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Reply

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1. Introduction

The three-dimensional location of Earth's bow shock under conditions of low Alfven (M_A) and magnetosonic (M_{ms}) Mach numbers, for which MHD effects are expected to modify previous gasdynamic and phenomenological models, is still of considerable interest. Recently, *Cairns et al.* [1995] (hereafter CFACPL) analyzed and discussed, in terms of existing theory, the theoretical implications of some terrestrial shock crossings for low solar wind M_A, M_{ms}, and ram pressure (P_{sw}) conditions. Even though typically $2 \lesssim M_{ms} \lesssim 4$, they showed that the standard model for the shock's standoff distance a_s needs substantial modification and/or the shock's shape itself varies significantly at low M_A and M_{ms}. Most previous workers [e.g., *Farris et al.*, 1991; *Russell and Zhang*, 1992] did not recognize that changes in shock shape with Mach number might explain unexpectedly distant shock locations. Comments and suggestions relating to CFACPL's paper have just been offered [*Russell and Petrinec*, this issue] (hereafter RP). This Reply paper constructively discusses these offerings. Specific criticisms of CFACPL's results and methodology are all refuted with simple explanations; nonzero values are emphasized for the focus offset, as well as the correctness of CFACPL's methodology. RP's suggestions, to use two-parameter conic section shock models and a recent model for a_s, are shown to be nonviable. We suggest, instead, the future use of three-parameter conic section shock models when practicable and the viability of a model for a_s based on MHD theory and simulations.

Before describing RP's statements we briefly summarize the results and methodology of CFACPL. (1) Some terrestrial shock crossings were shown to be both unusually distant (cf. the nominal shock location) and to be associated with changes in M_A, M_{ms}, and P_{sw}. (2) A three-dimensional paraboloidal shock model

$$x = a_s - b_s(y^2 + z^2) \qquad (1)$$

was used in a rotated GSE-like, Earth-centered, coordinate system with the x- axis aligned antiparallel to the solar wind velocity (relative to Earth). This model has two independent parameters, a_s (the standoff distance) and b_s (the flaring parameter). The terminator distance at $x = 0$ equals $\sqrt{a_s/b_s}$ (and is the semilatus rectum of a conic section with focus at the origin). (3) The standard, existing model for a_s as a function of P_{sw} and M_{ms} [*Formisano et al.*, 1971; *Farris et al.*, 1991] was reviewed and used; this model is obtained by phenomenologically substituting M_{ms} for the sonic Mach number M_S in *Spreiter et al.*'s [1966] avowedly gasdynamic result to yield

$$a_s \propto P_{sw}^{-1/6}\left(1 + 1.1\frac{(M_{ms}^3 + 3)}{4M_{ms}^2}\right) \qquad (2)$$

for an polytropic index of 5/3. Two existing models for b_s were described; the favored one, based on self-similarity, is a function of P_{sw} - see CFACPL's equation (4). It was argued that b_s should be a function of the solar wind Mach numbers, although such shape changes were not considered in previous analyses [e.g., *Farris et al.*, 1991; *Russell and Zhang*, 1992]. (4) The spacecraft locations and models for b_s were used to calculate the shock's apparent a_s values. (5) The standard model's predictions were then compared with the measured a_s and shown to be quantitatively inaccurate (too small) for $M_{ms} \lesssim 5$. (6) This result was interpreted in terms of either the standard model for a_s being inadequate at low M_A and M_{ms}, or b_s decreasing at low M_A and M_{ms}, or both effects simultaneously. It should be noted that this is the best and most conservative procedure currently available: ram pressure effects are normalized out for a simple, viable shock model, leaving Mach number effects to explain the differences inferred in the standoff distance and/or shock shape between the observations and the standard theoretical model.

