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On the Dissipation of Magnetic Fluctuations in the Solar Wind

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Abstract. Magnetic fluctuations in the solar wind show an $f^{-5/3}$ power-law spectrum below the Doppler-shifted proton cyclotron frequency but steepen to f^{-s} with $s \geq 3$ for higher frequencies. The origin of this steepening, however, remains unclear. The purpose of this study is to evaluate critically the often-employed assumption that the steepening is caused by dissipation via kinetic wave damping. For both Alfvén and magnetosonic waves and for a broad range of propagation angles, we show that the wave damping rate usually increases very strongly with wavenumber k. Consequently, the wave energy transfer always becomes slower than the damping at sufficiently high k, resulting in a strong cutoff in the power spectra rather than a steepened power-law. This result suggests that collisionless dissipation can not be the only physical basis for explaining the steepening. Furthermore, it casts serious doubts on the basic approach of treating magnetic fluctuations as an ensemble of linear waves.

1. Introduction

An ubiquitous feature of the solar wind is magnetic fluctuations, which are observed over a broad range of frequencies, from well below the proton cyclotron frequency Ω_p ($\sim 0.1 - 1$ Hz) to several hundred Hz [*Coleman*, 1968; *Gurnett*, 1991]. The observed power spectrum of the spacecraft rest frame frequency f typically shows a power-law $f^{-5/3}$ between ~ 0.001 and 1 Hz but steepens to roughly f^{-3} to several hundred Hz [cf. *Denskat et al.*, 1983; *Goldstein et al.*, 1994; *Leamon et al.*, 1998]. It is widely accepted that the $f^{-5/3}$ spectrum is the "inertial range" of MHD turbulence in the solar wind, presumably resulting from cascade processes from longer to shorter wavelengths.

The nature of the steepening near 1 Hz, however, has been a subject for intensive studies. This frequency is curiously close to the Doppler-shifted proton cyclotron frequency Ω_p. One school of thought, which is summarized nicely in Ghosh et al. [1996], appeals to change of invariants in controlling the flow of spectral energy transfer in the cascade process. In this picture, no dissipation is needed to explain the steepening. Furthermore, it is now believed that the MHD turbulence cascade is highly anisotropic, with a significant fraction of turbulent energy cascades mostly in a quasi-2D fashion, perpendicular to the background magnetic field $\mathbf{B_0}$ [*Shebalin et al.,* 1983; *Matthaeus et al.,* 1998]. How magnetic energy dissipates in this anisotropic energy cascade still remains an open question.

Another school of thought, which is referred to as the "linear damping approach" in this paper, is that this break indicates the onset of dissipation. In fact, using magnetic fluctuations to heat the solar wind plasma is a long-held hypothesis that is partly supported by the observed highly nonadiabatic proton temperature profile going away from the Sun [cf. *Richardson et al.,* 1995]. Several recent works have specifically used the collisionless damping of linear Alfvén and magnetosonic waves to account for this dissipation process [*Leamon et al.,* 1998; *Gary,* 1999; *Marsch,* 1999].

There is a subtle yet fundamental assumption in the linear damping approach, namely, that the observed magnetic fluctuations can be represented by an ensemble of linear waves near or in the dissipation range. Consequently, each linear wave, whether it is Alfvén or magnetosonic, will be damped according to its respective wave-particle interaction mechanism.

The main purpose of this study is to examine critically this linear damping approach. The basic idea is that, if the linear damping indeed serves as the dissipation mechanism, we should be able to calculate the turbulence power spectrum in the dissipation range which can then be compared with observations. We organize our paper as follows: In section 2, we describe an evolution equation for the turbulent energy $W(k)$ in wavenumber space,[1] following the study by *Zhou and Matthaeus* [1990]. We then calculate the collisionless damping rates for both Alfvén and magnetosonic waves with a wide range of parameters in section 3. In section 4, we solve the evolution equation for $W(k)$ by including three key physical processes: wave injection, cascade in k space, and collisionless damping as a function of k. The calculated $W(k)$, spanning both the inertial and dissipation ranges, is compared with the observed spectra. We then test the validity of the input physics assumptions. Implications of our results are discussed in section 5.

