LIGHT CURVE SOLUTIONS FOR ECLIPSING BINARIES IN NGC 188

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Abstract. We present light curve solutions for the W UMa-type eclipsing binaries EP, EQ, ER, ES, and V369 Cep in the old open cluster NGC 188. Using light curve solution parameters combined with reasonable mass estimates, we determine the distance modulus $V - M_V$ of the cluster. Our aim is to examine if current uncertainties in the cluster's distance and age can be resolved. Three binaries yield distance moduli close to $10\overset{m}{.}80$ ($\pm 0\overset{m}{.}08$), two others give values around $11\overset{m}{.}40$($\pm 0\overset{m}{.}09$). Depending on the amount of reddening, we find a weighted mean distance modulus for all five binaries between $11\overset{m}{.}01$ and $11\overset{m}{.}05$ ($\pm 0\overset{m}{.}06$), which lends modest support for the lower distance (1.65 kpc) and older age (10 Gyr) of the cluster.

Key words: open clusters, eclipsing binaries

1. Introduction

The galactic open cluster NGC 188 contains a large number of W UMa-type eclipsing binaries. Four of these were discovered by Hoffmeister (1964) and several more by Kaluzny and Shara (1987) as a result of a CCD survey in the B-passband. Light curves with relatively sparse phase coverage were obtained for EP, EQ, ER, ES, V369, V370, and V371 Cep. Kaluzny (1990) reobserved the cluster and obtained complete light curves in V for the same systems, with the exception of EP Cep. Apart from Edalati's (1994) solution of Kaluzny and Shara's (1987) B light curve of EP Cep, no other light curve solutions for binaries in NGC 188 have been published.

NGC 188 is generally considered to be an old cluster with an age of 10 Gyr, a color excess E_{B-V} of $0\overset{m}{.}08$, and a distance modulus $V - M_V = 11\overset{m}{.}1$ (VandenBerg, 1985). Recently however, Twarog and Anthony-Twarog (1989) argued that the cluster is considerably younger with an age of 6.5 ± 0.5 Gyr, a color excess $E_{B-V} = 0\overset{m}{.}12$ and a distance modulus $V - M_V = 11\overset{m}{.}5$. Following the ideas of Van Hamme and Wilson (1985), in this paper we explore an alternative method to help resolve these differing distance estimates. This method is based on the

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following reasoning. When the distance to certain types of eclipsing binaries is known, light curve solution and other pure photometric parameters can be combined to determine absolute masses. Basically, the idea is that if we know the distance, we know the luminosity, and any suitable temperature indicator (for example $B - V$) can then be used to calculate a star's absolute radius. In the Roche model for binary systems, the size of a lobe-filling component is related to the orbital semi-major axis, which gives the mass using Kepler's third law. Also needed is the mass ratio, which normally is obtained from radial velocities. However, in the case of semi-detached or over-contact systems, the mass ratio can be obtained from a light curve solution, and the above procedure can be applied *even if radial velocities are not available*. We can also turn the procedure around and determine the distance to the binary provided a reasonable estimate for the mass of at least one component can be made. Applying this procedure to the W UMa binaries in NGC 188, we first solve their light curves which yields relative radii and mass ratios. Then, making the reasonable assumption that contact or near-contact W UMa-type binaries have relatively un-evolved primary components, we estimate their masses using the observed $B - V$ color index and a suitable color-mass main sequence relationship. In principle, the distance to the system and hence to the cluster can then be determined.

Light curve solutions for 5 of the NGC 188 binaries and a description of the solution process are presented in Section 2. An outline of our method to determine the distance modulus of NGC 188 and a brief discussion of the results can be found in Section 3.

2. Light Curve Solutions

Light curve solutions for 5 W UMa-type binaries in NGC 188 (EP, EQ, ER, ES Cep, and V369 Cep) were made using the Wilson-Devinney program (Wilson and Devinney, 1971; Wilson, 1979). For EP Cep, only a sparsely phase-covered B-light curve was available. For the others, simultaneous B and V light curve solutions were made. Curve-dependent weights were selected based on standard deviations of least squares fits of Fourier series (up to 7θ sine and cosine terms) to the individual observations. These standard deviations are listed in Table 1, in units of light at phase $0\overset{\mathrm{P}}{.}75$. The differential corrections input parameter NOISE, which specifies the scaling of the level-dependent weights, was set to 1. This value corresponds to noise due to photon counting statistics (see Wilson, 1979). For each light curve, normal points were formed and weights assigned equal to the number of averaged individual points. The bins were very narrow, and, therefore, normal points were based on a maximum of 2 to 3 individual observations. However, most of the normal points consisted of just 1 individual observation. Temperatures for the primary components were selected based on the observed $B-V$ color index. For the albedo's and gravity darkening coefficients, canonical values for