RP's concerns are not with the data analysis or the final results, but primarily with the use of a paraboloidal model for the shock and whether this model can distinguish between changes in "shape" and size. It is worth emphasizing at this stage that RP make multiple untrue statements, including misquotations, and that it is not possible for this Reply to address every such occurrence. We concentrate on the most major items. Ordered as we discuss them, RP's comments and suggestions are the following: (1) the paraboloidal quantities a_s and b_s are not independent; (2) it is necessary to separate ram

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pressure and Mach number effects, and shape changes from size changes, and CFACPL's methodology does not do this; (3) the shock shape depends on the magnetopause shape; (4) CFACPL incorrectly conjecture that b_s does not depend on Mach number; (5) perhaps two-parameter conic section models, dependent on the eccentricity ϵ and the semilatus rectum K, would be a useful improvement over the paraboloidal model; and (6) perhaps MHD effects are not the cause of the standard model's failure at low Mach numbers, but that this failure results from an inadequate model for the magnetosheath thickness and the modified phenomenological model of *Farris and Russell* [1994] might be a viable replacement.

2. Responses

First, the paraboloidal model itself is reviewed and comment 1 refuted. Subsequently, the dependence of the paraboloidal model on the solar wind parameters and the magnetopause shape is considered and comments 2 - 4 dismissed. Suggestion 5 concerning conic section models is then generalized and made viable. Finally, suggestion 6 is addressed by showing that serious arguments exist against *Farris and Russell*'s [1994] model and by discussing an alternative model based on recent MHD work.

2.1. Two Independent Parameters for a Paraboloid

The form of (1) immediately shows the incorrectness of RP's contention (item 1) that a_s and b_s are generally not independent. There should be no need to prove this elementary fact graphically, although RP appear still not to grasp this point. Consider next the equation for a general conic section with focus on the x- axis at $x = x_0$, eccentricity ϵ, and semilatus rectum K:

$$R = K - \epsilon(x - x_0) \ , \qquad (3)$$

where R is the distance from the focus to a point on the curve and the x-y-z coordinates are defined relative to the origin. This generalizes RP's first equation, $R = K/(1 + \epsilon\cos\theta)$, with "origin" replacing "focus" in RP's text. Then, $x = K/(1 + \epsilon) + x_0 - (y^2 + z^2)/2K + O((y^2 + z^2)^2/K^3)$. Reducing to a paraboloid ($\epsilon = 1$) and comparing with (1) yields

$$a_s = x_0 + K/2 \quad , \quad b_s = 1/2K \qquad (4)$$

The parameters a_s and b_s for a paraboloid are therefore in general independent and RP's contention is again refuted. Only in the special cases that x_0 and K are not independent (a trivial special case that is not discussed further here) or that $x_0 = 0$, as assumed by RP, does a paraboloid have a single independent variable.

Is $x_0 = 0$ for Earth's bow shock? There are four arguments against this. A first, very direct argument involves existing published fits to a_s and b_s for Earth's bow shock (e.g., CFACPL) and the resulting values for x_0 and K calculated from (4) (see Table 1). For average solar wind conditions, Table 1 shows that the observed x_0 are $\sim$ 2.1 to 3.4 R_E, or $\sim 10 - 25\%$ of a_s, and so should not be ignored. A second argument involves self-similarity, which follows from writing the time-independent MHD momentum and energy equations as spatial gradients of terms of order unity plus terms that depend on M_A^{-2}, M_S^{-2}, and trigonometric functions of the angles between $\mathbf{B}_{sw}$ and the coordinate axes: for given M_A, M_S, and field orientation, the ram pressure then only affects the length scale of the shock-magnetopause interaction. Balancing P_{sw} against a dipole's magnetostatic pressure yields the standard result $a_s \propto P_{sw}^{-1/6}$ for Earth, whence $b_s \propto P_{sw}^{1/6}$ using self-similarity and (1) (CFACPL), and so $K \propto P_{sw}^{-1/6}$ and $x_0 \propto P_{sw}^{-1/6}$ via (4). Thus, in general, x_0 depends on P_{sw} (and M_A, M_S, ...) and is nonzero.

The remaining arguments against $x_0 = 0$ for Earth's bow shock are indirect. *Slavin and Holzer* [1981, and references therein] showed that three-parameter conic section fits (ϵ, K, x_0) to planetary bow shocks with $x_0 \neq 0$ are superior to two-parameter (ϵ-K) fits with $x_0 = 0$. Indeed, the best fit solutions found in subsequent work on the terrestrial and outer planets [e.g., *Slavin et al.*, 1985] all have $x_0 \neq 0$. Last, *Roelof and Sibeck* [1993] performed a detailed analysis of the magnetopause location using three-parameter conic section fits. They found, in general, that $x_0 \neq 0$ (their Figure 18a) and that substantial variations in x_0 occur for varying P_{sw} and IMF B_z component. Since $x_0 \neq 0$ for the magnetopause, it is intuitively unlikely that $x_0 = 0$ for the bow shock.

