A Magnetohydrodynamic Test of the Wang-Sheeley Model

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Abstract. The Wang-Sheeley relationship relates the solar wind speed at the Earth to the divergence rate of open magnetic flux tubes in the solar corona. This relationship is based on a statistically significant correlation between the flux tube divergence parameter \( f_s \) derived from a photospheric field-based potential field source surface model, and satellite observations of the solar wind speed. The fast solar wind emanates from regions of small magnetic divergence, while slow solar wind comes from regions of high magnetic divergence. Arge and Pizzo [2] improved the reliability of the method by relating the coronal flux tube expansion factor to the solar wind speed at the source surface instead of the satellite. We use a three-dimensional MHD model of the solar corona to further investigate the implications of the Wang-Sheeley relationship for solar wind acceleration. The results suggest what additional heating and momentum inputs may be necessary in an MHD model to obtain the observed relationship between flux tube divergence and solar wind speed.

INTRODUCTION

The Wang-Sheeley relationship relates the solar wind speed observed at the Earth to the amount an open coronal magnetic flux tube expands in a potential field source surface model. A coronal expansion factor, \( f_s \), is defined as:

\[
f_s = \left( \frac{r_s}{r_{ss}} \right)^2 \frac{B_s}{B_{ss}}
\]

where \( r_s \) and \( r_{ss} \) are the photospheric and source surface radii (typically \( r_{ss} = 2.5 r_s \)), and \( B_s \) and \( B_{ss} \) are the magnetic field strengths at these locations along the flux tube. The source surface is a sphere at which the field is presumed to be everywhere radial, mimicking the effect of a solar wind outflow on the coronal field.

Wang & Sheeley [13] found that the fast solar wind emanates from regions of small magnetic divergence, while slow solar wind comes from regions of high magnetic divergence. Wang and Sheeley [14] and Wang [15] argued that higher expansion factors are associated with regions of large mass flux. Thus for a uniform energy flux at the base of the corona, the high mass flux regions will have less energy per particle than the lower mass flux regions. This results in a decrease in the solar wind speed associated with increasing flux tube divergence.

Arge and Pizzo [2] improved the performance of the Wang-Sheeley relationship by relating the expansion factor to the solar wind speed at the source surface instead of the Earth’s orbit. Their new empirical relationship:

\[
v(f_s) = 267.5 + \frac{410}{f_s^{2/5}}
\]

applies strictly near the solar equator. The subsequent kinematic extrapolation to 1 AU, including an approximate treatment of stream interaction effects, is not of interest here. Our objective is to use their empirical relationship to better characterize the heating and momentum deposition in a global MHD coronal model.

Although it is established that the slow solar wind has a more complicated origin than the high speed wind, including a probable transient component and a contribution from low latitude coronal holes, we assume, as in Neugebauer et al. [7] that for low solar activity periods, a significant fraction comes from the edges of polar coronal holes. Admittedly, the details of the diverse nature of the slow solar wind are averaged over in the Wang-Sheeley empirical relationship. We nonetheless use the coronal magnetic field geometry from an MHD model to calculate solar wind speeds at the effective source surface with equation 2, and compare the results to solar wind speeds obtained directly from the MHD model. By doing this, we hope to learn more about both the physics behind the empirical relationship, and the acceleration terms needed in MHD simulations to model the solar wind.

THE SOLAR CORONA MODEL

We adopt a simple polytropic energy relation with \( \gamma = 1.05 \) (in order to achieve a trans-sonic solar wind). Following Mikic et al [6], it is known that polytropic models cannot account for the observed solar wind speeds using this low value of \( \gamma \), without increasing the temperature at the base of the corona to an unphysically high value,
FIGURE 1. Solar wind speed (right column) and Log of the magnetic field strength (left column) for each MHD case: polytropic (top), Alfvén waves $\delta V^2 = 35$ km/s (middle), and Alfvén waves $\delta V^2 = 70$ km/s (bottom).

and/or incorporating additional sources of heating and momentum. Since Alfvén waves may provide a potentially significant momentum source (Alazraki and Couturier [1]). We incorporate them into our MHD model by using the WKB approximation to evolve an effective Alfvén wave pressure. Here we assume that the scale size for variations in the corona are small over a typical Alfvén wavelength. The use of the WKB approximation has been validated by the analyses of Roberts [8] and Smith et al. [9] (see Usmanov et al., [12], for further discussion).

Our model is a three dimensional single fluid MHD model with a dipolar field geometry. We solve the following MHD equations (Mikic et al [6], Usmanov et al. [12]):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$  

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla P - \nabla p_w - \mathbf{J} \times \mathbf{B} - \rho \mathbf{g}$$

where $\rho$ is the plasma density, $\mathbf{v}$ is the plasma flow velocity, $P$ is the total thermal pressure (both electrons and ions), $\mathbf{J}$ is the current density, $\mathbf{B}$ is the magnetic field and $e$ is the energy density. The Alfvén wave pressure $p_w$, is governed by:

$$\frac{\partial e}{\partial t} + \nabla \cdot (e \mathbf{v}) = -P \nabla \cdot \mathbf{v}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

where $\epsilon$ is the Alfvén wave energy density $(\epsilon = 2p_w)$ and $\mathbf{v}_A$ is the Alfvén velocity.