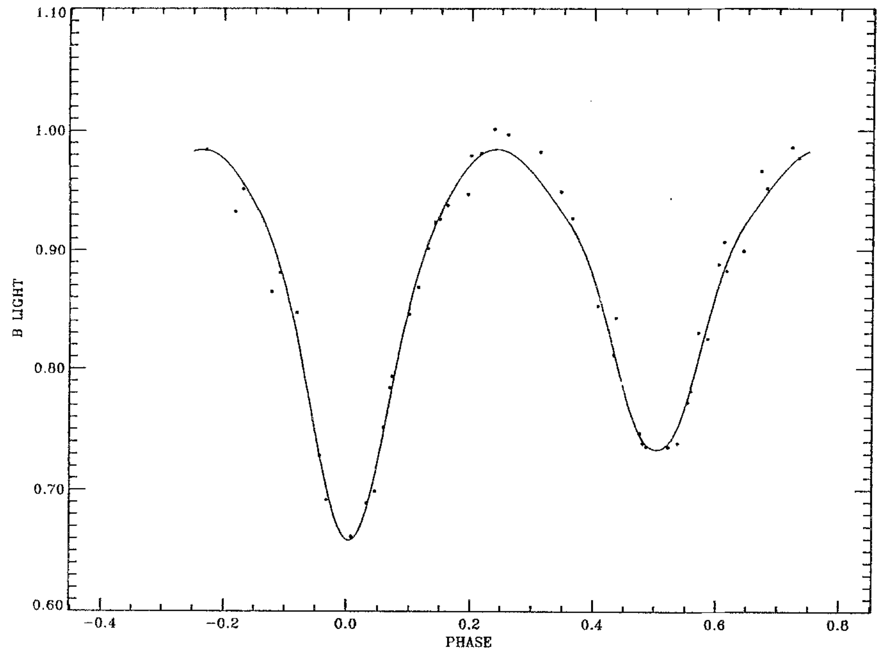

Fig. 1. Observed and solution B-light curve of EP Cep.

stars with convective outer layers were selected. A logarithmic limb-darkening law was used with coefficients obtained via bi-linear interpolation in the limb-darkening tables of Van Hamme (1993). For each binary, solutions were started using initial parameters corresponding to a detached configuration. Very quickly, the differential corrections program moved the parameters in the direction of a configuration in which either one star, or both, over-filled their respective limiting Roche lobes. Final solutions indicate that EQ, ER, and V369 Cep are over-contact systems with a relatively small fill-out parameter, f, of approximately 0.10. As is typical for late-G, early-K W UMa systems, a W-type vs. an A-type (Binnendijk's, 1970, terminology) solution yielded a better fit to the observations. For EP and ES Cep, a semi-detached solution with a lobe-filling less massive star gave the best fit. However, the primary, more massive, components fill 99% of their Roche lobe, so these systems are very near-contact, and marginal over-contact solutions would be nearly indistinguishable from those presented here. Also, since the EP Cep solution is based on just one sparsely covered light curve, we would like to consider any conclusions regarding EP Cep's configuration as preliminary. Edalati's (1994) also found a semi-detached solution for EP Cep, however with the more massive and larger star as the lobe-filling component. However, using Edalati's solution parameters, we were unable to fit the observations well.

Table 2 lists the solution parameters and standard deviations. Fixed parameters can be distinguished from adjusted parameters by their lack of mean errors. The observed and solution curves are shown in Figs. 1-9. Many light curves show

