



Atoms Through the Looking-Glass - a Relativistic Challenge

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Two Decades of EBIT spectra Five Decades after Parity Revolution

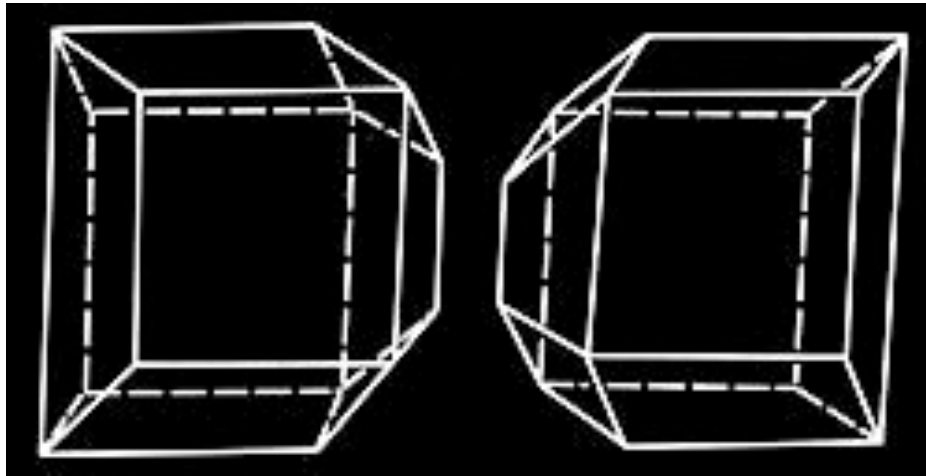
- Left-right - Parity
- Atomic Parity Non-Conservation
- Atomic calculations – ex: HFS, IS
- Relativistic calculations
- Nuclear distributions, EDM
- Bohr-Weisskopf effect - EBIT





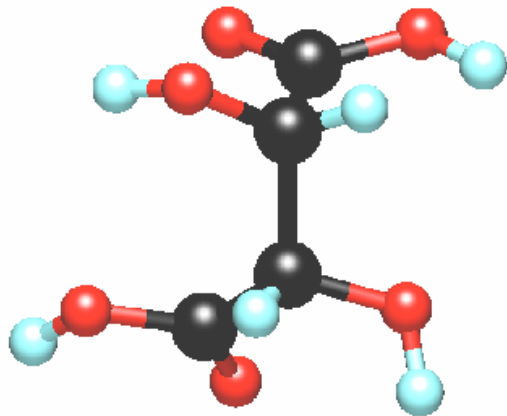
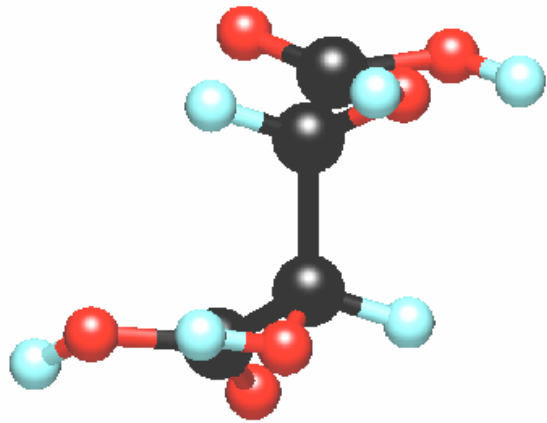
- Right-handed seashells
- Left-handed honeysuckles
- Righthanded bindweed ...





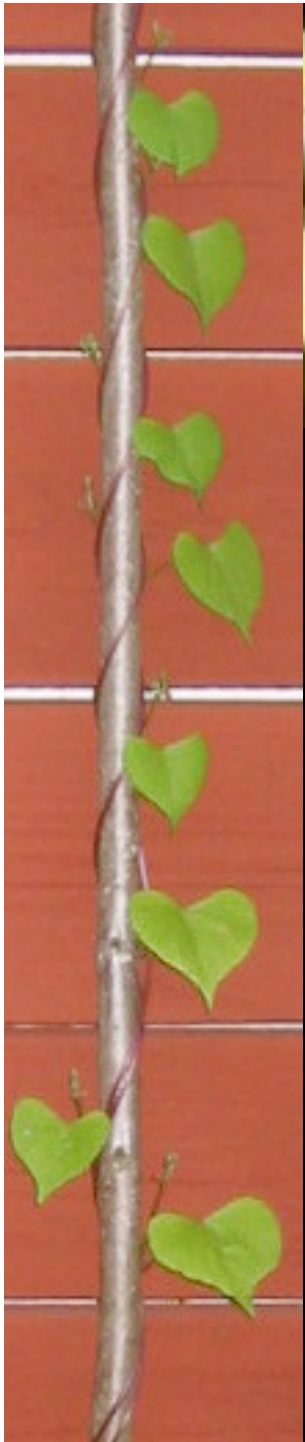
Pasteur

Parity

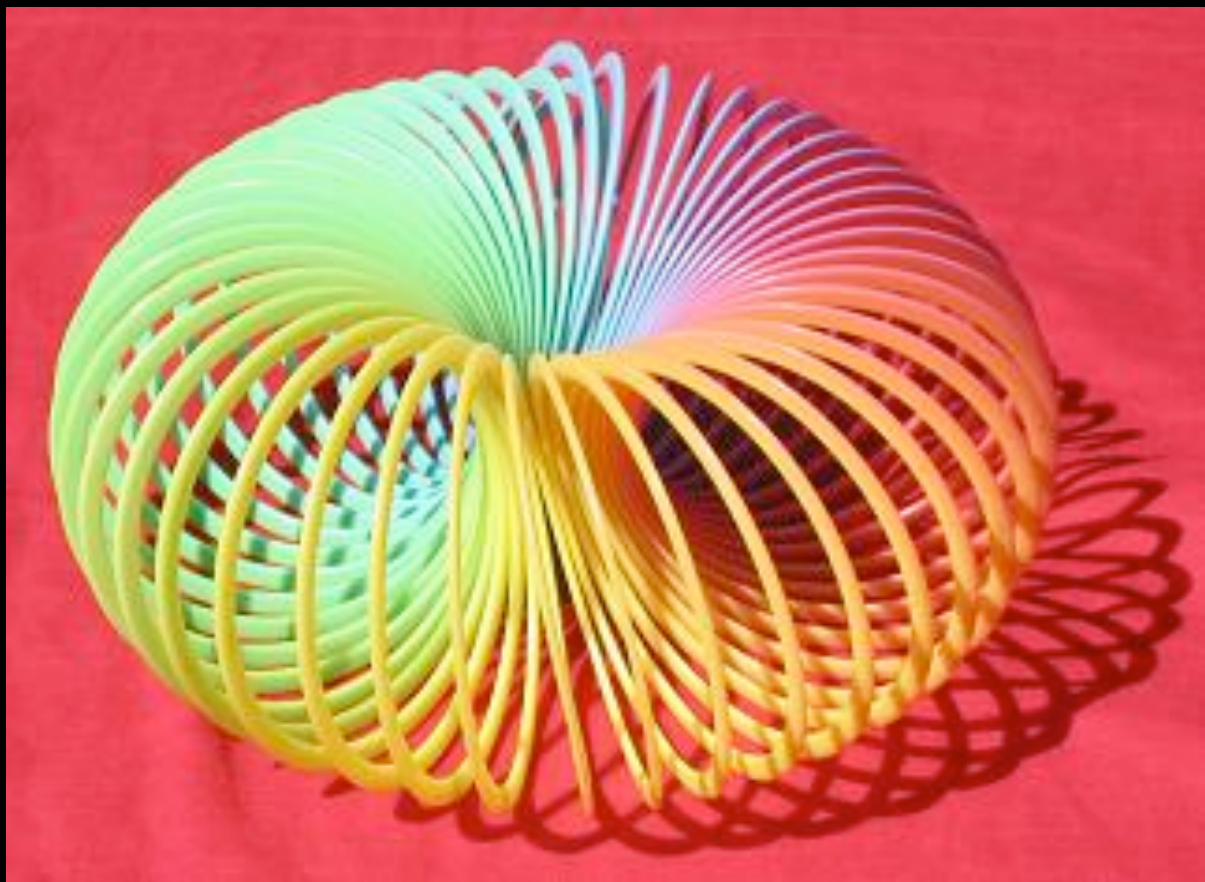


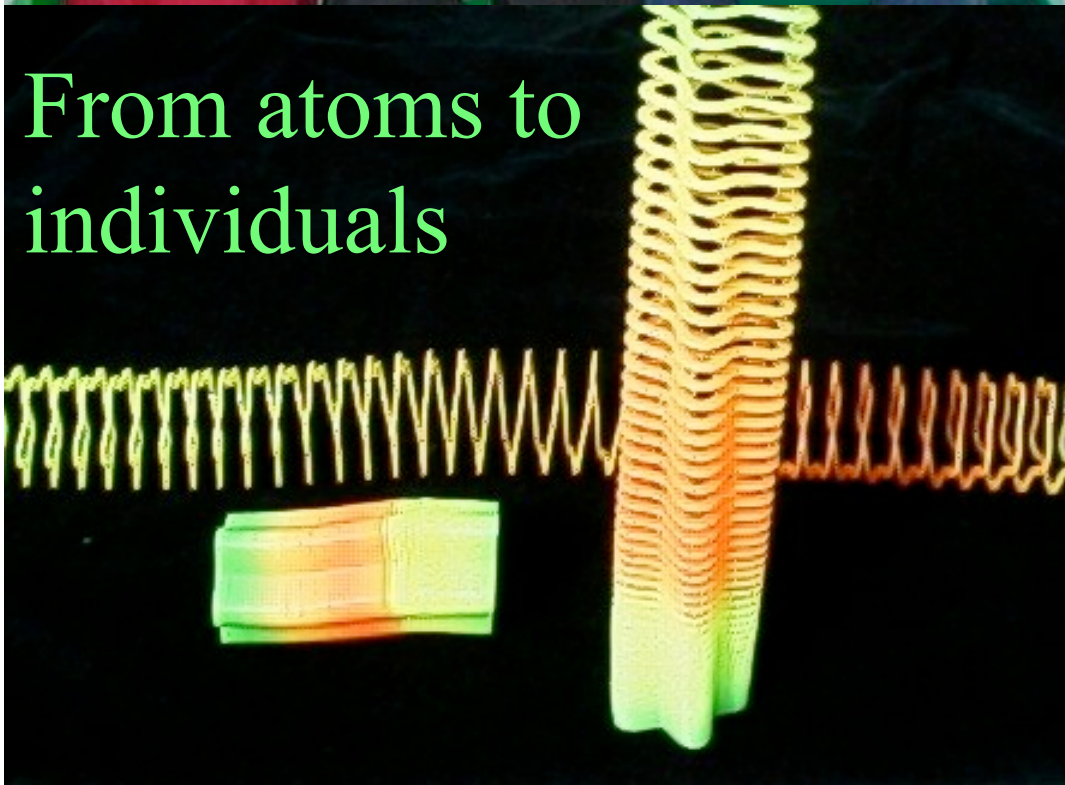
Life





Left-
or –
right-
handed
?







Inversion
Rotation

Left-
handed

-

Right -
handed

?