This assumption $x_0 = 0$, common in certain previous papers [e.g., *Farris et al.*, 1991] leads to systematic underestimation of the eccentricity ϵ for specified a_s and K. This can be seen by rearranging (3) with $y = z = 0$ into the form

$$\epsilon = \frac{K}{(a_s - x_0)} - 1 \ . \qquad (5)$$

Table 1. Parameters For Earth's Bow Shock

Data Source	a_s, R_E	b_s, R_E^{-1}	K, R_E	x_0, R_E
Filbert and Kellogg	14.6	0.0223	22.4	3.4
Etcheto and Faucheux	13.6	0.023	21.7	2.7
Farris et al.	13.7	0.0223	22.4	2.5
Errors for Farris et al.	±0.2	±0.0003	±0.3	±0.4

Positive, nonzero x_0 then lead to larger ϵ for measured a_s and K than if $x_0 = 0$. For example, taking $a_s = 13.5R_E$ and $K = 22.5R_E$ with $x_0 = 2.5R_E$ (see Table 1) yields $\epsilon = 1.05$, while the result for $x_0 = 0$ is $\epsilon = 0.67$. These results make it clear that the unexpectedly low value $\epsilon = 0.81 \pm 0.02$ found by *Farris et al.* [1991] for similar K and a_s may well be due to the assumption $x_0 = 0$. (See section 2.3 for another reason too.) We encourage Farris et al. to repeat their analysis using an (ϵ, K, x_0) conic section fit.

Consider now RP's repeated claim that previous work shows the near-Earth bow shock to be clearly elliptical ($\epsilon < 1$). This is blatantly untrue, as shown in *Slavin and Holzer*'s [1981] Table 4 which summarizes analyses of their data sets and reanalyses of *Fairfield*'s [1971] and *Formisano*'s [1979] data sets: of the 10 analyses considered, only 2 had $\epsilon < 1$ (with one of them having $\epsilon = 0.97$). (All 10 analyses led to $x_0 \neq 0$.) Additionally, as shown above, *Farris et al.*'s [1991] result $\epsilon = 0.81 \pm 0.02$ is plausibly due to the assumption $x_0 = 0$, since realistic $x_0 \sim 3$ in (5) imply $\epsilon \sim 1.0 - 1.3$. Experimentally, therefore, the available data favor $\epsilon \sim 1.0 - 1.3$. Thus it is entirely incorrect to claim, based on existing data and analyses with general x_0, that the near-Earth shock is significantly ellipsoidal; instead, existing data and analyses with $x_0 \neq 0$ argue that the near-Earth shock has $\epsilon \gtrsim 1.0$. Furthermore, as shown by CFACPL's Figure 11, for example, a paraboloid approximates the shock's shape quite well near to Earth.

At this time the physical reasons for x_0 being nonzero are not certain. The typical values $x_0 \sim 2 - 3.5$ for the bow shock in Table 1 are commensurate with typical L values for the ring current, and *Formisano et al.* [1971] have associated some distant shock crossings with distant magnetopause locations due to large D_{st} activity. It is therefore tempting, albeit possibly premature, to associate nonzero x_0 with the effects of the ring current and the Region 1 and 2 currents (linking the magnetopause, ionosphere, and plasma sheet), as suggested by Figures 20 and 21 of *Roelof and Sibeck* [1993].

2.2. Ram Pressure Versus Mach Number Effects and Shape Versus Size Issues

CFACPL properly separate the effects of changes in P_{sw} from M_A, M_{ms}, and M_S since MHD theory and existing observations indicate that the shock and the magnetopause respond self-similarly to P_{sw} variations and both (2) and the favored $b_s \propto P_{sw}^{1/6}$ model for the shock (see equation (4) of CFACPL) properly reproduce these variations. In particular, the apparent a_s values are calculated from the crossing locations and the $b_s \propto P_{sw}^{1/6}$ shock shape, and the theoretical model uses the accepted $a_s \propto P^{-1/6}$ variations: differences between the observed and predicted a_s are thus due to a_s and/or b_s varying with the Mach numbers in ways not accounted for by the standard models. Any suggestion that P_{sw} variations are not extracted properly is therefore incorrect, i.e., RP's item 2. To the contrary, CFACPL's methodology was specifically designed to isolate and normalize out P_{sw} effects, so that Mach number effects could be identified uniquely.

