (Figure 1, inset) show significantly different power-laws (exponents of -5.3 and -6.3) at the two spacecraft, indicating variation on that spatial scale, and possibly temporal variation on a scale of months. Such a variation is not unexpected since the real solar wind is not in exact dynamical equilibrium. Nevertheless, judging from the fact that theoretical prediction of $v^{-6.5}$ is intermediate between observed range of power-law indices, we find that the agreement is quite remarkable. For the sake of completeness we display the observed quasi-thermal noise spectrum in Figure 2, in which electric field fluctuation detected by STEREO on 30 November 2007 during the time interval from 07:29:00 to 08:21:00 is displayed.

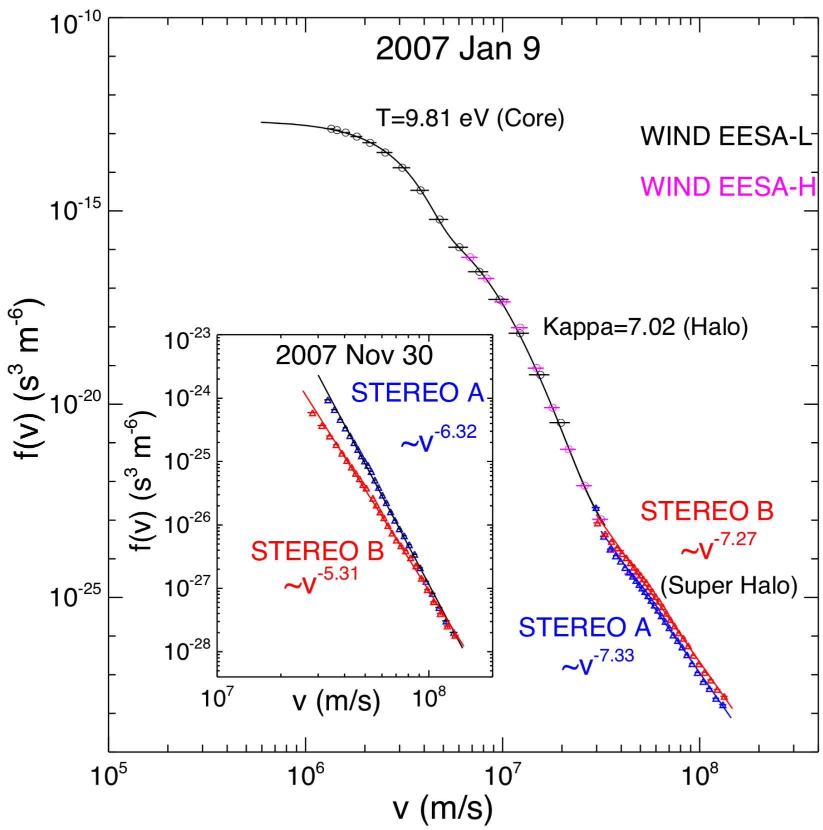

FIGURE 1. Omnidirectional electron velocity distribution function (VDF) measured from $\sim 10^6$m/s ($\sim$5eV) to $\sim 10^8$m/s ($\sim$60keV) during a quiet period in the interplanetary medium on 9 January 2007, The black line gives the Maxwellian fit to the solar wind (SW) core and Kappa fit to the SW halo, measured by the Wind spacecraft. The pink and blue lines are power-law fit to the solar wind superhalo measured by the STEREO A & B spacecraft. The three spacecraft are located within $\sim$140 R_E (0.06 AU) of each other, near L1, $\sim$200 R_E upstream of the Earth. The inset shows the superhalo electron spectra measured on 30 November 2007 by STEREO A & B, separated by $\sim$0.7 AU (20.6° ahead of, and 21.1° ecliptic longitude behind, the Earth, respectively).

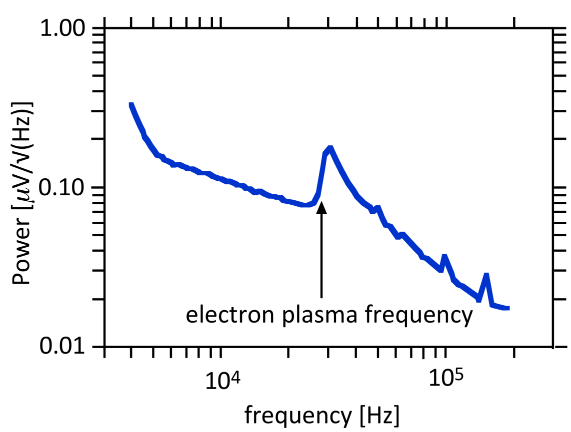

FIGURE 2. Quasi-thermal noise spectrum detected on 30 November 2007 during the time interval from 07:29:00 to 08:21:00, with a prominent enhancement at the local plasma frequency.

CONCLUSIONS

In the present paper we discussed the asymptotic steady-state solution to the self-consistent plasma turbulence equation. We argued that non-Maxwellian kappa-like electron VDF, which is the end result of beam-plasma and Langmuir turbulence process [6, 7, 8, 15], can be interpreted as the turbulent quasi-equilibrium [5]. Upon comparing the theory against the quiet-time solar wind electron VDF, we found a reasonable agreement between the theory ($v^{-6.5}$) and observation ($\sim v^{-5.0}$ to v^{-8}).

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