Parity Revolution 1956-1967

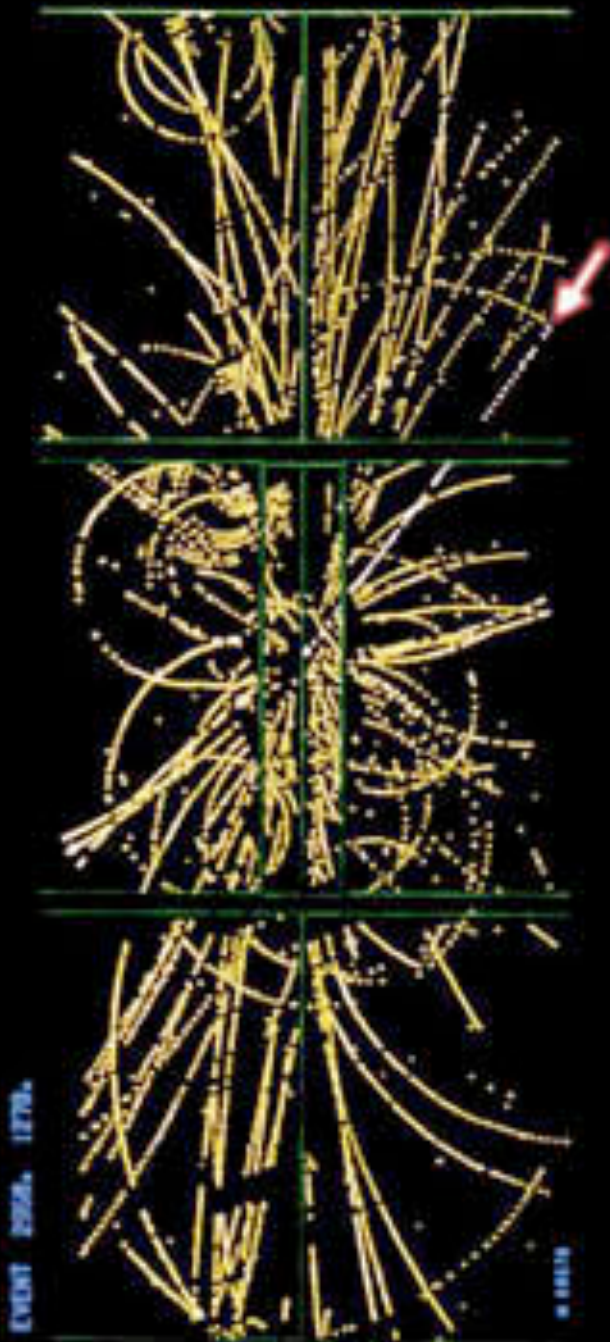
Lee and Yang

Wu et al:

Beta Decay of Co

W-boson – discovered at
CERN 1983

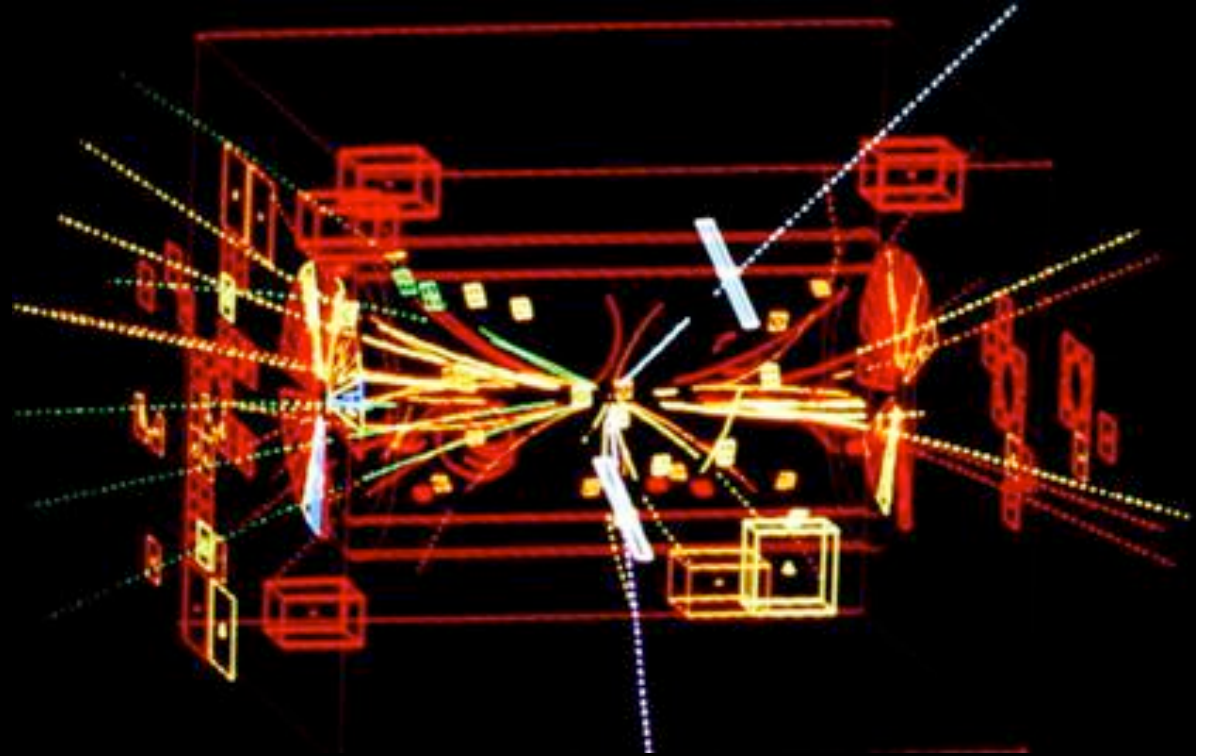
Atomic Parity Non-
Conservation (PNC)?



Neutral Weak Currents

—

Z- Boson
Discovered at
CERN in
1983



Z-boson exchange modifies the
electron-nucleus interaction

Z -boson :

As heavy as a Sr atom —
short range interaction

Search for atomic PNC

Bouchiat and Bouchiat, 1974

Z^3 enhancement (short range)

Possible for heavy atoms

Candidates:

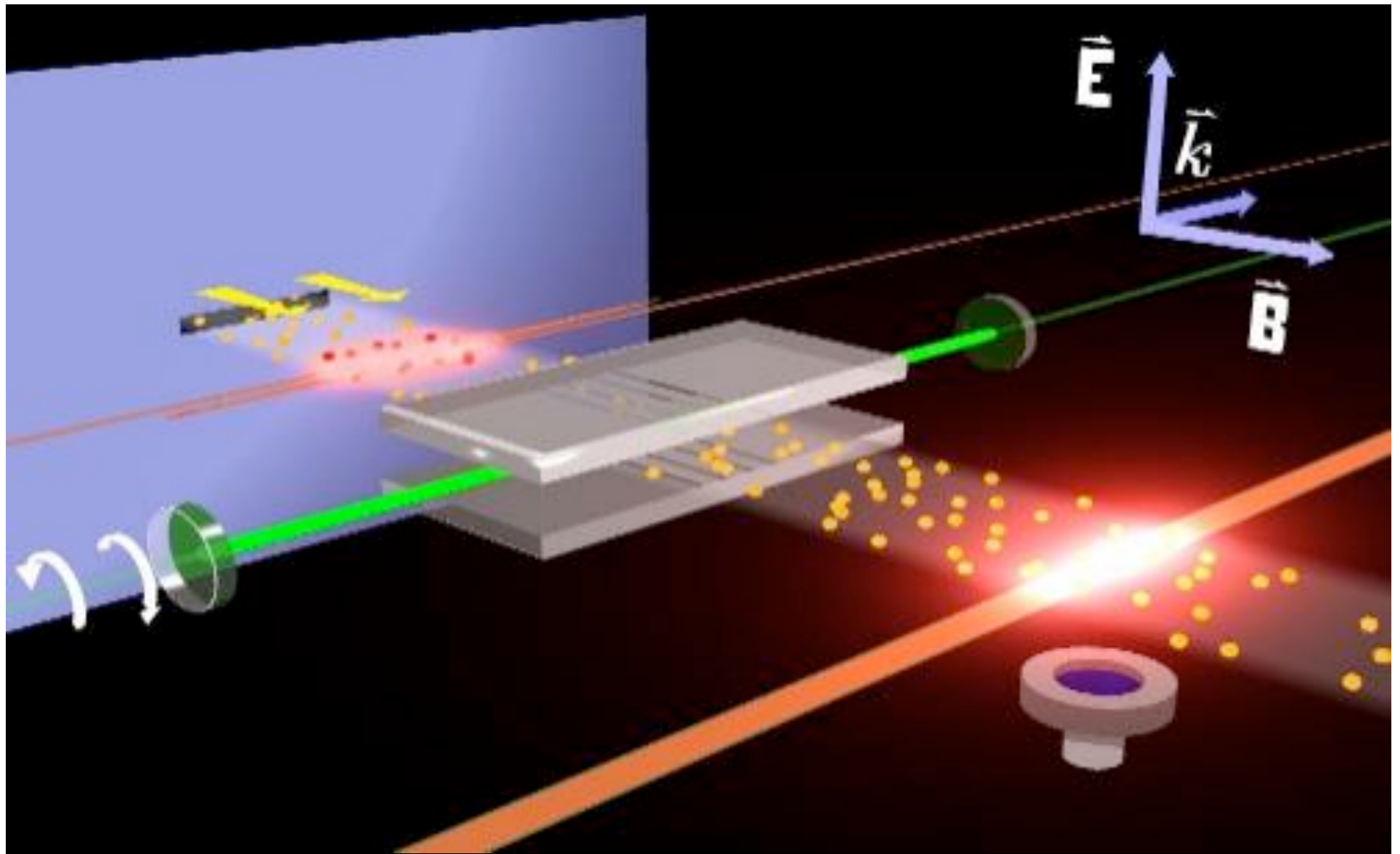
Cs, Bi, Tl, Pb, Yb, Sm, Fr ...

Paris, Oxford, Seattle, Novosibirsk,
Berkeley, Boulder ...

Relativistic effects, nuclear size ...

Z^3





Colorado setup

Atomic theory in the 70s

Energies

Transition
probabilities

MCHF / MCDF

CI

RPA

Hyperfine structure ...

- Atomic many-body calculations
- Core polarization, all orders
- MBPT
- All- orders –
- Coupled-Cluster
- Non-relativistic

Helium-like systems

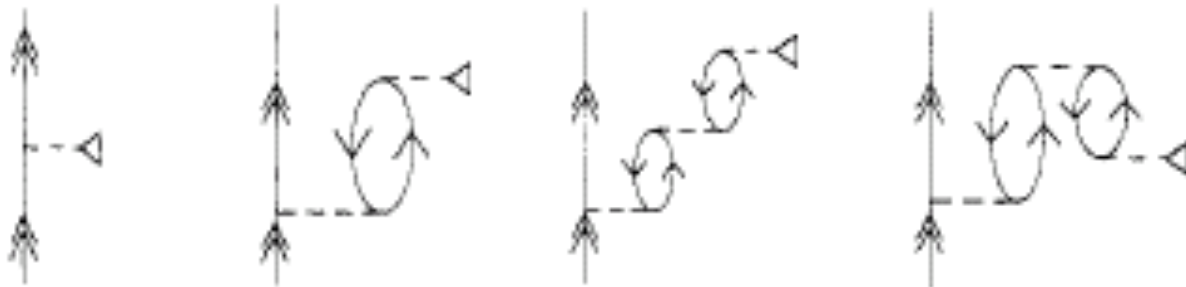
- Double CI – ”exact”
- All order pair equation – ”exact”
- Basis set questions / grid size + numerics

Essentially exact calculations possible

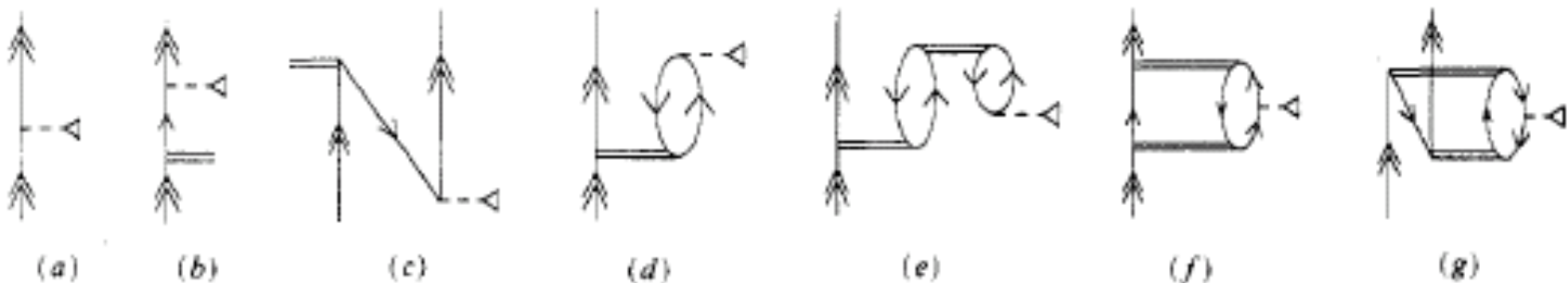
Hylleraas, Pekeris, Drake ...