The noun "shape" can be interpreted in several ways. Two relevant definitions, of the eight in the 1979 *Concise Oxford Dictionary*, are the following: "the configuration (or) external form ... produced by a thing's outlines", as in the shape being the shock's entire three-dimensional location; and a "specific form (or) embodiment", as in the surface being a paraboloid versus a hyperboloid etc. CFACPL primarily use "shape" in connection with the shock's global location (i.e., the first definition) and so refer to a paraboloid equation and the two parameters a_s and b_s. In this case, fixing the shock's nose using a_s the shock's global location or 'shape' is determined by b_s, which therefore acts as a shape parameter (and a size parameter since $K = 1/2b_s$). Since CFACPL discuss changes in shock shape primarily in terms of b_s, their usage is viable. RP instead use the second definition since ϵ is referred to as the shape parameter. This is also acceptable (although the assumption $x_0 = 0$ is not; see sections 2.1 and 2.3). Thus changes in shape can be modeled in terms of b_s and/or ϵ. Reasons for preferring one way of modeling shape over the other are given in section 2.3.

With respect to RP's item 3, the magnetopause shape can indeed affect both the shock's shape and standoff distance (e.g., by varying the factor 1.1 in (2)). However, self-similar variations of the magnetopause with P_{sw}, predicted by MHD theory and also observed [e.g., *Sibeck et al.*, 1991], alter the shock's shape self-similarly and do not modify the factor 1.1 in (2). Different IMF B_z components do modify the magnetopause shape [e.g., *Farris et al.*, 1991; *Sibeck et al.*, 1991] but the changes to (2) have not been quantified. CFACPL discussed their events in terms of existing theories.

Turning now to variations of b_s with M_A, M_S and M_{ms}, we note that nowhere did CFACPL conjecture that b_s is independent of Mach number; *RP's* item 4 is a blatant falsehood. Indeed, a major theme throughout CFACPL's paper, considered and expanded upon repeatedly, is that b_s should depend on the Mach numbers and that such variations in b_s can explain their results. Even casual reading of CFACPL will reveal this. The most favorable possible view of RP's item 4, and their attempt to support it, is that RP misread a portion of CFACPL's text and did not place it in context.

Expanding on how b_s varies with the Mach numbers, note that CFACPL specifically addressed (section 7.2) the maximum change in $b_s(P_{sw}, M_{ms})$ at low $M_{ms} \sim 2.4$, estimating $b_s(P_{sw}, M_{ms} = 2.4) = (0.60 \pm 0.22) \times b_s(P_{sw}, M_{ms} = 5.4)$. This implies that low M_A and M_{ms} effects cause substantial changes in shock shape. RP's remarks concerning CFACPL's Figure 10 show they have misread the text in section 7.2. That figure demonstrates, qualitatively, the significant effects of the shock shape depending on the MHD Mach numbers, while section 7.2 describes an attempt to estimate the maximal changes in a_s or b_s that separately bring the observations into agreement with existing models (while emphasizing that a_s and b_s most likely vary simultaneously and differently with M_A, M_S, etc. than the standard models). The α_0 curve in Figure 10 retains the standard shape b_s for the shock while a_s is enhanced, thereby having the same K but a different x_0 from the standard model, while the β_P curve has a different shape (b_s) from the nominal shock model but the value a_s predicted for the standard model, thereby having different K and x_0. Thus, by design, the α_0 curve can be translated onto the standard model, while the β_P curve has a different shape and cannot overlie the standard model. RP's initial text mislabelled the α_0 curve the (nonexistent) β curve and RP persist in misreading CFACPL's text and figure.

