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The role of nonlinear interaction in the formation of LF whistler turbulence upstream of a quasi-perpendicular shock

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Abstract. The role of the nonlinear interaction in developed low-frequency turbulence has been experimentally studied upstream of the ramp of a quasiperpendicular shock. The study has been carried out by application of the methods of bispectral analysis and wavelet decomposition to the Active Magnetospheric Particle Tracer Explorer Ion Release Module and Interball Tail Probe magnetic field data. It is shown that three-wave processes play a key role in the formation of the spectrum of the turbulence. Experimental results presented and previous theoretical considerations lead to the conclusion that energy is transfered from a narrow maximum frequency pumped by nonlinear dynamic processes to lower and higher frequencies.

1. Introduction

Low-frequency oscillations of the magnetic field upstream of the Earth's bow shock have been observed since the very first satellite experiments. Upstream of the quasiparallel part of the Earth's bow shock they are generated by the beams of ions reflected from the shock front. For quasiparallel shock geometry the magnetic field does not prevent the reflected ions from escaping into the far upstream, thus these waves can be observed at large distances (up to a few Earth radii R_e) from the front itself.

In contrast, upstream of a supercritical, quasiperpendicular shock, low-frequency oscillations in the frequency range $10^0 - 10^1$ Hz are observed only close to the shock, at distances which are of the order of the ion larmor radius (a few hundreds kilometers for the Earth's bow shock). These oscillations were identified as whistler waves [*Fairfield*, 1974]. Numerous theoretical models have been worked out for the generation mechanism of these waves [*Balikhin et al.*, 1997, and references therein]. All proposed models could be subdivided into two types. Models of the first type consider these waves to be a result of the numerous plasma instabilities. Other models attribute the generation of these waves to dynamics of the shock front and consider them as a part of the internal structure of a shock, some kind of nonlinear, nonstationary analogue of whistler precursors of a dispersive subcritical shock. Recently the joint wave number-frequency spectrum for these waves was experimentally determined on the basis of Active Magnetospheric Particle Tracer Explorer (AMPTE) United Kingdom Subsatellite (UKS) and Ion Release Module (IRM) measurements. That determination allowed the validation of the theoretical models proposed [*Balikhin et al.*, 1997]. That led to the conclusion that the observed waves can be generated as a result of the nonstationarity of the shock front itself [*Krasnosel'skikh*, 1985; *Balikhin et al.*, 1997], or via an instability of nongyrotropic proton distributions [*Wong and Goldstein*, 1988]. Both mechanisms can only explain wave spectra with a single maximum. Indeed, the experimental spectra obtained upstream of the ramp usually have one prominent maximum with $(\Delta f/f) < 1$. However, at frequencies below and above this primary maximum, other maxima are observed which are less prominent. The present paper provides quantitative evidence for the existence of a nonlinear interaction between waves observed in these main and secondary maxima. It shows that waves which correspond to the secondary maxima are the result of energy transfer from the main spectral component.

The importance of such a study is related to the main process which takes place at the front of a collisionless shock: the transfer of the kinetic energy of bulk plasma motion into other degrees of freedom. If the low-frequency turbulence upstream of the ramp of a supercritical quasiperpendicular shock consists of only whistler waves generated as a result of shock nonsta-

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tionarity and the product of their nonlinear wave coupling, then the spatial-temporal properties of this turbulence contain important information about the shock front nonlinear dynamics. However, in that case the role of this turbulence in the processes of energy redistribution would be limited by its participation in the reflection and acceleration of a portion of the incoming electrons [*Balikhin et al.*, 1989]. Alternatively, if the numerous maxima represent the result of competing mechanisms, then the waves which correspond to the secondary maxima could play a different role in energy redistribution. One of these secondary maxima could correspond, for example, to lower-hybrid waves which propagate almost perpendicular to the magnetic field and can be generated by the beam of reflected ions as suggested by *Vaisberg et al.* [1983]. The wave vectors of such waves should be directed almost perpendicular to the upstream magnetic field $k_{\|} << |k|$; therefore these waves can be in resonance both with ions via $\omega = k_{\perp} V_{i \text{ bulk}}$ and electrons $\omega = k_{\|} V_{e \text{ thermal}}$. Even a small level of such turbulence can provide an effective tool for transfer of momentum and a mechanism for the redistribution of energy at the shock front [*Vaisberg et al.*, 1983].