2. A Diffusion Approximation for Energy Transfer

Zhou and Matthaeus [1990] introduced a diffusion approximation to model the spectral energy transfer in wavenumber space. For isotropic turbulence the diffusion equation for the omnidirectional spectral density $W(k)$ is

$$\frac{\partial \tilde{W}(k)}{\partial \tau} = \frac{\partial}{\partial \tilde{k}}\left[\tilde{k}^2 \tilde{D}(k) \frac{\partial}{\partial \tilde{k}}\left(\tilde{k}^{-2}\tilde{W}(k)\right)\right] + \tilde{\gamma}(k)\tilde{W}(k) + S(k) , \tag{1}$$

where the last two terms of the right-hand side represent collisionless dissipation of the fluctuations and a source function $S(k)$ for wave energy injection, respectively [see also *Miller et al.,* 1996]. We use $S(k) = S(k_0)\delta(k-k_0)$ in this paper where k_0 is the injection wavenumber. We have written

[1] Throughout this paper we use the magnetic fluctuation energy W in wavenumber k space instead of frequency f, assuming that there is a single correspondence between the two quantities, though we consider a broad range of wave propagation angles.

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equation (1) in terms of the following dimensionless parameters: $\tilde{W}(k) \equiv (\omega_p/c)W(k)/U_B$ where $U_B = B_0^2/8\pi$, $\tau = \Omega_p t$, $\tilde{k} = kc/\omega_p$, $D(k) = v_A(\omega_p/c)^3\tilde{D}(k)$, $\gamma(k) = \Omega_p\tilde{\gamma}(k)$, ω_p is the proton plasma frequency and v_A is the Alfvén speed.

Even though equation (1) is derived from phenomenological and scaling arguments, it offers an attractive and tractable way of modeling the energy cascade process in wavenumber space. Assuming that damping is negligible for $k < k_d$, where k_d is the dissipation wavenumber beyond which $\gamma(k)$ becomes important (to be quantified), the spectrum $W(k)$ in this (inertial) range is mostly determined by $D(k)$, which depends upon the cascade phenomenology. For the Kolmogorov phenomenology, *Zhou and Matthaeus* [1990] proposed

$$D(k) = C^2 v_A k^{7/2}\left[\frac{W(k)}{2U_B}\right]^{1/2} , \qquad (2)$$

where C^2 is a dimensionless constant. Upon substituting this into equation (1) and assuming a steady state with no damping, we obtain the usual Kolmogorov spectrum in the inertial range $W(k) = W_0 k^{-s}$, where $s = 5/3$.

When damping becomes important for $k > k_d$, we can still look for power-law solutions to equation (1), as strongly suggested by the observations. Assuming that $W(k) = W_0 k^{-s}$ and using equation (2), we find, for a steady state and $k > k_d \gg k_0$,

$$\gamma = (s+2)(5-3s)\Lambda k^{(3-s)/2} , \qquad (3)$$

where $\Lambda = \Lambda(C^2, v_A, W_0)$ is a positive constant. In other words, a power-law spectrum of $W(k)$ in the presence of damping *requires* $\gamma(k)$ to be a specific power-law. Let $\gamma = -\gamma_0 k^\alpha$, then we must have

$$\left\{\begin{array}{lcl} s & = & 3-2\alpha \\ \gamma_0 & = & (s+2)(3s-5)\Lambda \end{array}\right. , \qquad (4)$$

to guarantee a power-law spectrum of $W(k)$ in the dissipation range. For example, for $s = 3$, the index α has to be 0, i.e., a constant damping rate and it has to have the right amplitude. For $s \geq 3$, α has to be negative, which is not consistent with most solutions of the linear Vlasov equation (as we will see immediately).