"RPA", Core polarization, Single excitations to all orders:

(For HFS –
only exchange)



+CORRELATION (double exc. all orders)



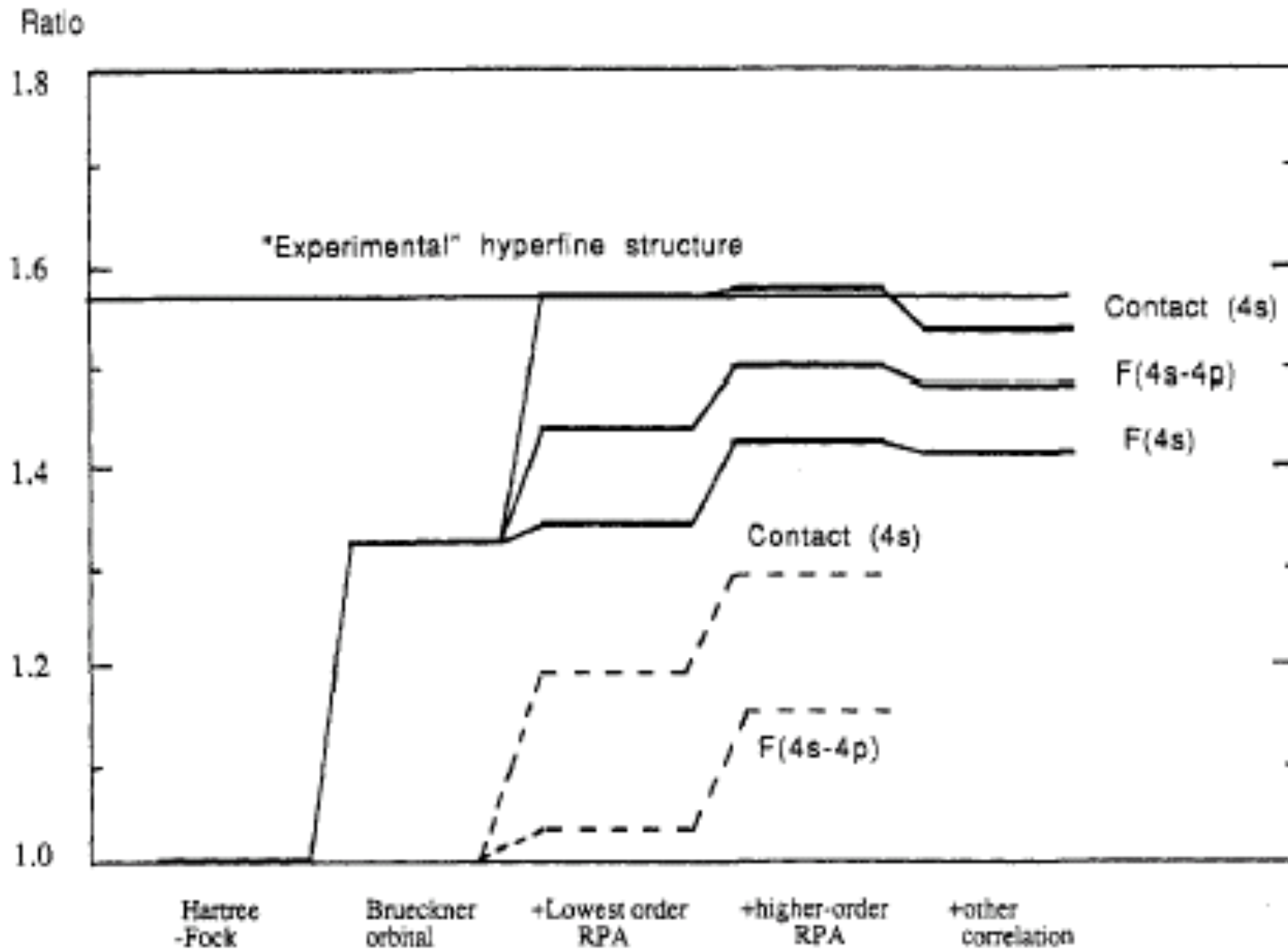
"BRUECKNER
ORBITALS"

"RPA"

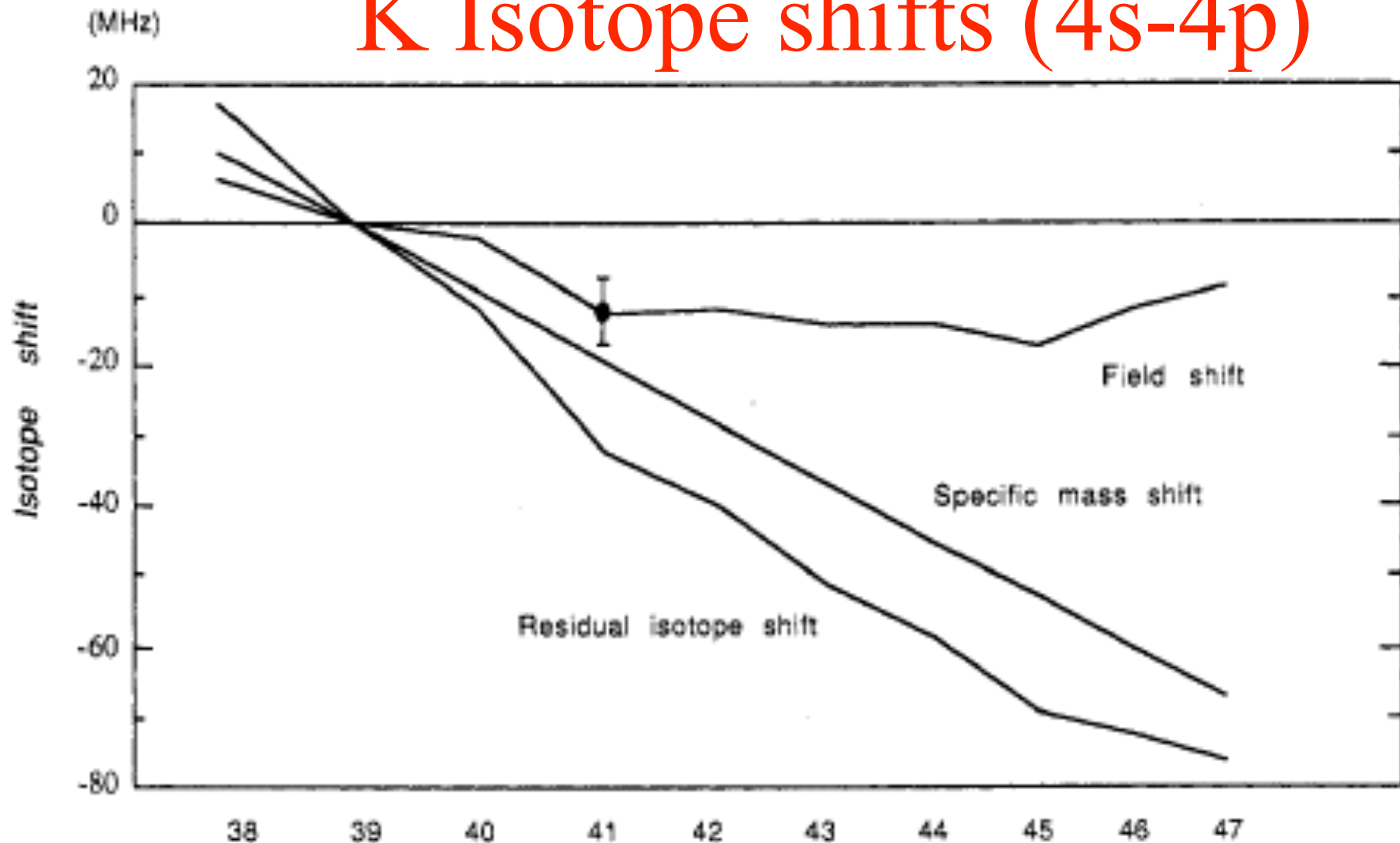
Other Correlation

Exempel: K – HFS + FS

(AMMP et al 1990)

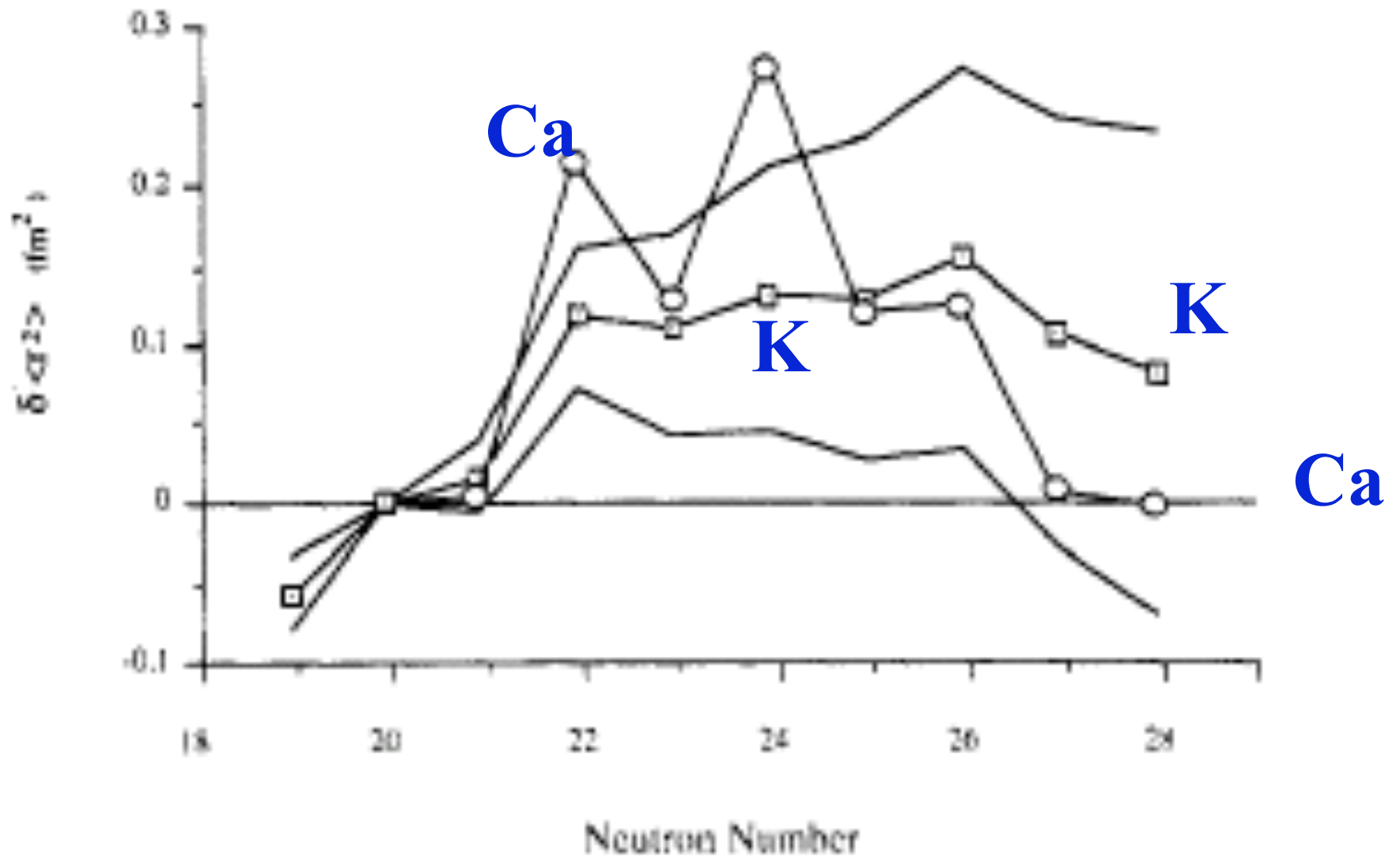


K Isotope shifts (4s-4p)



$$\delta\nu^{AA'} = \underbrace{\left(K^{\text{NMS}} + K^{\text{SMS}}\right)}_{\text{Nuclear recoil}} \left(\frac{1}{M_A} - \frac{1}{M'_A}\right) + F\kappa\delta\langle r^2 \rangle^{AA'}$$

K – Ca isotopes charge radii



Relativistic generalizations

The "Brown-Ravenhall Disease"

"Continuum dissolution"

$$mc^2$$

0

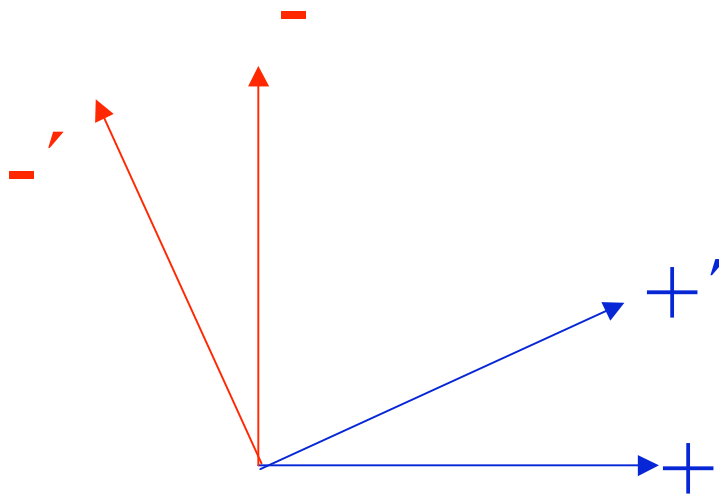
$$-mc^2$$

Positrons



Warning! A complete orbital set includes both positive and negative energy states!

Projection operators



What is
positive
?

”Free choice”

Free electrons – nuclear potential – Dirac–Fock?
”Furry picture”

Implementations of Projection operators

- Explicit insertion
- By construction of orbital set, keeping only positive energy part
- Implicit – through boundary conditions – relations upper/lower component (MCDF e.g.)

1986 (Trieste meeting)

Methods under development

Indiana, Göteborg, Novosibirsk, Oxford

...