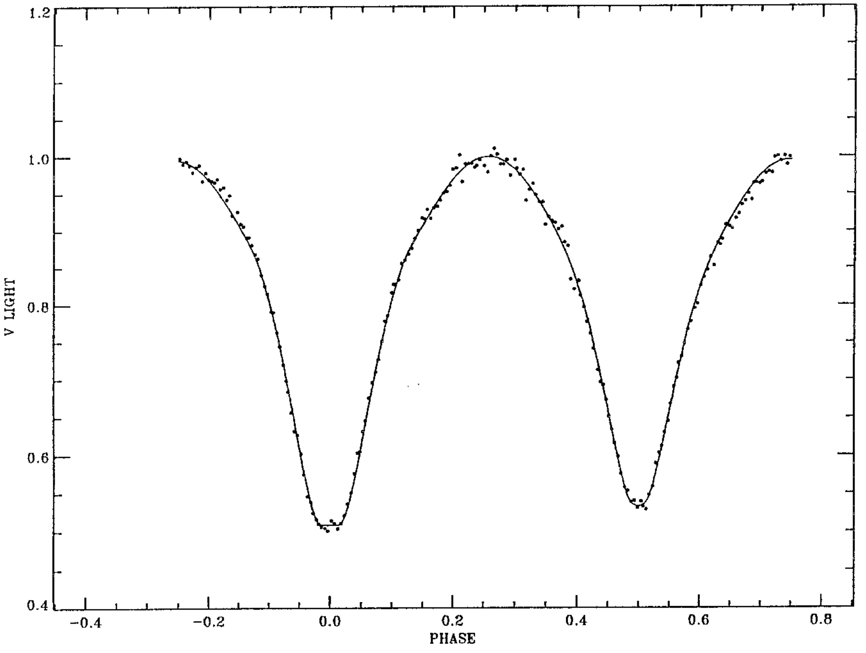

Fig. 2. Observed and solution V-light curve of EQ Cep.

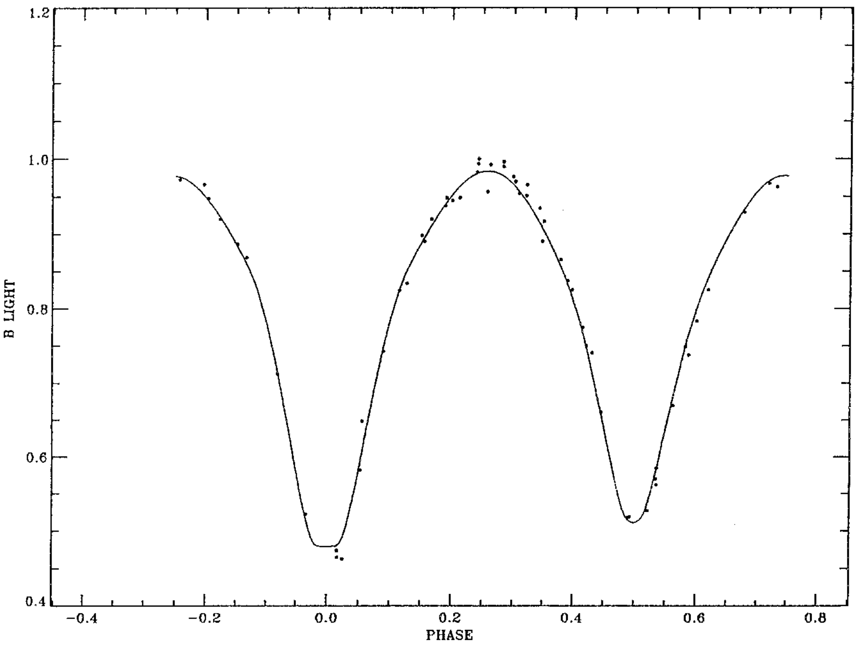

Fig. 3. Observed and solution B-light curve of EQ Cep.

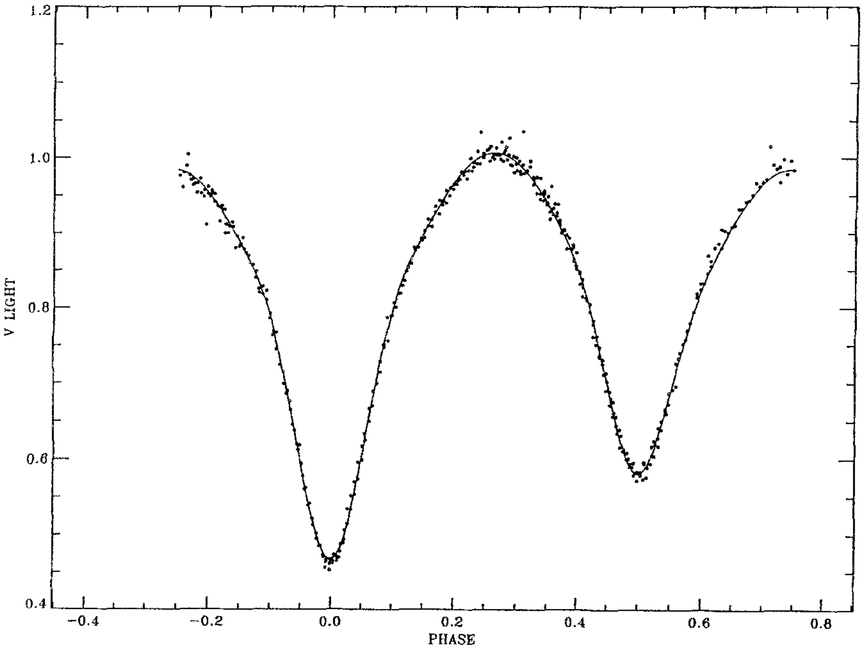

Fig. 4. Observed and solution V-light curve of ER Cep.