3. Physics of Collisionless Damping

Since the solar wind plasma is observed to be Maxwellian-like to a good approximation, we use the linear theory damping rates for $\gamma(k)$, under the assumption of a collisionless, homogeneous, magnetized electron-proton Maxwellian plasma with $T_e = T_p$. Two modes are of interest here: Alfvén and magnetosonic waves. The detailed damping physics depends on the proton β and propagation angle θ, where θ is the angle between $\mathbf{k}$ and $\mathbf{B}_0$,.

The primary damping for Alfvén mode is via the proton cyclotron resonance, which is very weak at long wavelength but becomes strong when $kc/\omega_p \sim 1$ [e.g., *Gary*, 1993, Figure 6.4; *Leamon et al.*, 1998, Figure 5]. By fitting the damping rates of different β_p and θ, we obtain a useful analytic expression

$$\frac{\gamma(k)}{\Omega_p} = -m_1\left(\frac{kc}{\omega_p}\right)^{m_2}\exp(-m_3\omega_p^2/k^2c^2) , \qquad (5)$$

where the m_j are fitting parameters. The upper panel of Figure 1 illustrates representative linear Vlasov solutions (solid dots) for $\gamma(k)/\Omega_p$ of the Alfvén mode at three different θ, and corresponding fits (solid lines) using equation (5).

Magnetosonic mode damping is via Landau resonance $\omega_r = k_\parallel v_\parallel$ [*Barnes*, 1966]. At oblique propagation, the damping is the strongest. At $kc/\omega_p \gtrsim 1$, this mode usually satisfies $\gamma \sim k^\alpha$ where $1 \lesssim \alpha \lesssim 3$, so that its short wavelength damping is similar to that of the Alfvén mode. This is illustrated in the bottom panel of Figure 1. At parallel propagation, however, damping is very weak and is limited to a finite range of k, as shown in the middle panel of Figure 1.

For the damping rates shown in Figure 1 that are calculated using $\beta_p = 0.5$, we consider them to be close to the lower limits of $\gamma(k)$ for the usual solar wind parameters. The damping rates for both Alfvén and magnetosonic waves increase with β_p. The key point we want to emphasize is the strong k dependence of $\gamma(k)$, i.e., $\gamma \propto k^\alpha$ with $1 \lesssim \alpha \lesssim 3$. This dependence will strongly damp out the waves at higher k.