L-extrapolation Helium

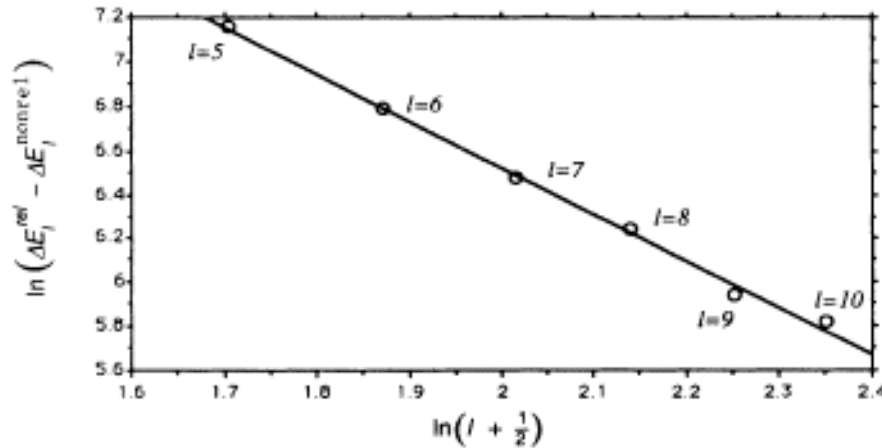


FIG. 2. Log-log plot of the relativistic corrections to the partial-wave increments of the Coulomb correlation energy of helium (from Table III).

Non-relativistic
Part L ⁻⁴

Relativistic
Part L ⁻²

2	-224 993
3	-553 90
4	-197 32
5	-8698
6	-4409
7	-2466
8	-1485
9	-947
10	-631
$\sum_{l=11}^{\infty}$	-1866 ^a
$\sum_{l=0}^{\infty}$	
Drake ^c	

ns and limits of the relativistic all-order Coulomb correlation evaluated relative to $E_0 + E_1 = -2.750 114 97$ a.u. using hydro-; the differences to nonrelativistic (nonrel) results from Ref. 8 .u.).

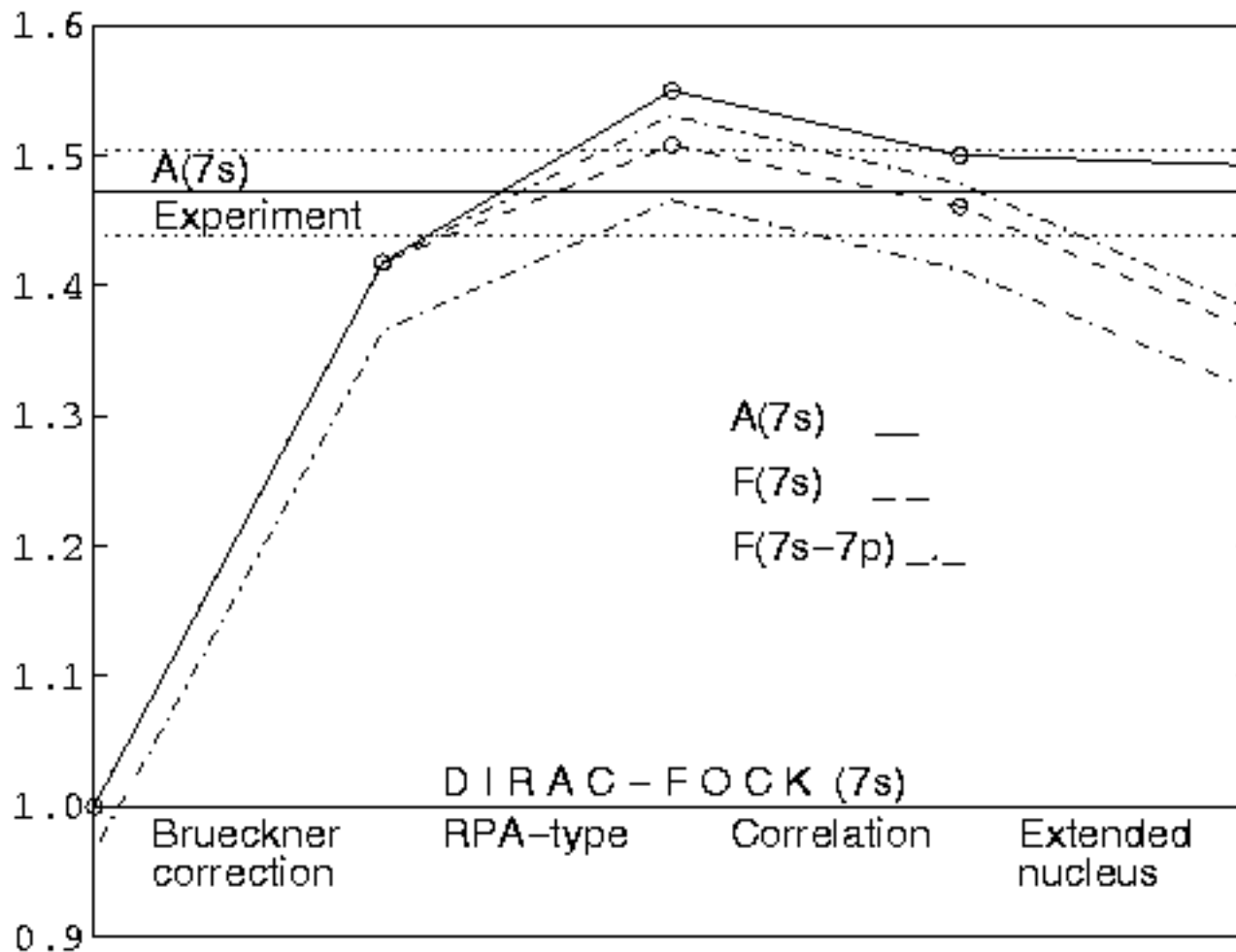
E_l^{rel}	$\Delta E_l^{\text{rel}} - \Delta E_l^{\text{nonrel}}$	$E_l^{\text{rel}} - E_l^{\text{nonrel}}$
-129 049 58	-2080.9	-2080.9
-150 535 37	168.4	-1912.4
-152 785 30	68.5	-1844.0
-153 339 20	33.2	-1810.8
-153 536 52	19.3	-1791.4
-153 632 50	12.8	-1778.6
-153 667 59	8.9	-1769.7
-153 692 25	6.5	-1763.3
-153 707 10	5.1	-1758.1
-153 716 56	3.8	-1754.4
-153 722 88	3.4	-1751.0
	36 ^b	
-153 741 54		-1715
-153 741 52		-1714

^aAdding the relativistic corrections from Eq. (29) to the nonrelativistic tail from Ref. 8.

^bFrom Eq. (29).

^cDeduced from Drake, Ref. 13 (see Table I).

Many electrons: Example Fr, HFS (A), IS (F)



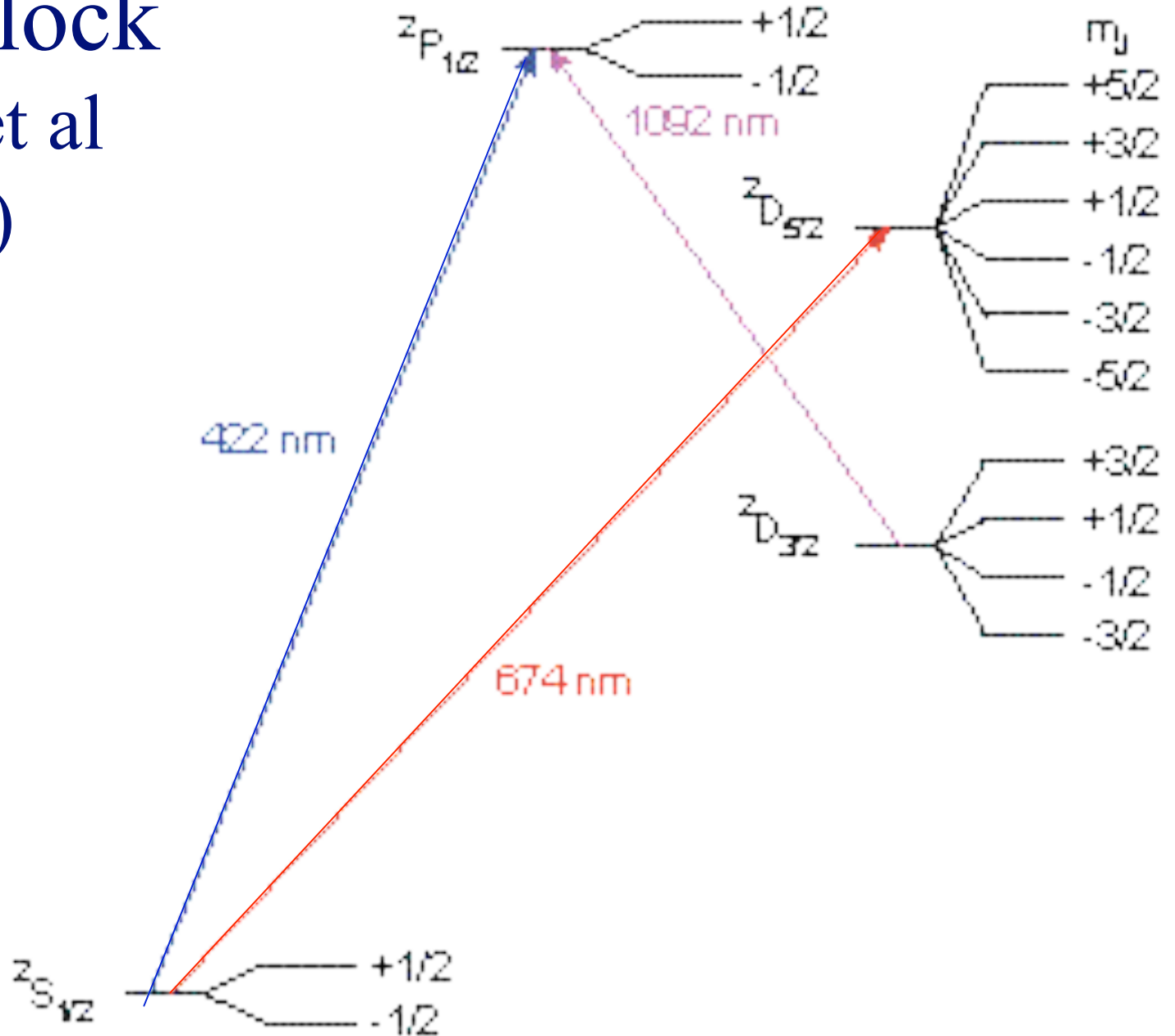
RMBPT –
Coupled
Cluster

Single and
double
excitations
(CCSD)

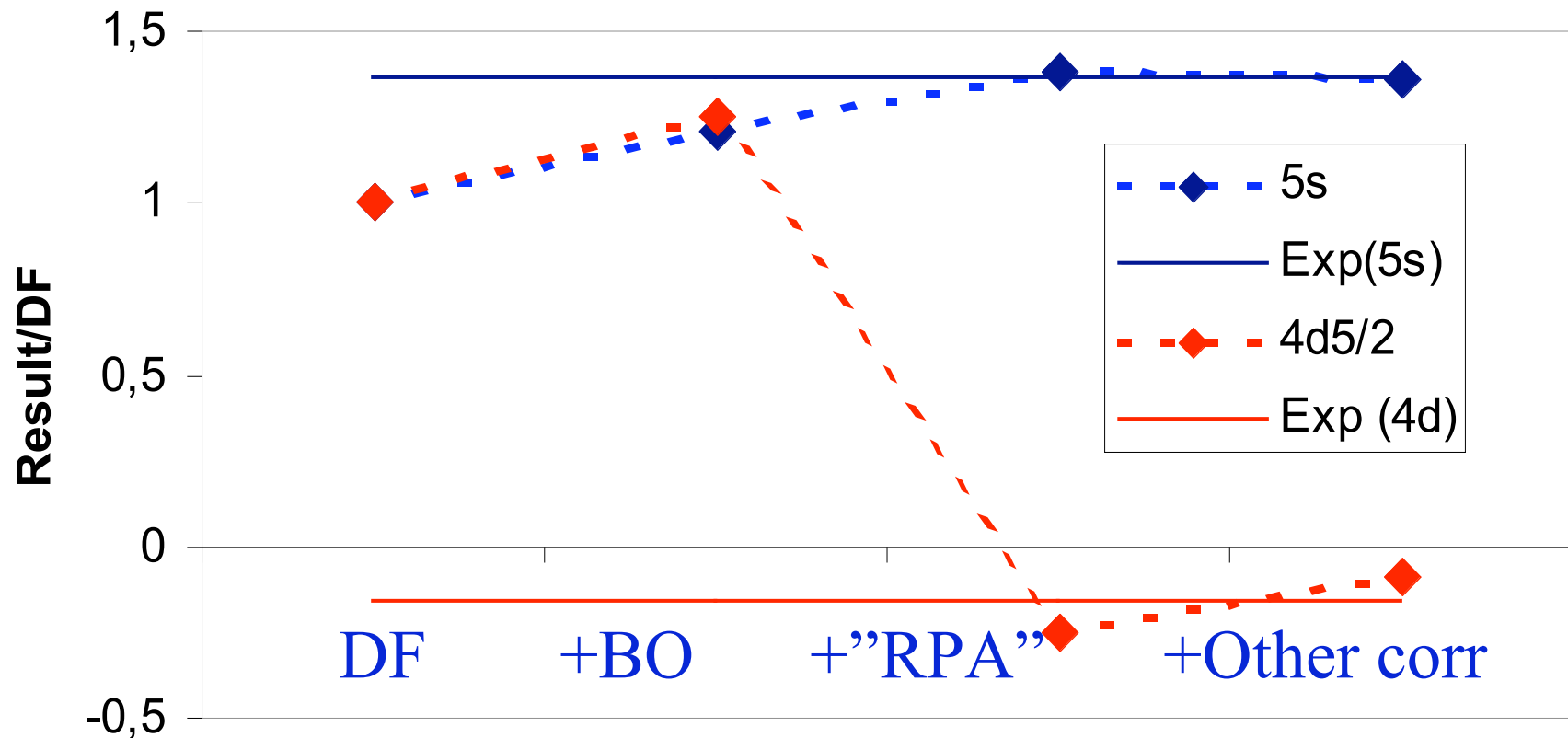
(AMMP 1999)

Sr ion clock (Klein et al NPL)

Hfs of
 $4d_{5/2}$?