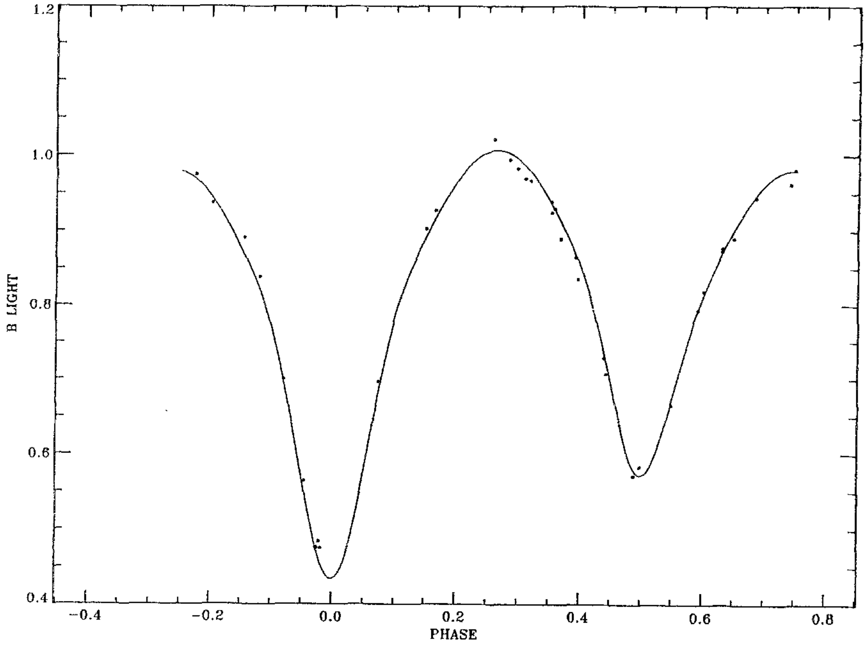

Fig. 5. Observed and solution B-light curve of ER Cep.

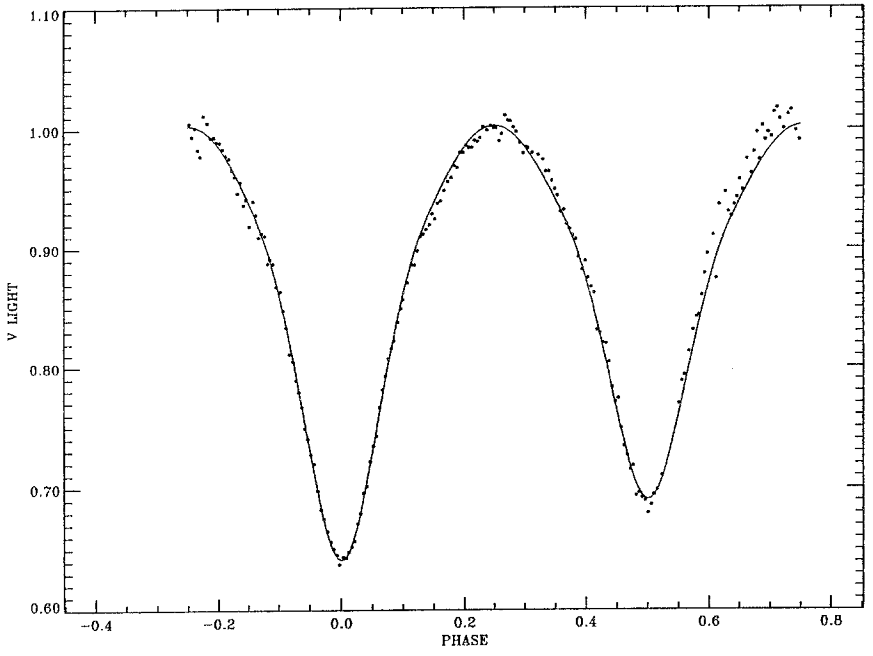

Fig. 6. Observed and solution V-light curve of ES Cep.