2.3. Use of ϵ and General Conic Sections

Fitting observational data to a general conic section model (3) with free parameters (K, x_0, ϵ) is a more general procedure than a paraboloidal fit. It is therefore preferable unless observational uncertainty or theoretical issues argue contrarily. Instead, however, RP suggest (item 5) using (3) with $x_0 = 0$; that is, a two-parameter model. This suggestion is not viable according to the arguments in section 2.1 and the material two paragraphs below. Moreover, RP's suggestion does not increase the number of fit parameters over the use of a paraboloid; instead, only the functional form of the model is changed.

We suggest that the future use of three-parameter fits to (3) would be, in theory, an improvement over the use of paraboloid models. There would then be a natural separation between size scales (K and x_0) and the shape parameter ϵ, as noted by RP. At present, however, it is important to realize two things. First, with $a_s(P_{sw})$ and $b_s(P_{sw})$ known, there is no benefit to using (3) for describing P_{sw} effects. Second, CFACPL performed their analysis when neither theoretical nor observational models existed for how b_s or K or x_0 depend (separately) on the Mach numbers. (Nor are we aware of such models now.) Since models already existed for $a_s(P_{sw}, M_A, M_{ms})$ and $b_s(P_{sw})$, and since paraboloid models often lie very close to hyperboloid models (e.g., CFACPL's Figure 10), CFACPL therefore adopted the conservative approach of normalizing out P_{sw} variations and testing existing models for how a_s and b_s vary with the Mach numbers. The resulting analyses produced clear evidence for changes in a_s and/or b_s at low Mach numbers not predicted by existing models. Once more advanced three-parameter shock models are available, improvements over CFACPL's conservative analysis should be possible.

The large variations in a_s and/or b_s at low Mach numbers found recently [*Russell and Zhang*, 1992; CFACPL; *Cairns and Lyon*, 1995] suggest that observers need to be aware of the dangers of overbinning their data (i.e., too large a range of M_A in each bin, especially at small M_A values), as well as the issue of statistical reliability (i.e., are there enough crossings in each bin?). A possible illustration of overbinning may exist in *Farris et al.*'s [1991] paper: despite the theoretical expectation that bow shocks be hyperboloids ($\epsilon > 1.0$) rather than ellipsoids [e.g., *Slavin and Holzer*, 1981; *Slavin et al.*, 1985], *Farris et al.*'s [1991] fit with $x_0 = 0$ for 351 crossings yielded $\epsilon = 0.81 \pm 0.02$. This may be due to averaging over too wide a range of solar wind conditions (i.e., the fit to multiple crossings of widely varying hyperboloids was an ellipsoid). Alternatively, this low value of ϵ may be due to the restrictive and inappropriate assumption $x_0 = 0$ made by Farris et al. (see section 2.1). We suggest that Farris et al.'s analysis be redone using general (ϵ, K, x_0) conic section fits.

2.4. Importance of MHD Effects and the Farris-Russell Model

The model of *Farris and Russell* [1994] (hereafter FR) for a_s involves a number of important steps. The first involves replacing *Spreiter et al.*'s [1966] relation $\Delta_{ms}/a_{mp} \propto X = \rho_{sw}/\rho_d$, with X given as a function of M_S by gasdynamic theory, that underlies the standard model (2) for a_s. (Here Δ_{ms} is the magnetosheath thickness, a_{mp} is the magnetopause standoff distance, and ρ_{sw} and ρ_d are the asymptotic plasma densities upstream and downstream from the shock.) Instead, FR suggested the replacement $\Delta_{ms} \propto M_2^2/(M_2^2 - 1)$, where M_2 is the downstream sonic Mach number. This was justified by noting that Spreiter et al.'s relation is recovered for large M_S and, they intuited, that the shock should move to ∞ as M_S and $M_2 \to 1$. Note that the shock motion argument is inconclusive: while the shock itself must disappear as $M_S \to 1$, analogies with water waves, etc. suggest that a bow wave structure located a finite distance from the obstacle is likely relevant for $M_S \lesssim 1$. It has also not been proven that FR's replacement is either valid at small M_S or unique. FR do not give an explicit MHD formula for a_s. On the basis of their previous work [e.g., *Farris et al.*, 1991] it is likely they envisaged making an "MHD" model solely by replacing M_S with M_{ms}. The analogous assumption in creating the model (2) [*Formisano et al.*, 1971; *Farris et al.*, 1991] has been shown to have little theoretical foundation [*Cairns and Grabbe*, 1994; *Cairns and Lyon*, 1995]. Instead, *Cairns and Grabbe*[1994] showed that a_s generally depends on M_A, M_S, and the magnetic field orientation and that these dependences cannot be subsumed totally into M_{ms}.

