Magnetic flux tubes inside the sun*

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(Received 12 November 1999; accepted 14 February 2000)

Bipolar magnetic active regions are the largest concentrations of magnetic flux on the Sun. In this paper, the properties of active regions are investigated in terms of the dynamics of magnetic flux tubes which emerge from the base of the solar convection zone, where the solar cycle dynamo is believed to operate, to the photosphere. Flux tube dynamics are computed with the “thin flux tube” approximation, and by using magnetohydrodynamics simulation. Simulations of active region emergence and evolution, when compared with the known observed properties of active regions, have yielded the following results: (1) The magnetic field at the base of the convection zone is confined to an approximately toroidal geometry with a field strength in the range $3 \times 10^4$–$10^5$ G. The latitude distribution of the toroidal field at the base of the convection zone is more or less mirrored by the observed active latitudes; there is not a large poleward drift of active regions as they emerge.

The time scale for emergence of an active region from the base of the convection zone to the surface is typically 2–4 months. (2) The tilt of active regions is due primarily to the Coriolis force acting to twist the diverging flows of the rising flux loops. The dispersion in tilts is caused primarily by the buffeting of flux tubes by convective motions as they rise through the interior. (3) Coriolis forces also bend active region flux tube shapes toward the following (i.e., antirotational) direction, resulting in a steeper leg on the following side as compared to the leading side of an active region. When the active region emerges through the photosphere, this results in a more rapid separation of the leading spots away from the magnetic neutral line as compared to the following spots. This bending motion also results in the neutral line being closer to the following magnetic polarity. (4) The properties of the strongly sheared, flare productive $\delta$-spot active regions can be accounted for by the dynamics of highly twisted $\Omega$ loops that succumb to the helical kink instability as they emerge through the solar interior.

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I. INTRODUCTION

The existence of magnetic fields on the surface of the Sun has been known indirectly for hundreds of years, dating back to Galileo’s original discovery of sunspots. It wasn’t until this century that it was shown experimentally that Galileo’s spots were associated with strong magnetic fields on the Sun. Where do the Sun’s magnetic fields come from? A great deal of research has been done to answer this question, with approaches ranging from detailed, high resolution observations of the Sun’s magnetic flux evolution, to studies of how magnetic activity varies with different stellar parameters for solar-like stars, to sophisticated mathematical models of the solar cycle dynamo. There is insufficient space here to review this vast subject. Instead, we will attempt to answer this question from a very narrow perspective, and review the evidence that magnetic fields in solar active regions originate from the base of the solar convection zone (roughly $2 \times 10^5$ km below the photosphere), with a roughly toroidal geometry, and with field strengths $\sim 3 \times 10^4$–$1 \times 10^5$ G. The fields we see at the surface in the form of active regions appear to be loops of flux which for one reason or another have emerged from this layer. We will discuss how many of the known properties of active regions can be explained by the dynamics of magnetic flux loops as they rise through the convection zone.

Solar active regions are bipolar, meaning that areas of strong magnetic field of one sign are paired with strong, adjacent fields of the opposite sign. This property suggests...
that active regions occur where the tops of magnetic flux loops pierce the solar photosphere (see Fig. 1). Bright x-ray or extreme ultraviolet loops connect the two opposite polarities through the solar corona.

Hale’s polarity law describes the observed orientation of active regions: On the average, active region bipoles are oriented nearly parallel to the east–west (E–W) direction, with the same orientation in the northern hemisphere, but with the opposite orientation in the southern hemisphere. This simple relationship suggests an approximately toroidal pattern to the Sun’s magnetic field geometry, with the toroidal field having opposite sign in the northern and southern hemispheres. The fact that active regions appear to be the tops of flux loops suggests that the toroidal field in the Sun is formed at a significant depth below the photosphere. The persistence of Hale’s polarity law for periods of several years during a given solar cycle indicates that the toroidal fields must be residing in a relatively stable region in the solar interior. Several theoretical arguments suggest that the only place where such fields could be stored for long periods without being disrupted by either buoyancy instabilities or by turbulent convective motions is below the solar convection zone. Yet if the fields are stored at any significant depth below the convection zone (i.e., in the radiative zone), they will be so stably stratified that they could never erupt to the surface on the time scale of a solar cycle. Therefore, the only viable region for the storage of the magnetic flux appears to be an interface layer separating the solar convection zone from the radiative zone, known as the “convective overshoot layer.” This region of the solar interior is now also known to coincide approximately with the transition from solid body rotation to the observed photospheric differential rotation pattern, the so-called “tachocline” deduced from inversions of the rotation profile in helioseismology. The presence of strong shearing motions, plus the right range of subadiabatic temperature gradients for flux storage, suggests that this is not only the region where toroidal fields are stored, but probably where they also are regenerated over the course of the solar cycle.