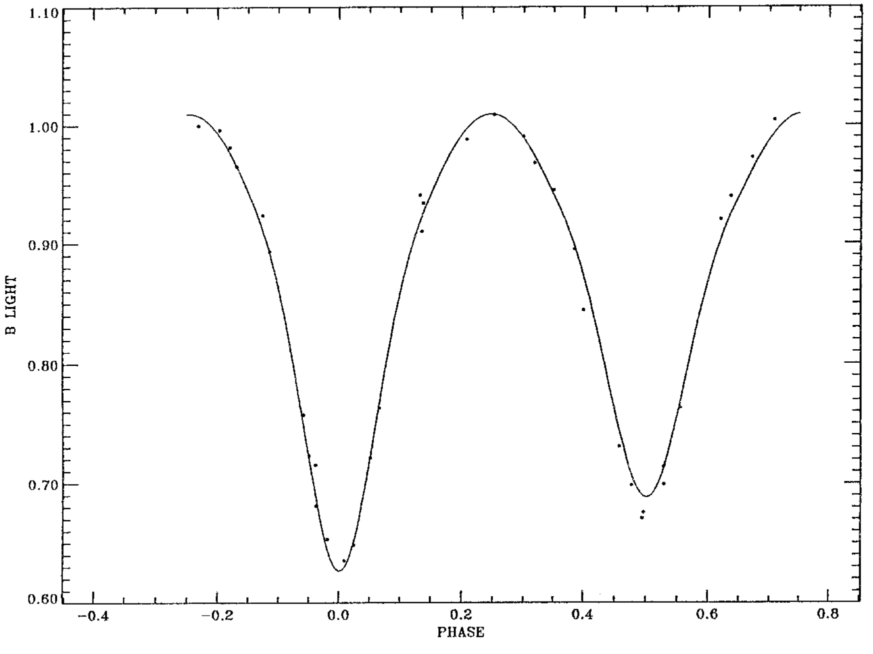

Fig. 7. Observed and solution B-light curve of ES Cep.

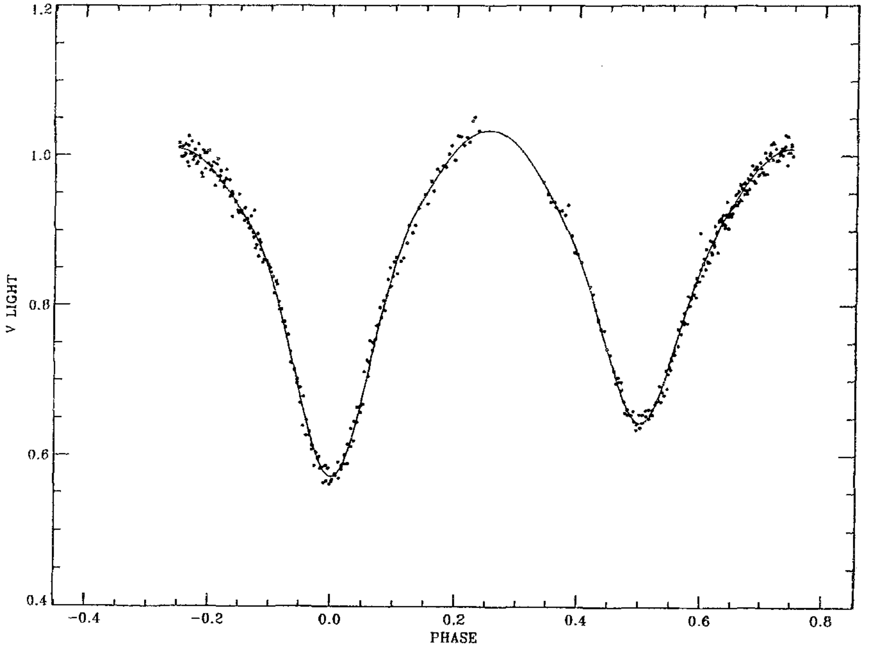

Fig. 8. Observed and solution V-light curve of V369 Cep.