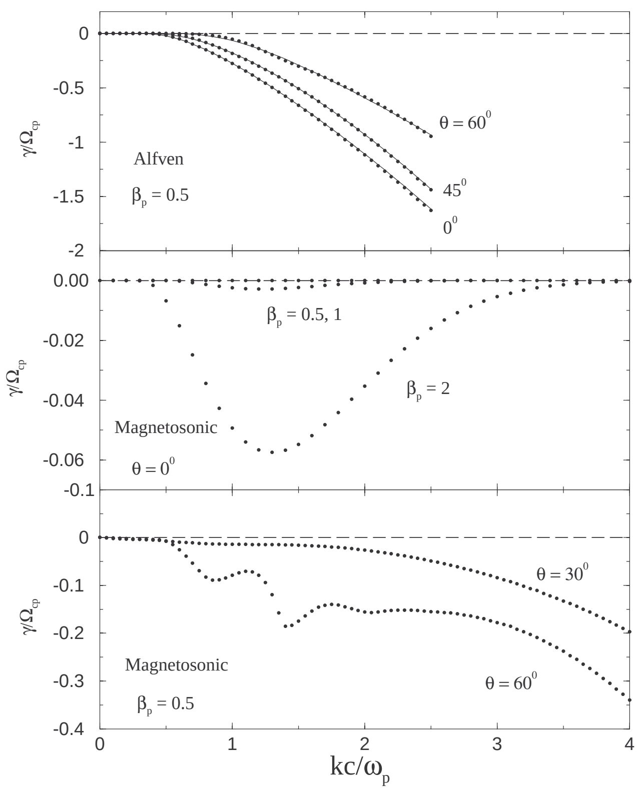

Figure 1. Linear Vlasov theory results. (*Top*) The damping rate of the Alfvén mode at $\beta_p = 0.50$ at three different angles of propagation as labeled (individual dots). The lines indicate the corresponding fits using Equation (5). At $\theta = 0^o$, $m_1 = 0.45$, $m_2 = 1.49$, and $m_3 = 0.49$; at $\theta = 45^o$, $m_1 = 0.34$, $m_2 = 1.68$, and $m_3 = 0.63$; at $\theta = 60^o$, $m_1 = 0.35$, $m_2 = 1.36$, and $m_3 = 1.66$. (*Middle*) The damping rate of the magnetosonic/whistler mode at $\theta = 0^o$ and $\beta_p = 0.50, 1$, and 2, respectively. The $\beta_p = 0.5$ curve is very close to 0. (*Bottom*) The damping rate of the magnetosonic/whistler mode at two different angles of propagation as labeled (individual dots). Although we stop our plots at $kc/\omega_p = 4$, the strong increase in $|\gamma|$ continues to at least $kc/\omega_p \sim 10$.

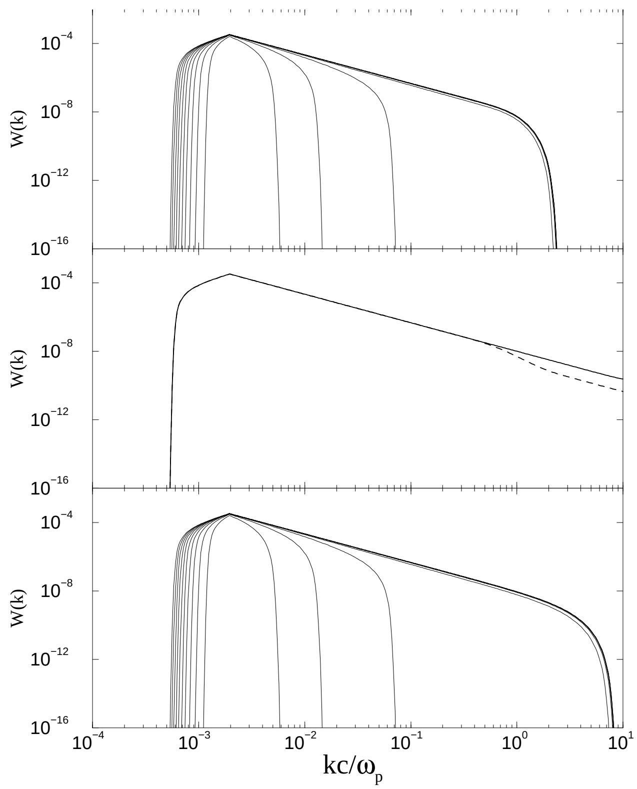

Figure 2. Computational model results from solving Equation (1). (*Top*) The temporal evolution of the power spectrum $W(k)$ as a function of wavenumber k, using the Kolmogorov diffusion coefficient (equation [2]) and the Alfvén wave damping rate of $\theta = 60^o$ and $\beta_p = 0.5$. The dark curve represents the steady state solution. (*Middle*) The steady state power spectrum $W(k)$ using the magnetosonic wave damping rates for $\theta = 0^o$ and $\beta_p = 0.5$ (solid line) and $\beta_p = 2$ (dashed line). (*Bottom*) Similar to the top panel except for using the magnetosonic wave damping rate at $\theta = 30^o$ and $\beta_p = 0.5$. The dark curve represents the steady state solution.

