JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 101, NO. A4, PAGES 7677–7678, APRIL 1, 1996

Comment on "Unusual locations of Earth's bow shock on September 24-25, 1987: Mach number effects" by I. H. Cairns, D. H. Fairfield, R. R. Anderson, V. E. H. Carlton, K. I. Paularena, and A. J. Lazarus

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In their recent paper, *Cairns et al.* [1995] conclude that when the magnetosonic Mach number is low there can be substantial changes in the standoff distance and/or shock shape. The purpose of this note is not to disagree with that statement which is in accord with our earlier work [*Russell and Zhang*, 1992] but to suggest a method whereby these two changes can be separated from each other so that their analysis can be made less ambiguous.

Over the forward part of the magnetosheath the flow is subsonic, and much of the shock remains in communication with much of the obstacle. Thus the shape of the obstacle plays an important role in determining the shape of the shock, and hence the forward part of the terrestrial shock is found to be elliptical in contrast to the hyperbolic shape it is expected to have far behind the Earth. The shape of the obstacle also plays a role in determining the standoff distance. A pointed obstacle would have a much smaller standoff distance than a sphere. A sphere would have a smaller standoff than an ellipse (with the Earth at the focus), etc. Physically, the standoff distance depends on the radius of curvature of the obstacle at the stagnation point in the flow and thus is insensitive to the choice of the origin of the coordinate system. *Spreiter et al.* [1966] gave a standoff formula as a function of Mach number for a specific shape, an elliptical, magnetosphere-like obstacle, with distance measured from the center of the Earth. All later work including that of *Cairns et al.* [1995] has followed this procedure of measuring the standoff distance from the center of the Earth rather than comparing with the radius of curvature of the obstacles thus making the standoff ratio coordinate-system dependent.

The reason the shock front sits anywhere is to allow all the shocked plasma to pass between the shock front and the obstacle. If the plasma is compressed less as when the Mach number drops to low values, then the shock must move away from the obstacle. The failing of *Spreiter et al.*'s [1966] formula is that it does not predict this motion correctly at very low Mach numbers. Since the shock should be at infinity when there is only an infinitesimal compression of the flow in order that the (slightly) shocked flow can move around the obstacle, and since this statement is true for a gas or a plasma, we need not look to an MHD effect to explain the failing of Spreiter et al.'s formula. *Farris and Russell* [1994] proposed an alternate formula equally valid for gas and magnetohydrodynamics that appears to redress this failing. More tests of this formula are certainly in order. However, in order to carry out such tests it is important to separate dynamic pressure effects from Mach number effects and shape changes from size changes. The methodology of *Cairns et al.* [1995] does not do this adequately.

The equation of a conic section is

$$R = K/(1 + \varepsilon \cos\Theta) \tag{1}$$

where R is the distance from the origin to the curve; K is the semilatus rectum, the point on the curve at right angles to the line joining the origin (at the focus) and the "nose" of the conic section; ε is the eccentricity of the ellipse; and Θ is the angle between the line joining the nose and the origin and the line joining the origin and any point on the curve. The parameters R and K have dimensions of length; Θ has dimensions of angle; and ε is dimensionless as befits a shape parameter. The eccentricity ε is greater than 1 for a hyperbola; equal 1 for a parabola; and less than 1 for an ellipse. *Cairns et al.* [1995] assume $\varepsilon=1$, despite the evidence from previous work that ε is significantly smaller than 1 [*Holzer et al.*, 1966; *Slavin and Holzer*, 1981; *Farris et al.*, 1991]. This formula may be rewritten for $\varepsilon=1$.

$$X = K/2 - (y^2 + z^2)/(2K) \tag{2}$$

If we equate this formula with that of *Cairns et al.* [1995a] with its focus at the Earth, we find that $a_s=K/2$ and $b_s=1/(2K)$, i.e., $a_s/b_s=K^2$ and $b_s=1/(4a_s)$. Thus, if the focus of the shock remains fixed, a_s and b_s are related and are not independent variables. Since we expect that the size of the shock should vary with the dynamic pressure because the obstacle size varies with dynamic pressure, it is not surprising that the parameter b_s is found to vary too. If a_s varies, then b_s must vary also, contrary to *Cairns et al.*'s [1995, p.49] conjecture. If one relaxes the assumption that the focus of the parabola is at the Earth, then the parabola may be fit to the data by translation along the solar wind flow direction. That solution is tried by the authors in Figure 10, curve α_o, but as can be seen by inspection the "shape" of the parabola does not change in this process even though a_s and b_s vary as the focus is moved. In fact, it is trivial to prove by substituting $x^1= (x-x_o)$ in their equation for a_s that, if b_s is constant, the act of changing a_s is equivalent to translation of the paraboloid. We note that

Paper number 95JA03711
0148-0227/96/95JA-03711$02.00

throughout their paper *Cairns et al.* [1995] solve for only one parameter assuming the value of the other. Nowhere do they solve for two or as they now advocate three parameters.

In order to separate the effects of shape changes, we recommend the use of the eccentricity as used by many previous authors [*Holzer et al.*, 1966; *Slavin and Holzer*, 1981; *Russell*, 1985]. Since it is dimensionless, it does not vary with the scale size as does the parameter b_s and will be much less sensitive to changes in dynamic pressure than b_s. We urge the authors to repeat their analysis to examine this shape parameter, ε. We do not recommend keeping the eccentricity fixed at unity as *Cairns et al.* [1995] do and changing the location of the focus of the parabola, since it is clear the eccentricity of the shock is not unity and can vary with Mach number. Only then will we be able to tell how dependent is the shape of the forward part of the shock on Mach number.

In the following reply, *Cairns et al.* [this issue] present new simulation results [*Cairns and Lyon*, 1995]. We note first that the shock location moves to very large standoff distances at low Mach numbers as expected [*Russell and Petrinec*, 1996], showing that the treatment of *Cairns and Grabbe* [1994] is an incorrect approach since their solution maximizes at an a_s/a_{mp} of 2.1 rather than going to infinity as the Cairns and Lyon simulation does. *Cairns et al.* [this issue] plot the shock position versus Alfvenic Mach number and not the magnetosonic Mach number. When the more physical comparison is made with magnetosonic Mach number, the simulation result is qualitatively the same as Farris and Russell's model but still on the high side. In contrast, the low Mach number gas dynamic simulation of *Spreiter and Stahara* [1995] is qualitatively similar to the Farris and Russell model but on the low side. Since the new MHD model appears higher than the other models even at the noncontroversial higher Mach numbers, we think the problem may be with the Cairns-Lyon model. Furthermore, inspection of Figure 2 of *Cairns and Lyon* [1995] suggests the problem is in the simulation itself. For a perpendicular shock, the fractional jump in density, in magnetic field strength, and the drop in velocity should all be equal, but they are not. The density has the correct jump for the stated conditions but the magnetic field jump and velocity drop are incorrect. If these differences are indicative of the uncertainty in the simulation, the differences shown between the Farris and Russell model and the Cairns-Lyon simulation are inconsequential and are very supportive of the Farris and Russell model. On a more minor note, we point out that the reason *Farris et al.* [1991] did not consider shape changes is that they restricted their attention to $M_{ms} \geq 4.5$.

Acknowledgment. This work was supported by the National Aeronautics and Space Administration under research grant NAGW-3477.

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(Received March 20, 1995; revised October 10, 1995; accepted November 27, 1995.)