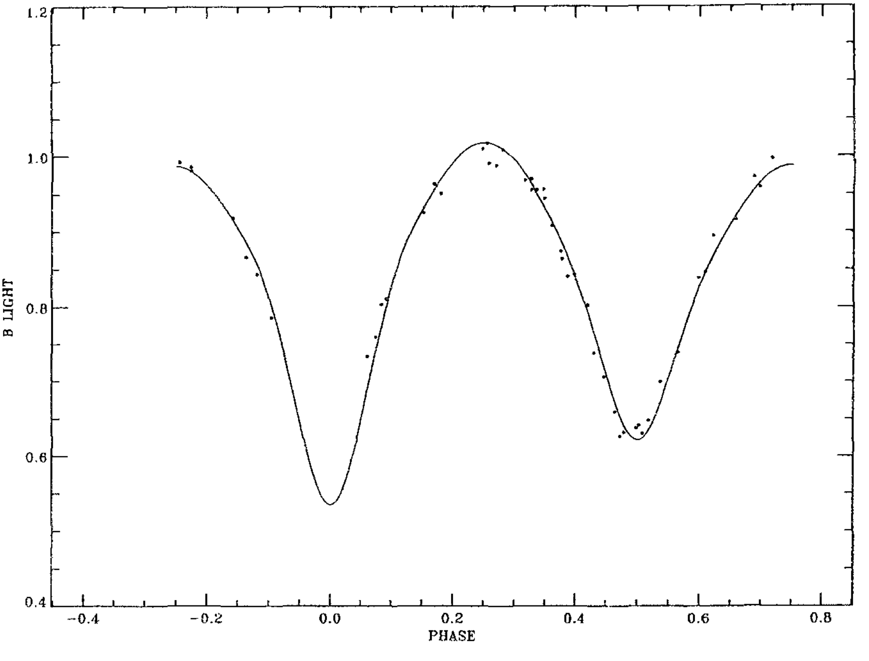

Fig. 9. Observed and solution B-light curve of V369 Cep.

asymmetries which were modeled using spots. Spot parameters are listed in Table 3. Various experiments with different spot types (hot or cool) and locations resulted in a preference for hot spots on the primary stars, visible around phase $0^{\mathrm{P}}.25$. Again, this is not an uncommon feature of late-type W-type W UMa binaries, as discussed in some detail by Samec et al. (1993). For ER and EQ Cep, the mass ratios obtained from the light curve solutions compare favorably with the spectroscopic mass ratios determined by Guinan (private communication) from just a few radial velocity measurements ($q_{\mathrm{spec}} = 0.56$ for ER Cep and $q_{\mathrm{spec}} = 0.45$ for EQ Cep).

TABLE I

Curve-dependent Standard Deviations.

System	σ_V	σ_B
EP Cep	...	0.0139
EQ Cep	0.0108	0.0147
ER Cep	0.0103	0.0100
ES Cep	0.0087	0.0096
V369 Cep	0.0109	0.0133

3. Distance Moduli

Following Van Hamme and Wilson (1985), the distance modulus $V - M_V$ of the primary component of a binary system can be written as

$$V - M_V = -39.19 + V + BC + 10 \log T + 5 \log r$$
$$+ \frac{5}{3} \log M_1 + \frac{10}{3} \log P + \frac{5}{3} \log(1 + q). \tag{1}$$

BC is the star's bolometric correction, T its effective temperature, r its relative radius (in units of the orbital semi-major axis), M_1 the mass of the primary in $M_{\odot}$, P the orbital period in days, and $q = M_2/M_1$ the mass ratio. The constant term is based on an absolute bolometric magnitude and effective temperature for the Sun of $4^{\mathrm{m}}.69$ and 5780 K, respectively. The variance of the distance modulus is obtained using the error propagation formula

TABLE II

NGC 188 Eclipsing Binary Light Curve Solutions.