Sr hyperfine structure



Theory:

1 MHz (AMMP, 2002)

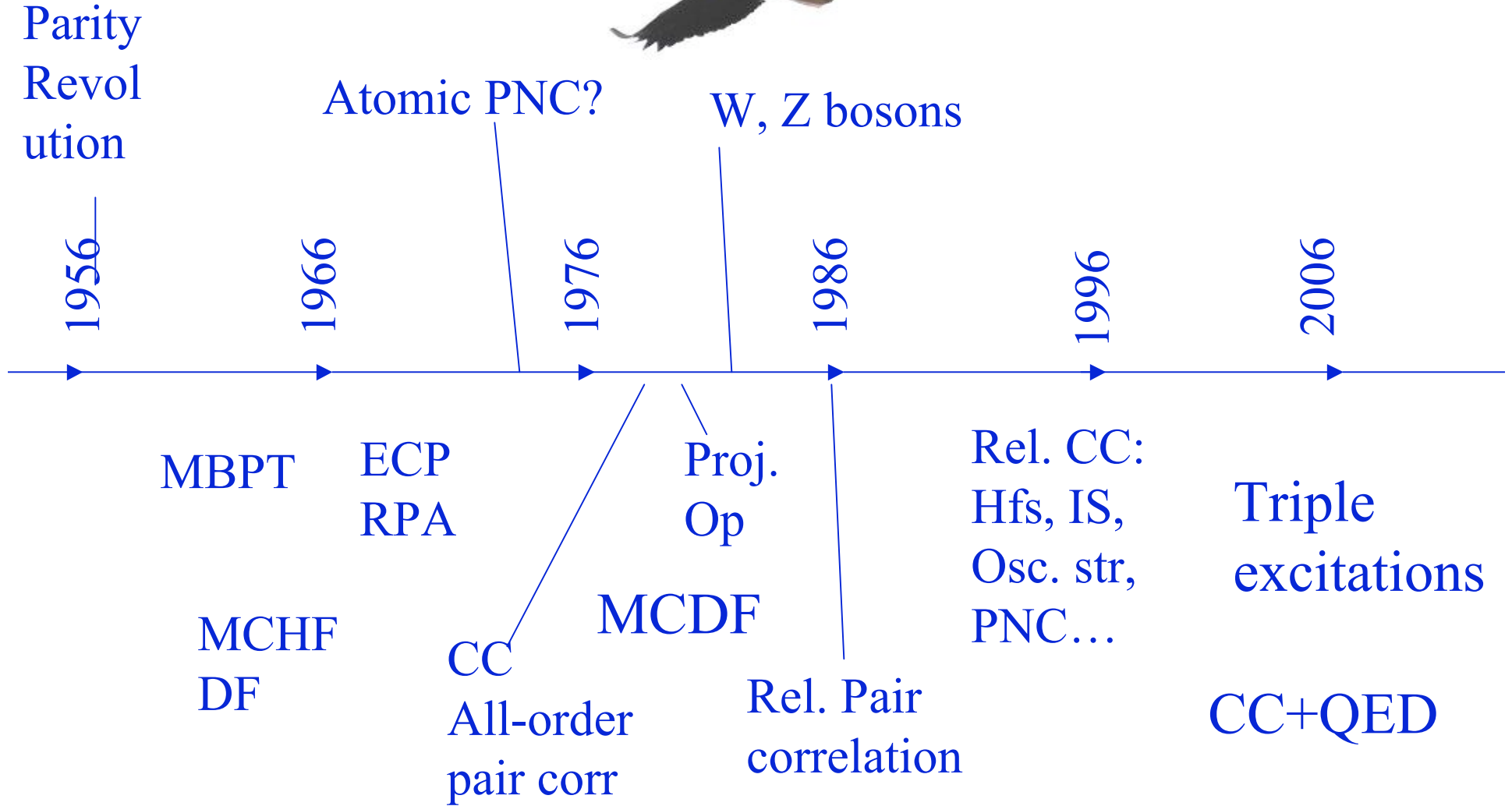
2.507 MHz (Yu et al, 2004)

-2.77 MHz (Itano, 2006)

$A(4d_{5/2})$

Exp: 2.1743(14) MHz
(Barwood et al 2003)

Timeline



Combination of CC and QED

Diagrammatic equation showing the expansion of a fermion-fermion interaction with a photon loop. The left side shows a thick horizontal line between two vertical lines labeled r and a on the left, and s and b on the right. A wavy line (photon) forms a loop between the thick line and the vertical lines. This is equal to a sum of four diagrams: 1) the thick line is straight, the wavy line is straight; 2) the thick line is straight, the wavy line is wavy; 3) the thick line is dashed, the wavy line is straight; 4) the thick line is dashed, the wavy line is wavy. The sum continues with an ellipsis and the word "folded".

Diagrammatic equation showing the expansion of a fermion-fermion interaction with a ghost loop. The left side shows a thick horizontal line between two vertical lines labeled r and a on the left, and s and b on the right. A wavy line (ghost) forms a loop between the thick line and the vertical lines. This is equal to a sum of four diagrams: 1) the thick line is straight, the wavy line is straight; 2) the thick line is dashed, the wavy line is straight; 3) the thick line is straight, the wavy line is wavy; 4) the thick line is dashed, the wavy line is wavy. The sum continues with an ellipsis and the word "folded".

Lindgren, Salomonson, Hedendahl 2006



PNC-interaction

$$H^{\text{PNC}} = G Q_W \gamma_5 \rho_n(r) / \sqrt{8} + \dots$$

Proportional to weak charge:

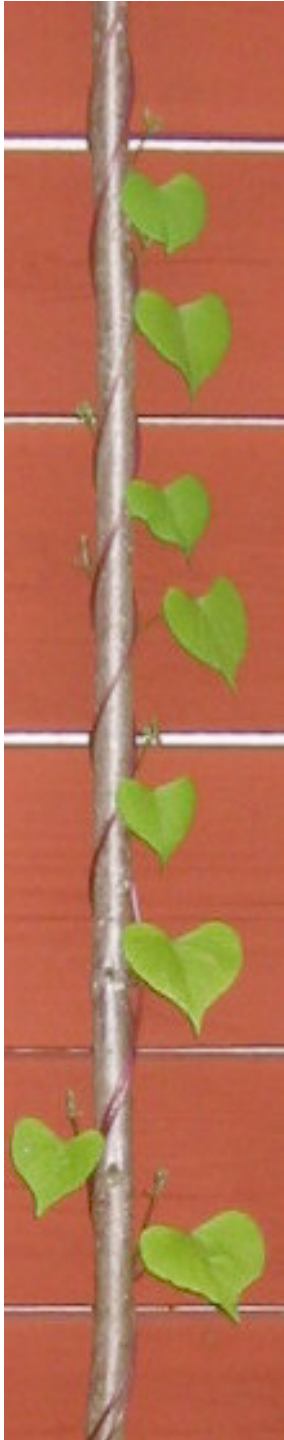
$$Q_W \cong (1 - 4 \sin^2 \theta_W) Z - N$$

$$\sin^2 \theta_W \cong 0.23$$

Study PNC in isotope chain (e.g. Fr)—
eliminates need for atomic theory?

But $\rho_n(r)$ may vary

Neutron distribution ?



Parity Non-Conservation in Cs

$$Q_W(^{133}\text{Cs}) = -72.61(28)_{\text{exp}} (73)_{\text{exp}}$$

Exp: Wieman et al (1999) + ...

Derevianko (2000, 2001) +
Blundell et al (1988, 1990, 1992),
Flambaum et al (1989, ...2000)

Including Breit interaction (0.9%) +
Estimate of correction for difference
neutron- proton distribution (-0.2%)

Standard Model

$$Q_W(^{133}\text{Cs}) = -73.20(13)$$



Neutron EDM ?

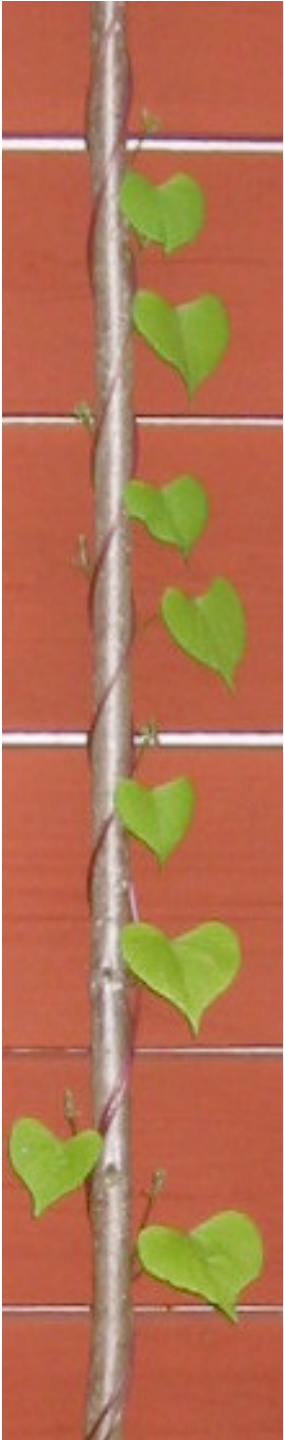
The argument against an electric dipole moment of a nucleus or elementary particle raises directly the question of parity.

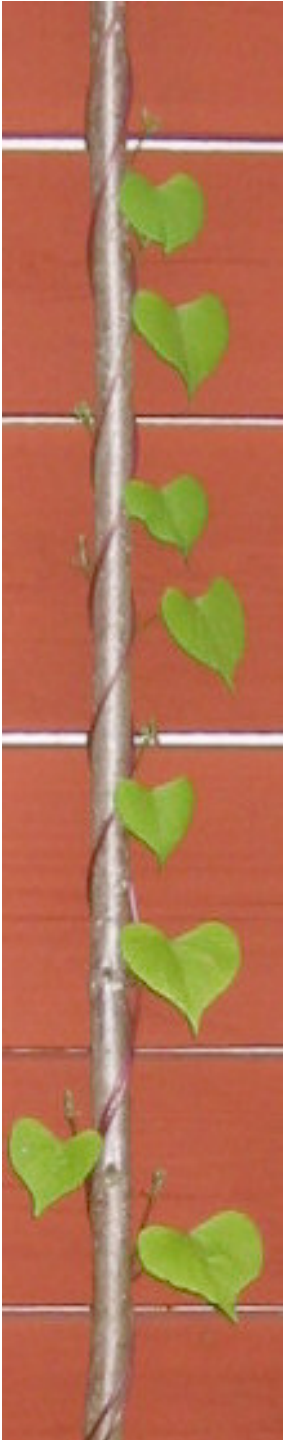
But there is no compelling reason for excluding this possibility.

The validity of the parity assumption **must rest on experimental evidence.** ...

The experimental evidence is not as conclusive as generally supposed.

(Purcell and Ramsey 1950)



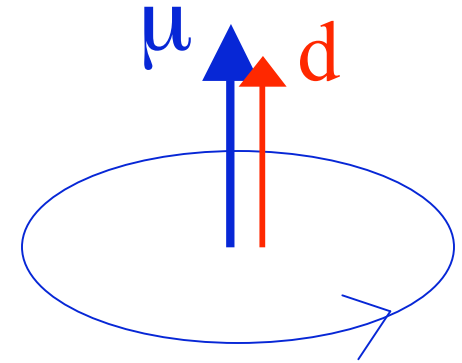


A nucleon with an electric dipole moment would show an asymmetry between left- and right-handed systems; in one system, the electric dipole would be parallel to the angular momentum, in the other, anti-parallel.