II. FLUX EMERGENCE: USING THE THIN FLUX TUBE APPROXIMATION

The picture of bipolar active regions as emerging magnetic flux loops (Fig. 1) suggests a simple theoretical approach as a first step: derive an equation of motion for a flux loop moving in a field-free background model of the solar convection zone. The simplest such model is the thin flux tube approximation. As originally formulated, the thin flux tube model is derived by simplifying the three-dimensional (3D) ideal magnetohydrodynamic (MHD) momentum equation for a thin, untwisted magnetic flux tube surrounded by field-free plasma, subject to three constraints: (1) when the tube moves, it retains its identity as a “tube” (i.e., it does not fragment, disperse, or diffuse away); (2) the tube is thin, in that its diameter is small compared to all other physical length scales in the problem, and (3) quasistatic pressure balance across the diameter of the tube is maintained at all times. If these conditions are true, one can derive an equation of motion for the 1D tube moving in a 3D environment. There are many reasons to question the viability of the thin flux-tube assumptions, but for now we will simply assume the model can be used. Certainly the observations suggest that active regions look and behave as though the magnetic field maintains a tube-like geometry, although there is the appearance that fragmentation of the tube has occurred by the time the active region reaches the photosphere (see Fig. 1). Solutions of the thin flux tube equation of motion, along with the induction, continuity, and energy equations, yield cross sectional averages of the magnetic field strength, thermodynamic variables, and velocity as a function of time and distance along the tube.

The thin flux tube equation of motion as commonly used is

\[ \rho_i \frac{D\mathbf{v}}{Dt} = F_B + F_T + F_C + F_D, \]

where

\[ F_B = g(\rho_e - \rho_i) \mathbf{v}, \quad F_T = B^2/8\pi \kappa. \]
\[ F_D = -\frac{C_D}{(m\Psi/B)^{1/2}} |v| v \; ; \; \; \; F_C = -2\rho \Omega \times v. \]  

\( F_B \) is the magnetic buoyancy force, \( F_T \) is the force due to magnetic tension (field line bending), \( F_C \) represents the Coriolis force (because \( D/Dt \) is the time derivative taken in the rotating reference frame), and \( F_D \) is an aerodynamic drag force resisting motion of the tube through the external, field-free plasma. The quantities \( \rho_i \) and \( \rho_e \) represent the mass density inside and outside the flux tube, respectively, and \( g \) is the local gravitational acceleration. The magnetic field \( \mathbf{B} \) points in the direction of the tube’s tangent vector \( \hat{s} \) (\( \hat{s} = \partial \mathbf{r}(s)/\partial s \)), and the curvature vector \( \mathbf{k} \) which gives the direction of \( \mathbf{F}_T \), is given by \( \kappa = \hat{s} \times \partial \mathbf{r}(s)/\partial s^2 \). The vector \( \mathbf{r}(s,t) \) denotes the tube axis position as a function of its own arc length \( s \) and time \( t \). In the expression for \( F_D \), \( v \) represents the normal component of velocity difference between the tube and the plasma outside the tube. The aerodynamic drag coefficient \( C_D \) is set (somewhat arbitrarily) to unity (see e.g. Ref. 6).