Parameter	EP Cep	EQ Cep	ER Cep	ES Cep	V369 Cep
Period (day)	0.2897419	0.3069030	0.2857299	0.3424579	0.3281916
ϕ	$0^{P}.0031 \pm 0^{P}.0015$	$0^{P}.4994 \pm 0^{P}.0002$	$0^{P}.4994 \pm 0^{P}.0002$	$0^{P}.0008 \pm 0^{P}.0003$	$0^{P}.5007 \pm 0^{P}.0003$
i	$74^{\circ}.9 \pm 2^{\circ}.5$	$84^{\circ}.96 \pm 0^{\circ}.37$	$79^{\circ}.63 \pm 0^{\circ}.16$	$70^{\circ}.99 \pm 0^{\circ}.15$	$74^{\circ}.96 \pm 0^{\circ}.16$
g_1	0.32	0.32	0.32	0.32	0.32
g_2	0.32	0.32	0.32	0.32	0.32
T_1	5100 K	5150 K	5200 K	5500 K	5300 K
T_2	4774 ± 46 K	5376 ± 7 K	5676 ± 8 K	5203 ± 14 K	5665 ± 11 K
A_1	0.5	0.5	0.5	0.5	0.5
A_2	0.5	0.5	0.5	0.5	0.5
Ω_1	2.33 ± 0.11	2.7738 ± 0.0094	2.888 ± 0.027	3.393 ± 0.052	2.862 ± 0.035
Ω_2	2.3124	2.7738	2.8877	3.3865	2.8621
q	0.233 ± 0.046	0.4632 ± 0.0046	0.530 ± 0.015	0.782 ± 0.029	0.506 ± 0.018
$x_1(V)$	...	0.795	0.778	0.778	0.778
$x_2(V)$	...	0.795	0.778	0.791	0.778
$y_1(V)$	...	0.130	0.200	0.200	0.201
$y_2(V)$	...	0.130	0.200	0.150	0.201
$x_1(B)$	0.852	0.852	0.847	0.847	0.847
$x_2(B)$	0.849	0.852	0.847	0.851	0.847
$y_1(B)$	0.028	0.045	0.098	0.010	0.098
$y_2(B)$	0.079	0.045	0.098	0.039	0.098
$L_1/(L_1+L_2)_V$	...	0.6184 ± 0.0024	0.5366 ± 0.0064	0.6210 ± 0.0097	0.5745 ± 0.0093
$L_1/(L_1+L_2)_B$	0.851 ± 0.027	0.6059 ± 0.0030	0.5108 ± 0.0066	0.6364 ± 0.0100	0.5554 ± 0.0097
$r_1(pole)$	0.471 ± 0.015	0.4260 ± 0.0010	0.4170 ± 0.0024	0.3759 ± 0.0034	0.4175 ± 0.0031
$r_1(point)$	0.598 ± 0.031	...	...	0.505 ± 0.013	...
$r_1(side)$	0.509 ± 0.019	0.4543 ± 0.0012	0.4440 ± 0.0028	0.3959 ± 0.0039	0.4443 ± 0.0036
$r_1(back)$	0.531 ± 0.018	0.4840 ± 0.0013	0.4755 ± 0.0027	0.4256 ± 0.0040	0.4740 ± 0.0035
r_1^{a}	0.504 ± 0.017	0.4564 ± 0.0012	0.4474 ± 0.0025	0.4005 ± 0.0037	0.4469 ± 0.0050
$r_1(\mathrm{lobe})^{b}$	0.5102	0.4489	0.4367	0.4017	0.4409
$r_2(pole)$	0.243 ± 0.014	0.2993 ± 0.0034	0.3123 ± 0.0094	0.3358 ± 0.0031	0.305 ± 0.012
$r_2(point)$	0.355 ± 0.011	...	...	0.475 ± 0.014	...
$r_2(side)$	0.253 ± 0.014	0.3131 ± 0.0042	0.327 ± 0.012	0.3517 ± 0.0033	0.319 ± 0.015
$r_2(back)$	0.286 ± 0.015	0.3494 ± 0.0072	0.365 ± 0.020	0.3834 ± 0.0032	0.354 ± 0.025
r_2^{a}	0.261 ± 0.055	0.3225 ± 0.0012	0.3369 ± 0.0023	0.358 ± 0.013	0.3280 ± 0.0034
$r_2(\mathrm{lobe})^{b}$	0.2615	0.3144	0.3255	0.3584	0.3217
f^{c}	...	0.11	0.15	...	0.084

[a] equal-volume radius
[b] Roche lobe equal-volume radius
[c] $f = (\Omega_{L_1} - \Omega)/(\Omega_{L_1} - \Omega_{L_2})$

TABLE III

Spot Parameters

System	Star	Longitude	Co-Latitude	Radius	Temperature Factor
EQ Cep	1	$20^{\circ}.03$	$90^{\circ}.00$	$12^{\circ}.66$	1.041
ER Cep	1	$20^{\circ}.90$	$98^{\circ}.10$	$10^{\circ}.60$	1.195
	1	$228^{\circ}.22$	$97^{\circ}.22$	$10^{\circ}.20$	0.880
V369 Cep	1	$101^{\circ}.10$	$90^{\circ}.00$	$21^{\circ}.53$	1.054

$$\sigma^2_{V-M_V} = \sigma^2_V + \left(\frac{\partial BC}{\partial(B-V)} + 10\frac{\partial \log T}{\partial(B-V)}\right)^2 \sigma^2_{B-V} +$$

$$\left(\frac{5}{3\ln 10}\right)^2 \left(\frac{9}{r^2}\sigma^2_r + \frac{1}{M_1^2}\sigma^2_{M_1} + \frac{4}{P^2}\sigma^2_P + \frac{1}{(1+q)^2}\sigma^2_q + \frac{6}{r(1+q)}\rho_{rq}\sigma_r\sigma_q\right). \quad (2)$$