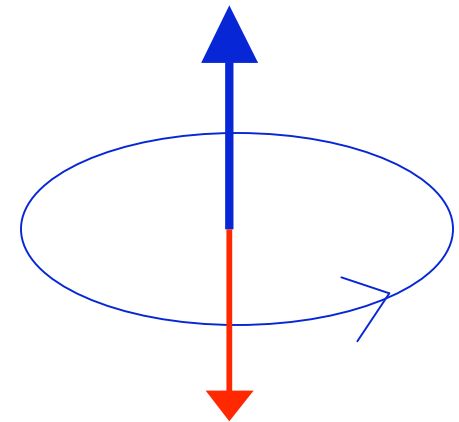
Purcell and Ramsey (1950)

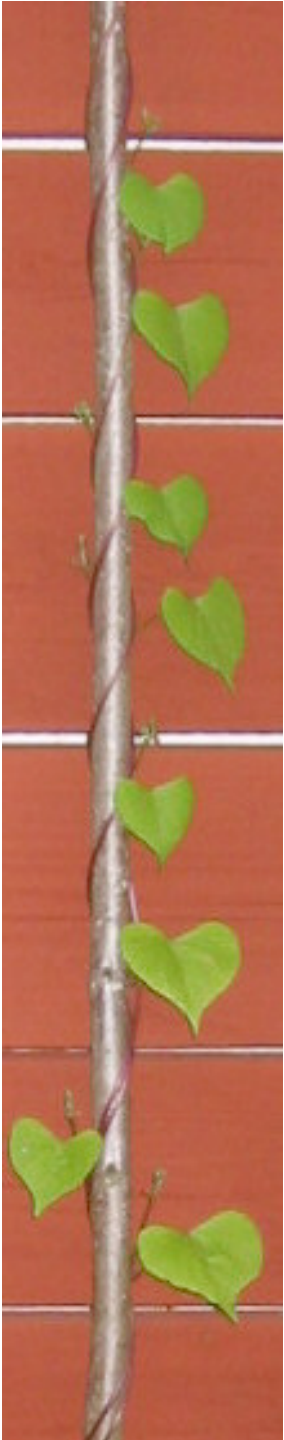
$$|D| < 3 \times 10^{-18} \text{ cm}$$

EDM ?



(P)





Neutron
EDM
Baker et al,
PRL 97,
131801
(2006)

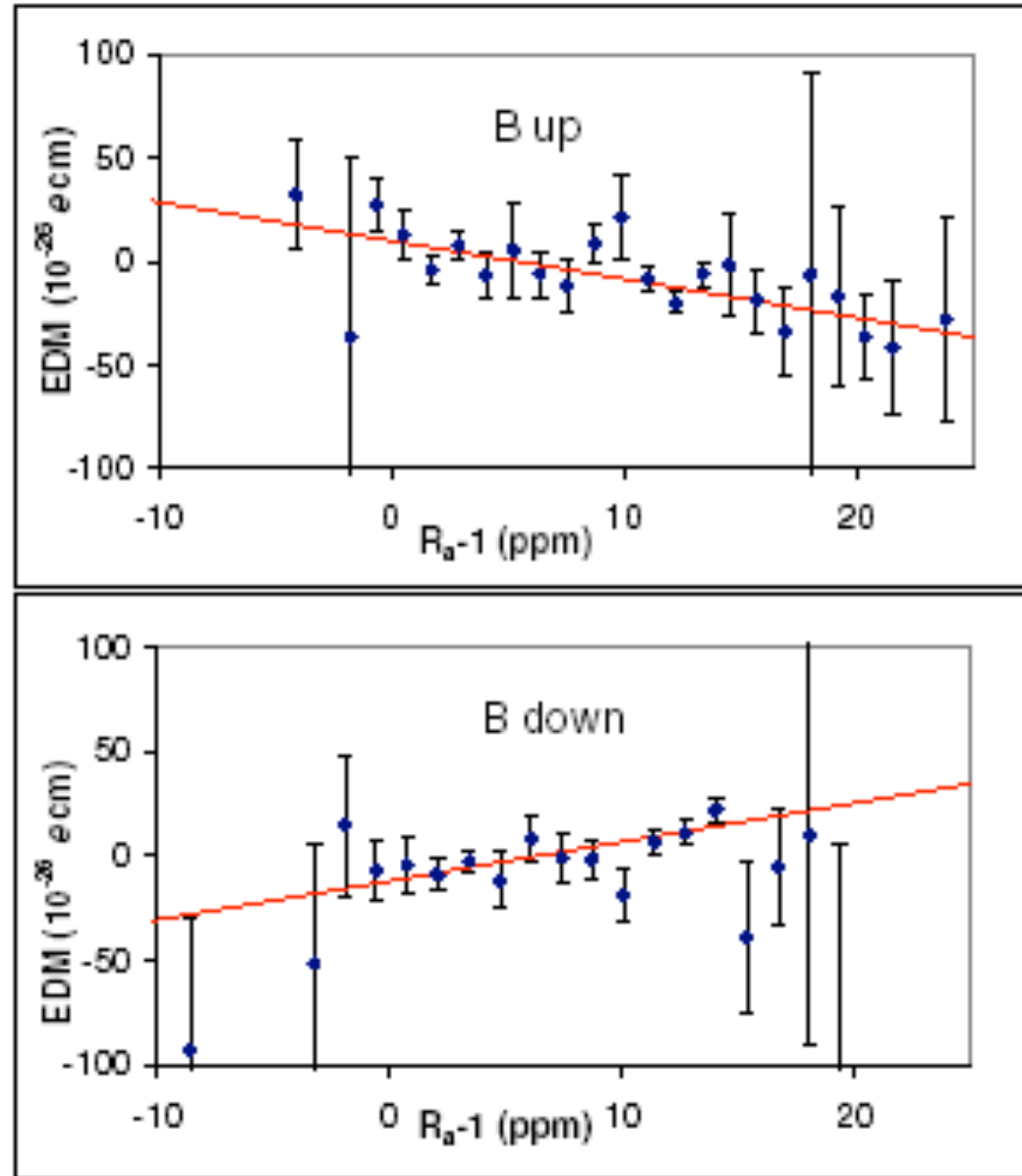
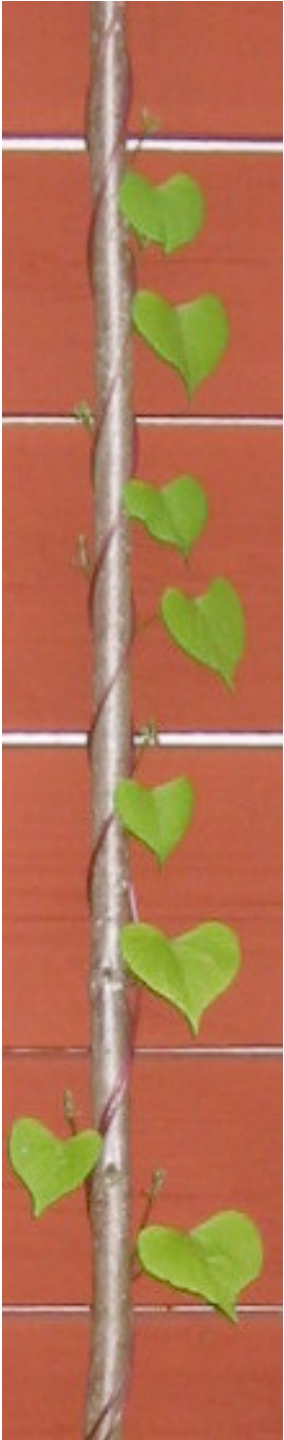


FIG. 2 (color online). Measured EDM (binned data) as function of the relative frequency shift of neutrons and Hg.



Neutron EDM limits

$$|D| < 3 \times 10^{-18} \text{ cm (1950)}$$

Purcell and Ramsey

$$|D| < 5 \times 10^{-20} \text{ cm (1957)}$$

$$|D| < 3 \times 10^{-24} \text{ cm (1977)}$$

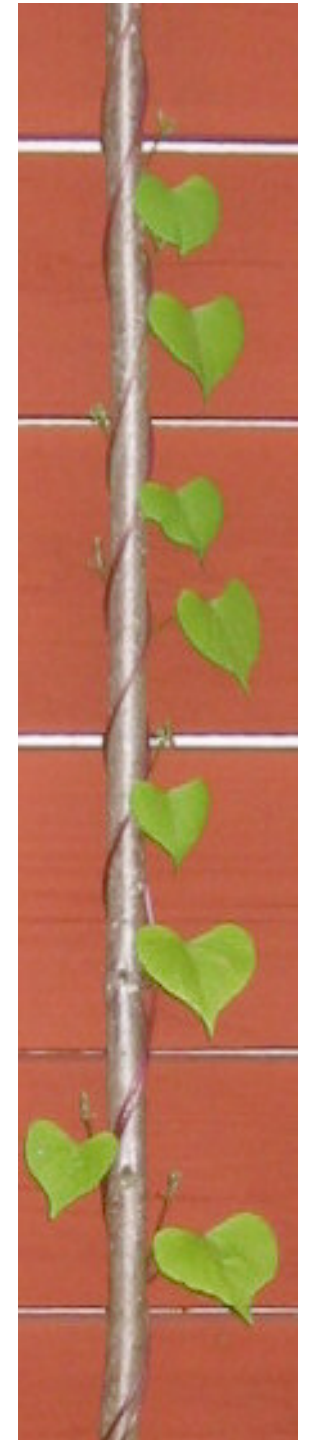
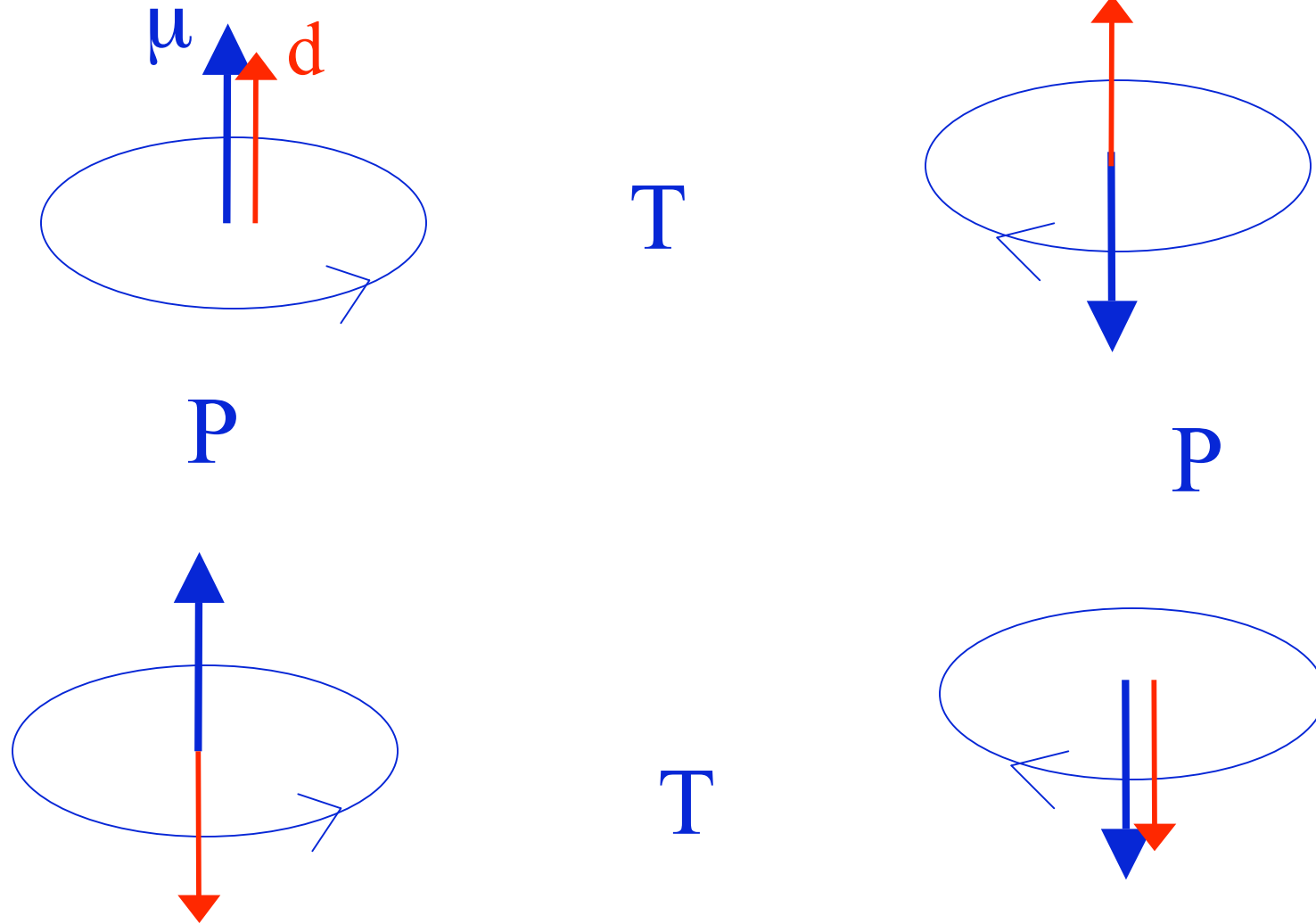
$$|D| < 6 \times 10^{-25} \text{ cm (1982)}$$

...

$$|D| < 2.9 \times 10^{-26} \text{ cm (2006)}$$

Why no
EDM?