**A. The latitudes of emerging active regions**

The first use of the thin flux tube model was to study the latitude distribution of magnetic flux emerging from the base of the convection zone to the Sun’s photosphere using a 3D, rotating model of the solar interior for the background plasma. For simplicity, the tubes were assumed to be in the form of axisymmetric flux rings, oriented in the toroidal direction. For flux rings with a field strength \( B_0 \) at the base of the convection zone of \( 10^4 \) G, the flux emerged parallel to the solar rotation axis and reached the photosphere at much higher latitudes than those corresponding to the observed latitudes of active regions. Only flux rings with \( B_0 \sim 10^5 \) G had latitudes of emergence that could be consistent with observations. This work was extended to nonaxisymmetric flux rings, yielding similar results. At the time these simulations were done, dynamo theorists predicted that the field strength near the base of the convection could be no greater than \( 10^6 \) G, based on arguments of equipartition between magnetic energy and the kinetic energy density associated with convective motions.

Other studies of the latitude of active region emergence basically confirmed these results, namely that if the field strength \( B_0 \) is close to the equipartition value \( (B_0 \sim 10^4 \) G), flux emergence occurs at very high latitudes. The thermal initial conditions used by Refs. 8, 9, and 10 were criticized as unrealistic and instead it was emphasized that one must start with flux loops in force balance in the convective overshoot layer. In their case, the equilibrium has an assumed toroidal symmetry, and its linear stability is then analyzed. Eruptive instability is found only if \( B_0 \) exceeds roughly \( 10^5 \) G, with azimuthal wave numbers of \( m = 1 \) or \( m = 2 \) being the most unstable, depending on the exact values of latitude and field strength (a more detailed analysis of the stability criteria for toroidal equilibria is presented in Refs. 15 and 16). For the flux rings which are unstable, and therefore result in erupting loops, the emergence is primarily radial, and there is no difficulty with flux emerging at latitudes which are too high. A different approach to the question of how magnetic flux is destabilized in the overshoot layer is to derive an energy equation for the transfer of heat by radiation into a magnetic flux ring in force balance. Over time, small entropy inhomogeneities in the flux ring will eventually evolve into flux loops that become buoyantly unstable under the influence of continued heating. In this case a variety of different field strengths are possible, provided the subadiabatic gradient (usually denoted by the quantity \( \delta = \nabla - \nabla_{ad} \)) is not too small in absolute value (\( \delta \) is negative when the gradient is subadiabatic). Latitudes of emergence that are consistent with observed active latitudes are found for values of \( B_0 \) down to \( 3 \times 10^4 \) G.

**B. Joy’s law**

“Joy’s law” is the phenomenon that has received the most attention with the thin flux tube model. This relationship describes the observed proportionality of the “tilt angle” of active regions with latitude. Joy’s law (the name for this relationship can be attributed to Zirin) is actually a slight deviation from Hale’s law. The observed average orientation of bipolar active regions is actually not quite toroidal, but instead is tilted slightly away from the azimuthal direction, with the polarity in the leading direction (the direction toward solar rotation) slightly closer to the equator than the following polarity. Further, the tilts are a function of latitude, with the size of the tilt angle roughly proportional to the latitude. The tilts are not large; at latitudes of \( 30^\circ \), the average tilt angle for spot groups is roughly \( 7^\circ \). Even though Joy’s law might seem like a subtle and inconsequential effect, it plays a very important role in several classical solar cycle dynamo models. In phenomenological solar dynamo models, the tilt from active regions is included as an empirical effect, and is the agent by which the poloidal flux from the previous cycle is reversed by the poloidal flux from the current cycle. While Joy’s law is incorporated into these models, the models do not address its physical origin.

The first study of Joy’s law used the thin flux tube approximation to evolve nonaxisymmetric perturbations of initially toroidal flux rings. The perturbations evolved into rising loops of flux which were then tilted in a manner consistent with Joy’s law. They found from their simulations that active region tilts could be explained by Coriolis forces acting on rising, expanding magnetic flux loops, and that if the field strength at the base of the solar convection zone was \( 10^5 \) G, that Joy’s law could be reproduced. Similar tilt angle results were reported in other studies. These studies employed a wide variety of different initial conditions for the rising loop calculations, but they were all able to reproduce Joy’s law for physically reasonable values of the field strength at the base of the convection zone. Active region tilt angles are one of the most robust predictions of thin flux tube models, with tilts depending primarily on latitude and the field strength at the base of the convection zone, and not on the details of the initial conditions used in the simulations. Reference 11 finds best agreement with observed tilts for flux loops originating in the lower part of the convective overshoot region, corresponding to field strengths of \( 1.2 - 1.4 \times 10^5 \) G, if the flux loops originate from unstable
equilibria. Ref. 13 finds good agreement for tubes which are destabilized by gradual radiative heating if the field strength lies in the range $4 - 10 \times 10^4 \, \text{G}$. They find the reverse of Joy’s law (i.e., the tilts go the wrong direction, in some cases) if $B_0 \lesssim 3 \times 10^4 \, \text{G}$.