Recent global gasdynamic (GD) [*Spreiter and Stahara*, 1995] and MHD simulations [*Cairns and Lyon*, 1995] (hereafter CL) provide an opportunity to test whether FR's model is viable. Elsewhere *Cairns and Grabbe* [1996] show that FR's model provides a very poor fit to *Spreiter and Stahara*'s [1995] GD simulation

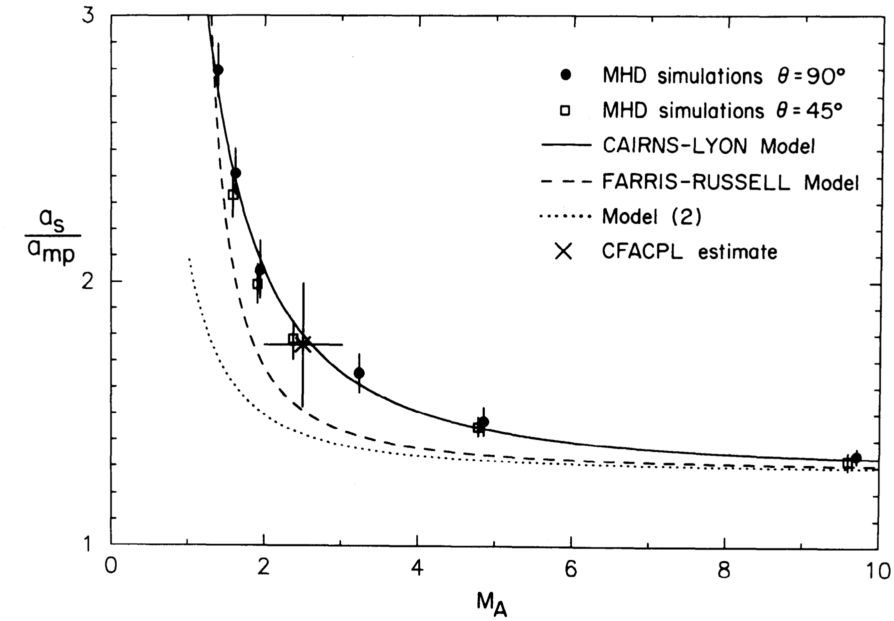

Figure 1. Comparison of models for the standoff distance with simulation results. The ratio of a_s to a_{mp} is plotted versus M_A for $M_S = 7.6$ and $\gamma = 5/3$. The circle and square symbols show the simulation results of *Cairns and Lyon* [1995] for $\theta = 90°$ and 45°, respectively. The dashed line corresponds to *Farris and Russell*'s [1994] model with M_S replaced by M_{ms} for $\theta = 90°$, while the solid and dotted lines show *Cairns and Lyon*'s [1995] model and the standard model (2) for $\theta = 90°$, respectively.

results, especially at low M_S where the best agreement is predicted. In particular, FR's model predicts a_s values $\sim$ 85% too high at $M_S = 1.15$ and $\sim$ 13% too high at $M_S = 6$. FR's model therefore appears unable to account for the GD simulation results.