EDM – Violates P and T



1987,
Stockholm

Will a neutron
EDM be
discovered?



K-mesons ...

Unless nature is malevolent against Norman Ramsey
... not necessarily in my life time



Ljungskile, Sweden, June 2006

TABLE I. Summary of limits (95% C.L.) set by the ^{199}Hg EDM and other experiments on model-independent and “naturalness” parameters.

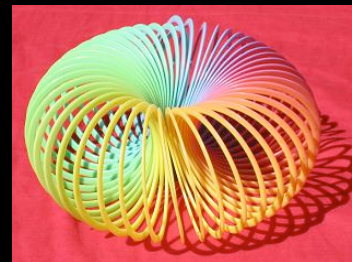
Parameter	Limit from ^{199}Hg	Best other limit		Ref.
$\bar{\theta}_{\text{QCD}}$	1.5×10^{-10}	6×10^{-10}	n [3]	[6,11]
\tilde{d}_d (cm)	7×10^{-27}	1.1×10^{-25}	n [3]	[6,12]
C_T	1×10^{-8}	5×10^{-7}	T1F [13]	[14]
C_S	3×10^{-7}	4×10^{-7}	T1 [4]	[14]
$\mathcal{E}_q^{\text{SUSY}}$	2×10^{-3}	1×10^{-2}	n [3]	[1]
$\mathcal{E}^{\text{Higgs}}$	$0.4/\tan\beta$	$0.7/\tan\beta$	T1 [4]	[1]
x^{LR}	1×10^{-3}	1×10^{-2}	n [3]	[1]

Romalis et al 2001

Parity Non-Conservation and Nuclear Structure ?



- EDM: ($\langle r_d^2 \rangle - \langle r_c^2 \rangle$)
(Schiff moments) +
- Isotope Chains PNC– neutron distribution
- Anapole moments ...



What is known about nuclear distributions?

Nuclear Charge Distributions

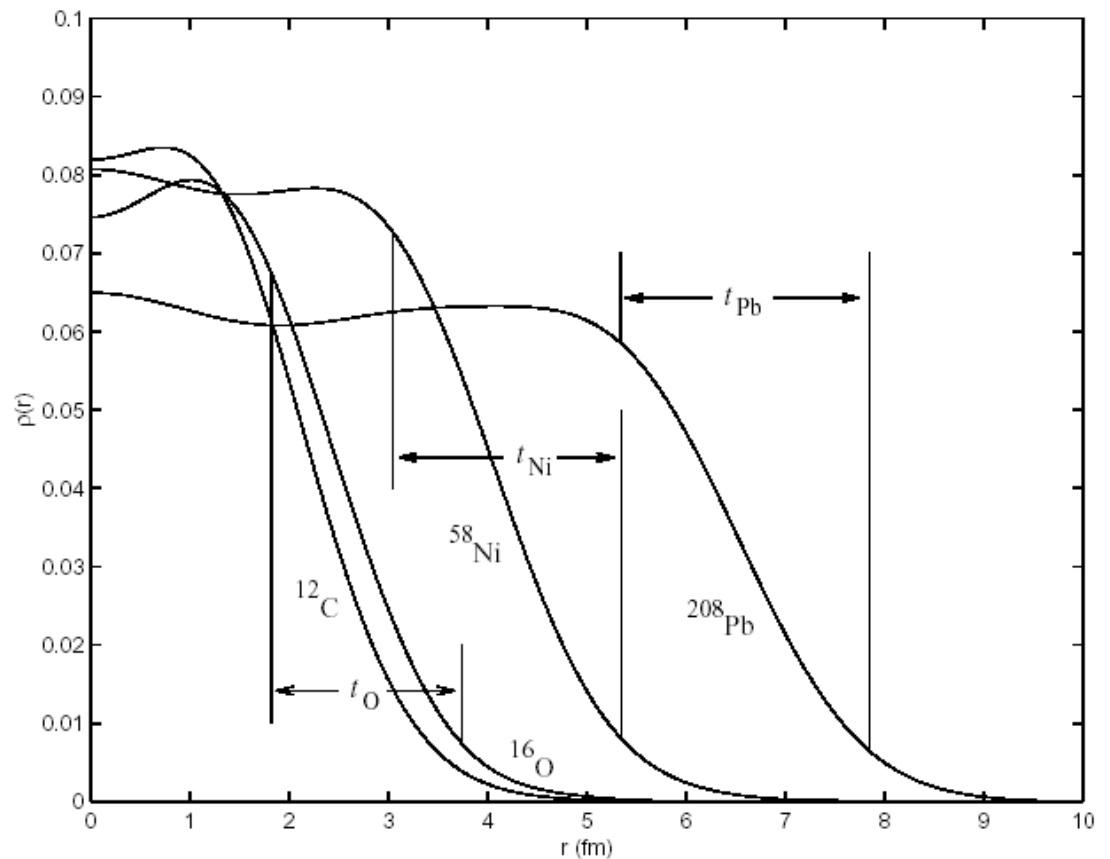
First approximation:

Homogeneous
distribution:

$$R = R_0 A^{1/3}$$

$$R_0 = 1.2 \text{ fm}$$

$$\langle r^2 \rangle = 3 R^2 / 5$$



Fermi Distribution

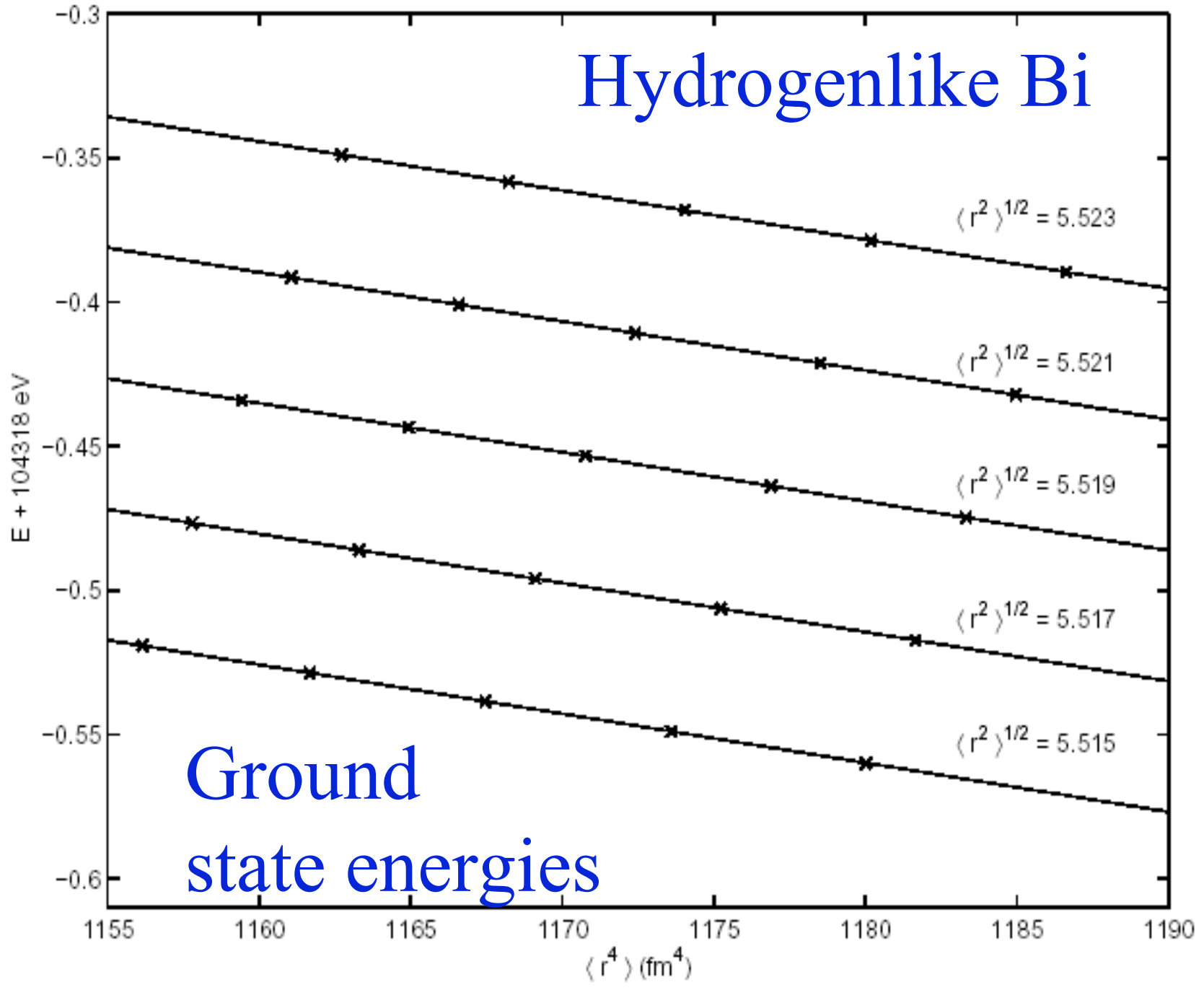
$$\rho(r) = \frac{\rho_0}{1 + e^{(r-c)/a}}$$

$$\begin{aligned} \langle r^2 \rangle &\approx \frac{3}{5}c^2 + \frac{7}{5}\pi^2 a^2, \\ \langle r^4 \rangle &\approx \frac{3}{7}c^4 + \frac{18}{7}\pi^2 a^2 c^2 + \frac{31}{7}\pi^4 a^4, \\ \langle r^6 \rangle &\approx \frac{3}{9}c^6 + \frac{11}{3}\pi^2 a^2 c^4 + \frac{239}{15}\pi^4 a^4 c^2 + \frac{127}{5}\pi^6 a^6 \end{aligned}$$

Homogeneous
nuclear distribution, radius c

”Thickness
parameter”, a

Hydrogenlike Bi



Ground
state energies

Francium Hyperfine anomalies (Grossmann et al 1999 Sprouse 2006)

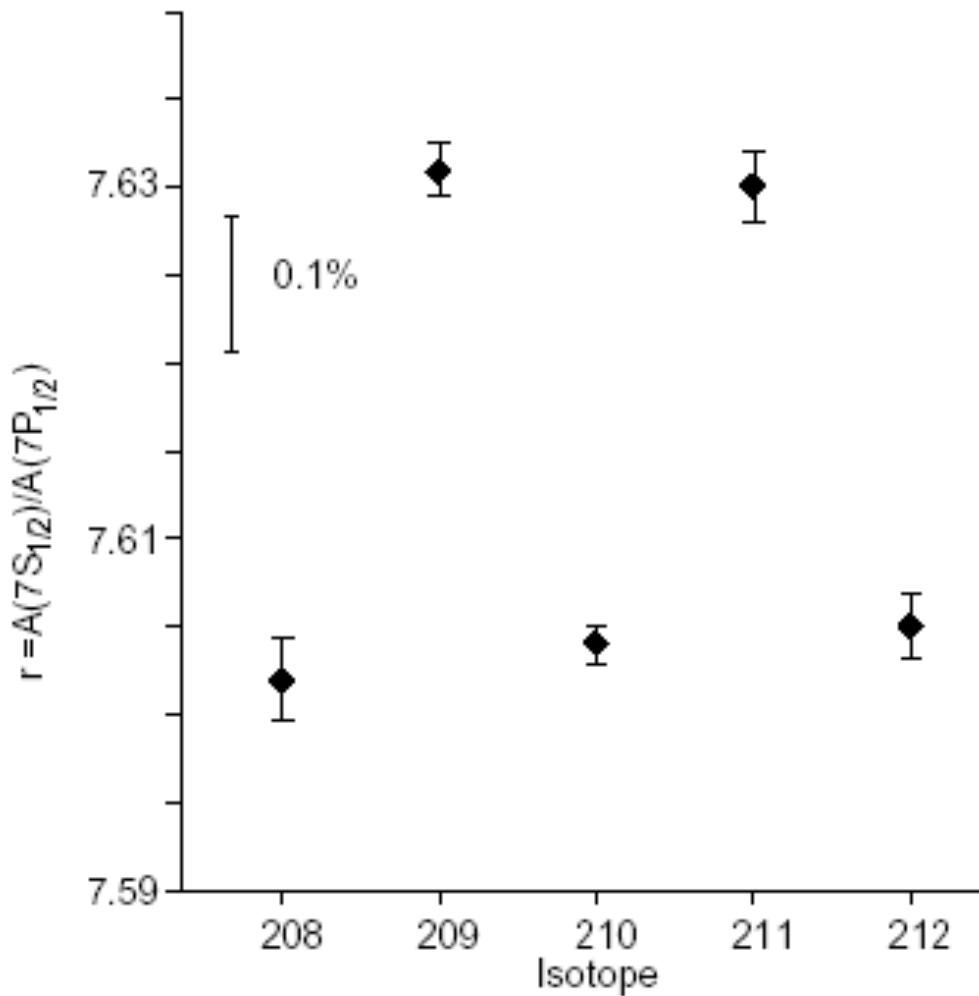


Figure 13. Ratio of hyperfine A magnetic dipole constants of $7S_{1/2}$ and $7P_{1/2}$ states observed for five different Fr isotopes. The s states come from the work of Coc *et al* [39] and the p states come from [28].