If the Coriolis force can account for Joy’s law, it should be possible to explain the phenomenon in a simple fashion without resorting to detailed flux tube simulations. A simple cartoon model has been presented in Ref. 10, in which the buoyancy-driven rise of a flux tube is balanced by an aerodynamic drag force, and leads to a rising, diverging loop geometry. The Coriolis force then acts on the diverging velocity field to twist this loop into a backward “S” shaped geometry (in the northern hemisphere) when the projected loop is viewed from above. If this Coriolis force is then balanced with a magnetic tension force opposing the twisting motion, one can show that $\alpha \propto \Phi^{1/4} \sin \theta$, where $\alpha$ is the tilt angle, $\Phi$ is the magnetic flux in the rising tube, and $\theta$ is latitude. This simple analysis not only yields Joy’s law, but also predicts a new relationship between tilt and magnetic flux that can be tested with observation.

This prediction was tested\textsuperscript{18} by examining the tilt angles of a large number (24,701) of spot groups observed at Mt. Wilson over many decades during the 20th century (see Ref. 24). While white light spot group measurements contain no magnetic information, there are proxy relationships that can be used.\textsuperscript{25} The separation distance $d$ between the poles of spot groups is a reasonable proxy for the magnetic flux $\Phi$. Tilt angles were indeed found to be an increasing function of both latitude and $d$,\textsuperscript{18} with the data consistent with $\alpha \propto d^{1/4} \sin \theta$. These results can also explain the $d$ variation in the “residual tilt.” Reference 18 argued that the best fit to the data from flux tube simulations is obtained for $B = 2 - 3 \times 10^4 \, \text{G}$, based on the simulations of Ref. 10. On the other hand, those simulations had initial conditions which were not consistent with flux tubes initially in force balance, as Refs. 12 and 14 have noted. The simulations of Ref. 13 assume a more physically self-consistent destabilization of the flux tubes. In this case, a best fit of the data is obtained with field strengths in the range $4 - 10 \times 10^4 \, \text{G}$, i.e., at significantly higher values than those quoted in Ref. 18.

The observed $d$ dependence of active region tilts also argues against the explanation of Joy’s law proposed by Ref. 19 that the tilt angle simply reflects the underlying orientation of the magnetic field at depth. Reference 19 suggested that such an orientation would occur because of differential rotation with latitude. If this were the only origin of active region tilts, tilt angles would have no dependence on the amount of magnetic flux in an active region. While an underlying orientation that is not purely toroidal cannot be ruled out, the observed $d$ dependence of tilt suggests that the dominant tilting occurs from Coriolis forces.

C. Tilt angle dispersion

A further result regarding active region tilts was shown in Ref. 18: the behavior of the tilt angle “noise” as a function of $d$ and $\theta$. There is actually a substantial scatter of individual active region tilts around the average Joy’s law behavior. What is the origin of this? Is it measurement error, or does this tell us something about flux tube dynamics in the Sun? In an effort to understand the scatter, Ref. 18 first measured the level of scatter as a function of both latitude and polarity separation distance $d$. They found that the degree of tilt scatter $\Delta \alpha$ was independent of latitude (in contrast to the latitude dependence of the mean tilts), but $\Delta \alpha$ did depend on $d$, roughly as $d^{-3/4}$. Next, they investigated the measurement error in the tilt angle by analyzing the algorithm used to define the tilt angle of a spot group in terms of the positions of the constituent spots. They found measurement errors which were significantly smaller than the levels of scatter shown by the spot group dataset, and concluded that the tilt scatter has a physical origin. Reference 18 proposes that the scatter is due to the buffeting of flux loops by turbulent motions as they rise through the convection zone.