A similar conclusion follows on comparing CL's simulation results with FR's model (with M_S replaced by M_{ms}). Figure 1 shows the simulated a_s values, normalized by a_{mp}, as a function of M_A for MHD runs with quasi-perpendicular magnetic field orientations $\theta = 45$ and 90°, $M_S = 7.6$, and polytropic index $\gamma = 5/3$ (CL). The dashed line shows FR's prediction for $\theta = 90°$ and identical M_S and γ (the $\theta = 45°$ curve lies below that for $\theta = 90°$). As expected, the model agrees well at large $M_A \sim 10$. FR's model is clearly not viable for the important range $1.5 \lesssim M_A \lesssim 5$, being well below the error bars for the simulation. (The model's viability at $M_A \lesssim 1.5$ is unknown.) Note that $a_{mp} \sim 10\ R_E$, so that an error of 30% in Figure 1 is an unacceptable error of $\sim 3R_e$ in a_s. CL's additional important MHD simulation results include the recovery of the observed three-dimensional shock location for average solar wind parameters, good agreement between the density jumps X and Rankine-Hugoniot theory, and close reproduction of *Spreiter et al.*'s [1966] linear relation $\Delta_{ms}/a_{mp} = 1.1X$ for quasi-gasdynamic (GD) runs with $M_A \gg M_S$. CFACPL's observational estimate (star symbol) for the maximum a_s value for $M_{ms} \sim 2.4$ agrees well with the simulations.

It thus appears necessary to discard FR's model due to its lack of consistency with both GD and MHD simulations. The model (2) with $M = M_{ms}$ for $M_A \lesssim 5$ should also be discarded, as CFACPL (and *Russell and Zhang* [1992]) concluded on observational grounds. In contrast, the solid line shows CL's model, which uses the relation $\Delta_{ms}/a_{mp} = 3.4X - 0.6$ found in their MHD simulations and uses MHD theory to specify $X(M_A, M_S, \theta)$ following *Cairns and Grabbe* [1994]. (The different Δ_{ms} - X relations in CL's quasi-GD and MHD simulations also argue that substituting M_{ms} for M_S does not yield a correct MHD theory.) Very good agreement exists between the simulation results and associated model. CL also showed that θ effects are important and cannot be subsumed into M_{ms} effects alone. In summary, whereas RP's suggested model suffers severe difficulties, CL's model appears viable and should be tested observationally.

In passing, we note some incorrect claims in the last paragraph of RP's Comment that are not related to the paper of CFACPL. First, CL's result that a_s increases at low M_A over and above the prediction (2) does not prove that *Cairns and Grabbe*'s [1994] approach, that $\Delta_{ms} \propto X(M_A, M_S, \theta)$, is incorrect. Indeed, to the contrary: CL's model appears very successful in Figure 1, yet differs from Cairns and Grabbe's model only in that relation $\Delta_{ms}/a_{mp} = 1.1X$ found in gasdynamic simulations has been replaced by the relation $\Delta_{ms}/a_{mp} = 3.4X - 0.6$ found in CL's MHD simulations. Second, CL's simulations do not show that $a_s/a_{mp} \to \infty$ as $M_A \to 1$ contrary to RP's claim. Instead, (1) CL's simulations have $M_A \gtrsim 1.5$ and $a_s/a_{mp} \lesssim 3$, and (2) CL's model, which agrees very well with the available simulations, has $a_s/a_{mp} \to 3.8$ (not ∞) as $M_A \to 1$. Third, the simulation model recovers the result $\Delta_{ms}/a_{mp} \sim 0.25 \sim X$ in the gasdynamic limit. Fourth, unpublished analyses show that the velocity and magnetic field jumps in CL's simulations are consistent with the MHD Rankine-Hugoniot conditions, as found for the density jumps. Fifth, the differences in Figure 1 between FR's model and CL's simulations are large compared with the error bars. Finally, changes in shock shape undoubtedly exist even for $M_{ms} \gtrsim 4.5$.

3. Summary

RP's comments concerning the number of independent parameters in paraboloid shock models, the separation of ram pressure and Mach number effects, the dependence of b_s on Mach number, and size versus shape issues are all incorrect. It is demonstrably wrong to assume, as RP do, that Earth's bow shock has zero focus offset x_0; this assumption is inconsistent with published shock data and can lead to unrealistically low values of ϵ. RP's suggestion to use two-parameter (ϵ, K) conic section shock models (with $x_0 = 0$) instead of paraboloids is thus not viable. We suggest that three-parameter (ϵ, x_0, K) conic section models should eventually be a theoretical improvement over paraboloid models. Further work is necessary to realize this improvement since the variations of x_0 and K with Mach number must be specified. Finally, *Farris and Russell*'s [1994] model for a_s, suggested relevant by RP, is inconsistent with recent gasdynamic and MHD simulations. Instead, we suggest that CL's model for a_s appears viable and should be tested observationally.

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