Ratio
 $A(7s)/A(7p_{1/2})$

Odd-even
staggering

Tl hyperfine anomalies

(A-M M-P 1995)

$$\Delta (6p_{1/2}) = -1.04 \cdot 10^{-4} \quad (\text{Lurio Prodell 1956})$$

$$\Delta (6p_{3/2}) = 16.26 \cdot 10^{-4} \quad (\text{Gould 1956})$$

$$\Delta (7s_{1/2}) = -3.4(18) \cdot 10^{-4} \quad (\text{Hermann et al 1993})$$

Single particle:

$$\Delta(ns_{1/2}) / \Delta(np_{1/2}) = 3.5$$

$$\Delta(np_{3/2}) = 0$$

Many-body effects.

CCSD:

consistent with
experimental data

”Breit Rosenthal” effect on HFS - Nuclear Charge Distribution

Correction $(1 - \varepsilon_{\text{BR}})$ due to charge distribution:

$$\varepsilon_{\text{BR}} = b_2 \langle r_c^2 \rangle + b_4 \langle r_c^4 \rangle + b_6 \langle r_c^6 \rangle$$

Parameters depend on l and j , but very weakly on n

But Breit-Rosenthal effect is insufficient ...

Bohr-Weisskopf effect: Extract $\delta \langle r_m^2 \rangle$

Bohr-Weisskopf effect:

Correction to hfs interaction for distributed nuclear magnetic moment

$$h_{\text{BW}}^{\text{hfs}} = \int \left\{ \eta(R - r) \left[g_s \left(\frac{\mathbf{S}_n}{r^2} - \sqrt{10} (\mathbf{S}_n \mathbf{C}_n^2)^1 \frac{r}{R^3} \right) + g_L \mathbf{L}_n \left(\frac{1}{r^2} - \frac{r}{R^3} \right) \right] \times \boldsymbol{\alpha} \right\} \rho_m(R) R^2 dR$$

$$\epsilon_{\text{BW}} = a_2 \langle r_m^2 \rangle + a_4 \langle r_m^4 \rangle + a_6 \langle r_m^6 \rangle$$

Tl result (1995)

$$^{203:205}\delta \langle r_c^2 \rangle = 0.115(3) \text{ fm}^2$$

(From muonic data, Engfer et al At Data Nucl. Data Tables 1974)

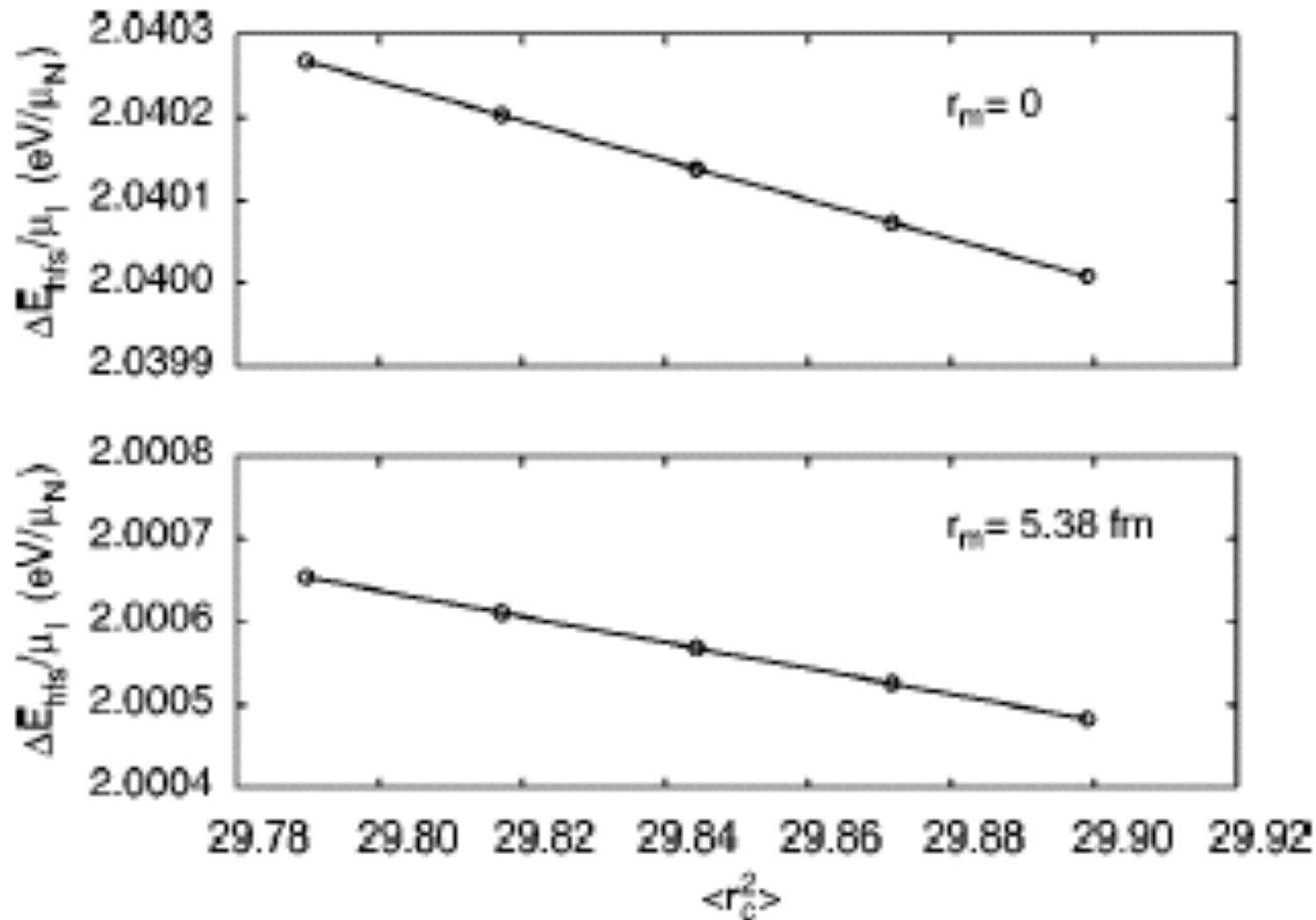
gives

$$^{203:205}\delta \langle r_m^2 \rangle = 0.26(2) \text{ fm}^2$$

About equal contributions to
the hyperfine anomaly in Tl

Evidence for different distributions

Breit-Rosenthal and Bohr-Weisskopf effects are not independent



Tl

Hydrogen-like Tl

(Beiersdorfer et al 2001)

^{203}Tl :

$$\varepsilon_{\text{BW}} = 2.212\% \quad \langle r_{\text{m}}^{1/2} \rangle = 5.83(14)\text{fm}$$

^{205}Tl :

$$\varepsilon_{\text{BW}} = 2.248\% \quad \langle r_{\text{m}}^{1/2} \rangle = 5.89(14)\text{fm}$$

Larger than charge radii –

Challenging nuclear theory to reproduce

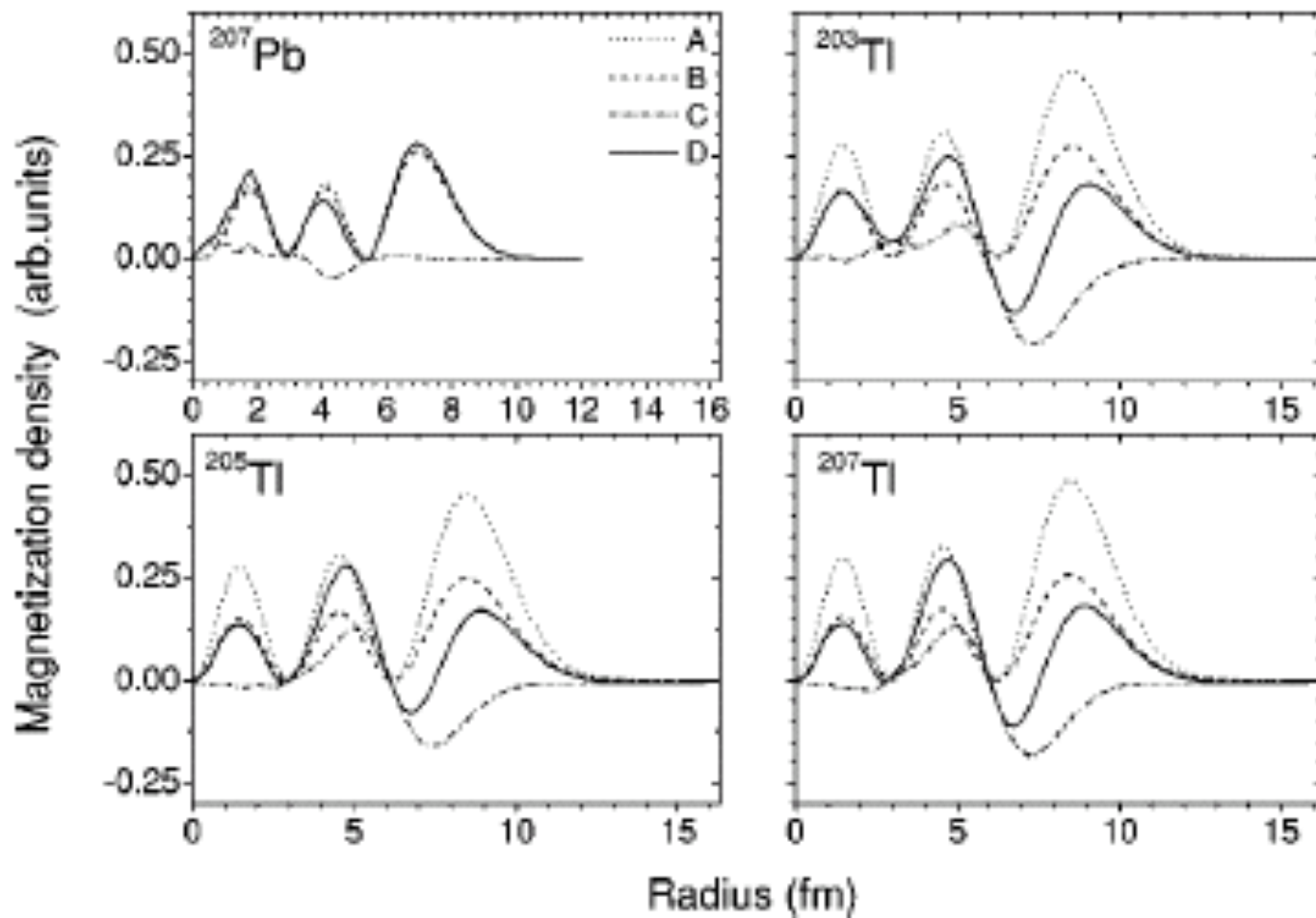


FIG. 3. Calculated magnetization distribution for ^{207}Pb and $^{203,205,207}\text{Tl}$. Line *A* shows the single-particle result, line *B* has been normalized with the spectroscopic factor of the single particle, line *C* shows the contributions of diagonal and off-diagonal elements in DCM, and line *D* is the total magnetization.

Tomaselli
et al, 2002

Experiment =

$$\text{QED} + \frac{\text{atom}}{\text{nucleus}} * \text{nucleus} + \text{new physics?}$$

Nuclear Challenge!

Atomic Physics at the Interface to Nuclear and Particle Physics



(INT, 1999)

Many-body methods also connect
condensed matter theorists and
quantum chemists (+TlF, YbF, PbO ...)

Many-body Physics