Reference 27 performed a detailed analysis of the convective buffeting hypothesis, by considering an initially horizontal flux tube rising through a turbulent medium. The velocity amplitude and the eddy size are assumed to be determined by a mixing length model of the convection zone. They found that the $d$ dependence of the tilt fluctuations from the theoretical models could be closely matched to the observed tilt scatter\textsuperscript{18} for physically reasonable values of the magnetic field strength and for mixing length parameters close to the accepted solar values. The theoretical $d$ dependence derived from two effects: tubes with larger magnetic flux (and hence larger $d$) rise faster and are perturbed less by turbulence, and the larger footpoint separation averages over more distortions in the tube’s axis. Values of $B_0$ that seemed consistent with the observed behavior were in the range $2 - 4 \times 10^4 \, \text{G}$. The flux tubes in this study were not initially in force balance, and thus the criticisms of Refs. 12 and 14 apply. It is not clear at this time how this might change the field strength that results from a best fit to the observations.\textsuperscript{28}

D. Active region asymmetries: inclinations, spot group motions

Thin flux tube models have had great success\textsuperscript{12,29} in explaining the differences in magnetic field inclination and spot motions between the leading and following sides of active regions.

The tendency of plasma in an emerging flux loop to preserve its angular momentum results in a retrograde motion of the emerging flux loop when viewed from the Sun’s rotating reference frame. This will distort the shape of the flux loop, yielding a leading leg that has a shallow angle of inclination relative to the horizontal direction, and a following leg that is more steeply inclined. If one imagines what happens to such a distorted loop as it rises through the photosphere, two observationally testable phenomena are immediately apparent: there will be a more rapid motion of the leading polarity sunspots away from the emerging flux region as compared to the motion of the following polarity spots, and the field inclination near the following polarity will be steeper than the leading polarity. This property of spot motions is well known (e.g., Refs. 30 and 31). The field incli-
nation predictions have been confirmed indirectly through the observation that the magnetic neutral line appears closer to the following polarity spots. The predicted field inclinations could be tested directly with ground or space based vector magnetographs, such as the Imaging Vector Magnetograph at the University of Hawaii, or the vector magnetograph planned for Solar-B.

III. BEYOND THE THIN UNTWISTED FLUX TUBE APPROXIMATION

A. Testing the thin flux tube assumptions with 2D MHD simulations

Given the large number of published results that use the thin flux tube approximation, there is an increasing interest in understanding its range of validity. References 33 and 34 showed with 2D MHD simulations that an initially buoyant, untwisted magnetic flux tube with axial symmetry will break apart into two counter-rotating elements which then repel one another. References 35, 36, and 37 showed that if the magnetic field is twisted, the tendency to break apart is lessened; if the tube has a degree of twist that exceeds a certain critical value, the tube no longer fragments. The critical degree of twist can be determined in an approximate fashion by equating the Alfvén speed of the azimuthal component of the field with the flux tube’s rise speed through the solar interior. At face value, these results suggest that a flux tube must have some minimum degree of twist to maintain its tube-like nature. References 36 and 37 have also shown via MHD simulation that the aerodynamic drag formulation for the force of the background plasma on a thin flux tube, which is commonly used in the thin flux tube models, is approximately correct. On the other hand, Ref. 38 argues that if the effective Reynolds number of the plasma in the simulations is increased to much greater (and more realistic) values, asymmetric vortex shedding results in forces on the tube which cannot be included as a simple aerodynamic drag term. Clearly a deeper understanding of when the thin flux tube approximation can and cannot be used is necessary.

B. 3D MHD simulations of flux tube fragmentation

We have recently used the 3D anelastic MHD code ‘‘ANMHD’’ to study how rising Ω shaped loops may fragment during their rise through a simplified, gravitationally stratified model convection zone. An example of what happens to an initially cylindrical tube that starts out near the base of the simulation box is shown in Fig. 2. The tube is given an initial entropy perturbation that results in it being buoyant in the center, but held down near the ends. In this case, the tube’s initial magnetic field is untwisted. Figure 2 shows the magnetic configuration as a volume rendered image after the center of the tube has risen roughly two-thirds of the way toward the top of the box. Near the apex of the tube, most of the magnetic flux has been rolled up (‘‘fragmented’’) within two oppositely rotating vortices (the magnetic fragments are visible as the two separated bright ridges near the center of the figure) connected above by a thin, curved convex magnetic sheet (the sheet is not visible in the figure). We have performed a large number of such simulations, varying the amounts of initial magnetic twist, and also varying the degree of curvature at the apex of the Ω loop.