4. Numerical Calculations

We used a Crank-Nicholson method to solve equation (1) numerically until a steady-state is attained. We choose $B_0 = 10^{-4}$ Gauss and $v_A/c = 10^{-4}$. The fluctuation energy is injected at $k_0 c/\omega_p = 0.002$ at a rate of 10^{-15} erg cm^{-3} s^{-1} through the source function $S(k_0)$. The Kolmogorov diffusion coefficient $D(k)$ (equation [2] with $C^2 = 1$) is used. We have used a large set of damping rates, depending on the choice of the modes, β_p and propagation angle (cf. Figure 1).

Figure 2 summarizes our main results where $W(k)$ indeed follows a $k^{-5/3}$ power-law in the inertial range but always shows an exponential cutoff (or a steep roll-over) towards higher k, in contrast to an observed steepened power-law. In the top panel of Figure 2, we have used the Alfvén wave damping rate at $\theta = 60^o$ with $\beta_p = 0.5$. The spectrum cascades successively to higher k until damping cuts it off. In fact, as discussed in §2, $\gamma(k)$ given in equation (5) does not satisfy the conditions given in equation (4) for obtaining a power-law. The exponential cutoff in $W(k)$ is easily understood from the fact that the energy transfer rate in wavenumber space via diffusion always becomes slower than the strongly increasing damping rate.

For the magnetosonic waves, as long as there is appreciable damping, the resulting $W(k)$ is very similar to the Alfvén case. This is shown in the bottom panel of Figure 2, where $\theta = 30^o$ and $\beta_p = 0.5$. Since the damping rate in this case is smaller than that of the Alfvén wave, the "cutoff" k in $W(k)$ is larger. But the qualitative behavior of strong roll-over in $W(k)$ remains.

For the magnetosonic waves with very small damping (i.e., $\theta = 0$ and $\beta_p < 1$), there is no break in the steady state power spectrum $W(k)$. This is shown in the middle panel of Figure 2. For $\beta_p = 2$ where a low level of damping is present within a small range of k (cf. the middle panel of Figure 1), the steady state power spectrum $W(k)$ shows a drop but quickly recovers, continuing on with a $k^{-5/3}$ power-law since $\gamma(k)$ becomes negligible again.

One caveat in relating Figure 2 to the observations is that the observed power spectrum represents a summation over all directions of wave propagation, whereas our calculations are one-dimensional, even though we have considered the damping as a function of θ. We do not expect, however, that a summation of our results over different θ could yield a power-law spectrum in the dissipation range.

To summarize, among all the mode choices, propagation angles, and proton β, we find no regime of parameters that yield a broken power-law behavior in $W(k)$, going from the inertial to the dissipation range. Instead, we always find that $W(k)$ exhibits a strong roll-over in the high wavenumber regime ($kc/\omega_p \geq 1$). This is in sharp contrast to the often-observed power-law spectra.

5. Conclusions

We have used a simple model of Kolmogorov energy transfer in wavenumber space of magnetic fluctuations to calculate the dissipation range power spectrum in collisionless, homogeneous, isotropic, magnetized plasmas. Dissipation is provided by the linear Vlasov theory. We find that $W(k)$ shows a steep cutoff in k rather than a steepened power-law. The key physical reason for this result is the mismatch between the energy cascade rate that is governed by the diffusion coefficient $D(k)$ and the dissipation which is determined by the collisionless damping rate $\gamma(k)$. Except for the magnetosonic waves propagating parallel to $\mathbf{B_0}$ in $\beta_p < 1$ plasmas, the damping increases very strongly with k so that it completely damps out the spectral energy input from the cascade. Generally, if magnetic fluctuations can be regarded as an ensemble of linear modes, these modes will be so strongly damped near kc/ω_p and beyond, that one would not expect to obtain a "residual" power-law extending to much shorter wavelengths (or higher frequencies), in contrast to the observations. These results cast serious doubts on the fundamental assumption of treating magnetic fluctuations as an ensemble of linear waves.

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