For r we use the equal-volume radius of the primary component. The correlation coefficient between r and q, ρ_{rq}, is obtained via the correlation between q and the surface potential Ω. The apparent V-magnitude and standard error (corrected for interstellar extinction) are the mean and standard deviation of the magnitudes observed around phase $0^{\rm P}\!.75$. This magnitude is for the system of two stars combined. The magnitude of the primary component can be calculated by taking into account the fraction of the total light at phase $0^{\rm P}\!.75$ contributed by each individual star according to the light curve solution. Effective temperatures and bolometric corrections, as well as numerical estimates of the partial derivatives in Eqn. (2), were computed using a cubic spline interpolation in the appropriate calibrations listed in Lang (1992). A typical standard deviation of $0^{\rm m}\!.013$ for the $B-V$ color index is assumed in accordance with Johnson and Morgan (1953). Masses were computed from $B-V$ colors using the equation

$$M = 1.594 \pm 0.040 - 0.787 \pm 0.064(B-V), \quad (3)$$

which is a weighted least-squares straight line fit of mass vs. $B-V$ for main sequence stars with $B-V > 0.50$, compiled by Andersen (1991) and Popper (1993). A 15% standard deviation for M_1 is used in Eqn. (2) to reflect the width of the main sequence color-mass relationship. Before attributing single-star properties to over-contact binary stars, based on color indeces, the $B-V$ colors needed to be corrected for the luminosity transfer which takes place between components of an over-contact binary. We used the method of Mochnacki (1981), modified so as to account for a change not only in temperature but also in radius. The part U of the secondary's luminosity which comes from the primary component, expressed as a fraction of the luminosity of the primary, can be computed using Mochnacki's Eqn. (7). The amount by which the effective temperature would increase if no luminosity were transferred to the secondary is given by

$$\Delta \log T = \frac{1}{4}\log(1+U)\left(1 - 2\left(\frac{\partial \log R}{\partial \log L}\right)\right), \quad (4)$$

which is the analogue of Eqn. (8) in Mochnacki (1981). The partial derivative in Eqn. (4) was estimated to be 0.30, based on the $\log R$ - $\log L$ slope of a fitted straight line to the empirical radius-luminosity data in Andersen (1991) for main sequence stars with spectral type later than A8.

Distance moduli for each binary, and for two different reddening values, are listed in Table 4. The results are not very sensitive to the amount of reddening. Three binaries give a mean distance modulus of $10^{\rm m}\!.80 \pm 0^{\rm m}\!.08$, two others give

TABLE IV
NGC 188 Distance Moduli

System	$E_{B-V} = 0.08$	$E_{B-V} = 0.12$
EP Cep	10.78 ± 0.15	10.83 ± 0.15
ER Cep	10.70 ± 0.13	10.78 ± 0.12
ES Cep	10.76 ± 0.12	10.80 ± 0.11
EP, ER, ES mean	10.745 ± 0.076	10.800 ± 0.071
EQ Cep	11.26 ± 0.12	11.28 ± 0.13
V369 Cep	11.44 ± 0.12	11.56 ± 0.12
EQ, V369 mean	11.350 ± 0.085	11.431 ± 0.088
Mean (all)	11.014 ± 0.057	11.049 ± 0.055

values around $11\overset{m}{.}40 \pm 0\overset{m}{.}09$. These numbers compare with the values $11\overset{m}{.}1$ derived by VandenBerg (1985) and $11\overset{m}{.}5$ by Twarog and Anthony-Twarog (1989). We must conclude that our method is not able to unambiguously distinguish between the lower and higher distance estimates. However, a weighted mean for all five systems yields a result about one σ below VandenBerg's estimate. Therefore, our results slightly favor a distance of 1.65 kpc to NGC 188, and, implicitly, a 10 Gyr age. The largest contribution to the quoted errors, calculated using Eqn. (2), comes from the adopted uncertainty of 15% in the absolute masses. Determining the distance to a cluster based on parameters of member close binaries would improve considerably if these uncertainties could be reduced. We strongly recommend obtaining new multi-color light curves with good and complete phase-coverage, as well as radial velocities. These would help to reduce uncertainties in the absolute masses and radii and provide a powerful and independent cluster distance indicator.

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