We found that the critical twist necessary to prevent fragmentation is far lower for the Ω loop geometry than it is for the 2D limit identified earlier. In the limit of zero curvature, however, the 2D critical twist limit is recovered. We found that even for cases of no initial twist, the separation between the two magnetic fragments at the apex of an emerging Ω loop with some curvature is substantially reduced from that of the corresponding 2D behavior.

A further result is that in all the cases studied so far, reconstructed ‘‘magnetograms’’ of the magnetic field seen in a horizontal plane of the simulation (which may resemble what one would see in the photosphere) show that flux emergence results in bipolar crescent shaped structures in the early stages of emergence, which then separate from one another, with the individual magnetic polarities becoming tighter and more compact as emergence continues.

C. δ-spot active regions

While most active regions seem to exhibit only slight amounts of magnetic twist, there are a few active regions with unusual morphology and orientation (δ-spot active regions), which seem to be very highly twisted. δ-spot active regions are also responsible for the largest solar flares; similarly, active regions with coronal loop structures that are ‘‘sigmoidal,’’ also an indication of strong active region mag-
netic twist,\textsuperscript{41} are associated with large coronal mass ejections. From the ‘‘space weather’’ perspective, \( \delta \)-spot active regions are clearly of great importance.

\( \delta \)-spot active regions emerge with opposite magnetic polarities jammed together—sunspot umbrae of opposite magnetic polarity are frequently contained within a single penumbra. \( \delta \) spot regions often emerge highly tilted away from the E–W direction, and frequently appear to rotate as they emerge. Observations with vector magnetographs show that the magnetic field in \( \delta \) spots is strongly twisted; the transverse field along the magnetic neutral line is strongly sheared away from the direction that a potential field extrapolation would indicate. Analysis of the observed development of \( \delta \)-spot regions over time has convinced many observers that the magnetic field has a kinked or knotted geometry below the photosphere\textsuperscript{42–44}.

These results have motivated a great deal of theoretical work on the possible relationship between \( \delta \)-spot active regions and magnetic flux tubes that are sufficiently twisted to become kink unstable. Reference 45 did a comprehensive linear stability analysis of the kink mode for an infinitely long, pressure confined twisted flux tube with cylindrical geometry, in a high \( \beta \) plasma. Given the axial field component \( B_\theta(r) = B_\theta(1 - \alpha r^2 + \cdots) \), the azimuthal component \( B_\phi(r) \) of the tube is given in terms of a twist parameter \( q \) as \( B_\phi(r) = q r B_\theta(r) \). Reference 45 showed that if the twist \( q \) exceeds a critical twist \( q_{cr} \), the tube is kink unstable. \( q_{cr} \) depends only on the second order term of the Taylor expansion of \( B_\phi(r) \): \( q_{cr} = \frac{\sqrt{\alpha}}{\pi} \). Reference 45 gives a complete prescription for computing all the growth rates, range of unstable wavenumbers, and eigenfunctions of the unstable modes. They also showed that as a twisted flux tube rises through the solar interior, a tube that is initially stable to the kink mode may become unstable as it emerges, due to the fact that flux tube expansion will decrease the axial field more rapidly than the azimuthal component of the field, and will therefore decrease the kink stability threshold \( q_{cr} \) while \( q \) remains roughly constant.

Fully compressible 3D MHD simulations of the kink instability in a high \( \beta \) plasma\textsuperscript{46} confirmed the growth rates predicted by Ref. 45, and also showed that the kink mode saturates in roughly ten linear growth times scales, at amplitudes of roughly 30% of the tube radius for the fastest growing modes. Reference 47 then showed that if several unstable modes are excited simultaneously, as seems likely in the highly turbulent convection zone, that the modes can interfere constructively and produce complex, large amplitude knot-like geometries, with a strong resemblance to the inferred shape of \( \delta \)-spot active regions. ‘‘Magnetograms’’ reconstructed from the simulations show that many of the observed \( \delta \)-spot properties are reproduced.