The equations are solved in three-dimensions in spherical coordinates $(r, \theta, \phi)$, using the Zeus-3D (Stone and Norman, [10] and [11]) code developed at the Laboratory for Computational Astrophysics at NCSA. Each case is axially symmetric (e.g. independent of $\phi$) with a resolution of $75 \times 30$ in the $r$ and $\theta$ directions. The radial
points are non-uniformly spaced; resolution ranges from 3.2 × 10^6 km near 1 \( R_s \), to 3.1 × 10^5 km at 15 \( R_s \). The azimuthal points are uniformly spaced from 0 to \( \pi \) with an angular resolution of 6°.

The initial values of the hydrodynamic variables at 1 \( R_s \) are set to \( \rho_0 = 1.67 \times 10^{13} \text{ kg/m}^3 \) and \( T_0 = T_i = T_e = 1.6 \times 10^6 \text{ K} \). We assume a dipole field strength of 1 G at the equator and hold \( B_r \) constant at the inner boundary (are parameters are similar to those used in earlier simulations by Linker and Mikic [5]). The values of \( v_r \), \( v_\theta \) and \( B_\theta \) are calculated at this boundary. In addition, the Alfvén wave energy density at the lower boundary is assumed to be given by:

\[
\varepsilon_0 = \rho_0 \left\langle \delta v^2 \right\rangle (1 - \sin^4 \theta) \tag{8}
\]

where \( \left\langle \delta v^2 \right\rangle \) is the mean square amplitude of the velocity fluctuations. The angular dependence, which is somewhat arbitrary, is adopted so as to maintain a closed streamer belt. The radial dependence is initialized with the one-dimensional solution to equations 3-7, found by Usmanov et al. [12].

We consider three heating/acceleration scenarios:

1. A simple polytropic model \( \gamma = 1.05 \) and \( \varepsilon_0 = 0 \) (no Alfvén waves).
2. A polytropic model, with Alfvén waves having mean square amplitude velocity fluctuations of 35 km/s at the lower coronal boundary.
3. A polytropic model, with Alfvén waves having mean square amplitude velocity fluctuations of 70 km/s at the lower coronal boundary.

In case 2 the 35 km/s mean square amplitude velocity fluctuations are the same value used by Usmanov et al. [12] and near the upper limit of the non-thermal velocity fluctuation amplitudes inferred from observations by Hassler et al. [3].

### RESULTS

The model was initiated with a radially symmetric hydrodynamic Parker solar wind solution combined with an initial dipole magnetic field configuration. The system was allowed to relax for 110 hours of solar time. Based on the resulting field configuration, the source surface used for the Wang-Sheeley wind speed calculations was taken to be at 5 \( R_s \), where the last closed field line occurs in the MHD coronal model.

Figure 1, shows contours of the solar wind speed and magnetic field strength for each of the three cases. The flow speeds are highest in the polar regions, decreasing toward the equator above the closed field line region. A stagnation region exists inside of the closed field. The presence of the Alfvén waves significantly increases the plasma flow speed near the poles over the simple polytropic case. The flow speed near the ecliptic is also increased over the polytropic case, though not to the same extent. The current model does not have sufficient spatial resolution to fully resolve the flow details above the closed field region. Thus while it is sufficient for the purposes of this analysis over most of the 5 \( R_s \) sphere, we cannot make conclusions concerning the first open flux tubes. The resolution will be improved in future runs.

The magnetic field configurations are similar for cases 1 and 2 despite the different flow speeds. The last closed field line in each case occurs just inside 5 \( R_s \), beyond this distance the field lines are open. These results imply that the magnetic field configuration is not very sensitive to our addition of Alfvén waves. This assertion is further supported by examining the calculated expansion factor \( f_s \) (figure 2) at the source surface for these cases. The magnetic field divergence in case 3 is slightly different than the other cases but the magnitude of the field has not significantly changed. A few more field lines are open in this case due to the higher flow speed, and the effective source surface has moved out slightly. These differences are further reflected in the \( f_s \) calculation. This difference however, does not significantly change the speed predicted by the Wang-Sheeley relationship.

The flow speeds calculated at the source surface by the improved Wang-Sheeley relationship are compared to the flow speed results from each MHD case in figure 3. The Arge and Pizzo [2] Wang-Sheeley relationship results, which were essentially the same in the 3 cases, yield higher predicted wind speed values at the source surface than any of the MHD models. The gradual drop off in the flow speed with colatitude for the Alfvén wave cases (compared to the Wang-Sheeley and polytropic
CONCLUSIONS

We have used an MHD model of the solar corona to compare solar wind speeds from such a model to those predicted by the Wang-Sheeley empirical relation. The implications of this study are:

- The magnetic field configuration is not very sensitive to the details of the plasma flow speed. This implies that a simple polytropic model can be used to understand many effects related to the magnetic field configuration.
- Alfvén waves of reasonable amplitude introduced at the base cannot produce wind speeds in agreement with the Wang-Sheeley predictions. Therefore, another acceleration process that acts between the lower corona and the source surface seems to be needed.

Here we only examine the effects of Alfvén waves within the WKB approximation on the solar wind speed. Other heat sources and sinks (such as Spitzer conductivity, or optically thin radiative losses) may also dramatically affect the calculation. If the Alfvén waves were to undergo a turbulent cascade and transfer energy to the high frequencies, then ion cyclotron resonances may provide additional acceleration (see Hollweg, [4] for further discussion). However, exact expressions for these processes, especially within an MHD model context, are not well known. The Wang-Sheeley relationship provides an additional constraint that can be used to tune representations of physical processes within an MHD model.

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