Simulations of kink unstable flux tubes in a gravitationally stratified model of the solar interior by Refs. 48 and 39, using a 3D anelastic MHD code confirm the overall picture of Ref. 47. They further show that the apparent spot-group rotation of the kink instability is enhanced as a result of the greater expansion of the kink at the apex as compared to the legs, and that the kink instability greatly enhances the buoyancy of the tube. By including gravity, Refs. 48 and 39 were able to show directly from the simulations (see, e.g., Fig. 3 and 4) that the morphology of the kink unstable \( \Omega \) loop strongly resembles the behavior and appearance of \( \delta \)-spot active regions. Figure 3 shows the time evolution of a buoyant, rising \( \Omega \) loop which is also kink unstable. Figure 4

\[ \text{FIG. 3. The emergence of a kink unstable flux tube rising in a gravitationally stratified model of the solar interior. This result is from a 3D MHD simulation described by Fan et al. (Ref. 48) with a resolution in } x, y, z \text{ of } 128 \times 128 \times 256. \text{ The left panels (a) show the tube as seen from the side, while the right panels (b) show the tube as viewed from above. The times in are given in units of } a/\nu_a, \text{ where } a \text{ is the initial tube radius and } \nu_a \text{ is the initial Alfvén speed along the tube axis. The horizontal plane shown in panel (3a) is the plane used to evaluate the ‘‘vector magnetogram’’ shown in Fig. 4.} \]

\[ \text{FIG. 4. A ‘‘vector magnetogram’’ constructed from the kink mode simulation shown in Fig. 3, at the time shown in panel (3a) of Fig. 3, and at the height indicated by the plane seen in that panel. The contours indicate the vertical component of the field, while the arrows indicate the direction and magnitude of the horizontal component of the field. Note that the bipolar structure has been rotated by over 90° from the initial tube direction, and that the horizontal magnetic field along the magnetic neutral line is strongly sheared. These are properties commonly seen in observations of } \delta \text{-spot active regions.} \]
shows a "vector magnetogram" constructed from a horizontal cut through the simulation when the tube has risen most of the way through the simulation box [see Fig. 3(a)]. The vertical field configuration (contours) shows a large rotation of more than 90° away from the initial Hale orientation of the tube, while the horizontal field (arrows) shows strong shear along the neutral line (the zero contour separating positive from negative vertical field). As noted above, the rotation and shear are common features of observed δ-spot active regions.

IV. CONCLUSIONS

Over the past decade, the use of flux tube dynamics to study and interpret the observed properties of solar active regions has been extremely productive, and has resulted in a much more holistic and physical picture of solar activity and solar magnetic fields.

Thin flux tube models of emerging active regions have been quite successful in reproducing many observed properties of active regions, and in deriving constraints on the magnetic field generated by the solar cycle dynamo. The interplay between theory and observation has been particularly rich and rewarding, as illustrated with the thin flux tube model of active tilts: Once the Coriolis force acting on rising flux loops was proposed as the source of tilts, theory predicted that tilts should also depend on the amount of magnetic flux, which was then confirmed observationally. Observations then showed that tilt dispersion was a function of active region size, but not latitude, suggesting a connection with convective motions. A theoretical model was developed which was then found to be consistent with the observed behavior of tilt dispersion.

On the other hand, the conditions of applicability of the thin flux tube model are at this point uncertain. Much more study with 3D MHD simulation will be necessary before we understand when the thin flux tube approximation applies and when it does not. The physics of flux tube fragmentation and its relationship to observed active region morphology appears to be a promising area of future research.

The linkage between kink unstable flux tubes and δ-spot active regions seems very promising, but there is much that remains to be done. For example, the kink hypothesis for δ-spot active regions predicts that the twist and writhe in δ-spot active regions should have the same sign. Does it? We will only find the answer to this question once a large number of observations of δ-spot active regions are carefully examined.