In the present paper the nonlinear interactions are studied by means of a bispectral analysis. This provides a measure of the extent of statistical dependencies of the three spectral components by examination of their phase coherence [*Kim and Powers*, 1979]. This technique has been successfully applied in the studies of turbulence in space, laboratory and numerical plasmas [*Kim et al.*, 1989; *Lagoutte et al.*, 1989; *Ohnami et al.*, 1993]. In particular, significant results by means of bicoherence were obtained in the study of nonlinear wave-wave interaction between signals from a ground-based transmitter and narrowband ELF emissions in the subauroral ionosphere using Auroral 3 and ISIS measurements [*Tanaka et al.*, 1987; *Ohnami et al.*, 1993].

The traditional estimators of bicoherence are based on Fourier decomposition. Such techniques require long intervals of stationary data. However, the average duration of observation of low-frequency waves upstream of the shock ramp is about 1 min [*Fairfield*, 1974]. The intervals of stationarity for this turbulence are even shorter. Therefore a more robust estimator based on the continuous wavelet transform (CWT) [*Dudok de Wit and Krasnosel'skikh*, 1995] has been used in the present study.

Data obtained from the AMPTE IRM and Interball Tail magnetometer in the vicinity of the Earth's bow shock are analyzed in the present paper. The magnetic field experiments on AMPTE IRM (PI H. Lühr) and Interball are described by *Lühr et al.* [1985] and *Klimov et al.* [1995]. The analysis of the data from two different magnetometers helps to eliminate the possibility that nonlinearities are introduced by the instrument itself. The shock normals were calculated using the model described by *Farris et al.* [1991].

2. Bispectral Analysis

The goal of the analysis performed was to identify whether the data contain signatures which indicate the presence of nonlinear three-wave coupling processes. Three wave-coupling is a nonlinear process in which $(\omega_1, \vec{k}_1) \leftrightarrow (\omega_2, \vec{k}_2) + (\omega_3, \vec{k}_3)$ and is usually characterized by the presence of three peaks in the Fourier spectrum corresponding to frequencies ω_1, ω_2, and ω_3 where $\omega = 2\pi f$ and $\vec{k}$, are the angular frequency and wave vector respectively, and f is the frequency. The frequencies and the corresponding wave vectors must satisfy the resonance conditions [*Sagdeev and Galeev*, 1969]:

$$\begin{aligned} \omega_1 &+ \omega_2 = \omega_3 \\ \vec{k}_1 &+ \vec{k}_2 = \vec{k}_3 \end{aligned} \quad (1)$$

It is worth noting that the resonance condition for the frequency should be satisfied in any inertial frame. A direct consequence of (1) is that the phases ϕ_i of interacting waves should obey the following relation:

$$\phi_1 + \phi_2 - \phi_3 = \text{const} \quad (2)$$

If such a phase relation is statistically established, this is a definite sign of a nonlinear interaction between the corresponding waves [*Kim and Powers*, 1979].

The bicoherence function is a tool to validate this phase relation for a stationary signal. Let us consider some real, stationary signal $X(t)$. The bispectrum $\mathcal{B}(f_1, f_2)$ of $X(t)$ is defined as

$$\mathcal{B}(f_1, f_2) = < X(f_1) X(f_2) X^*(f_1 + f_2) >$$

where $X(f_i)$ is the Fourier component at frequency f_i, the asterisk denotes complex conjugation, and brackets denote ensemble averaging. The bicoherence function $b(f_1, f_2)$ is the normalized bispectrum [*Kim and Powers*, 1979]:

$$b^2(f_1, f_2) = \frac{|\mathcal{B}(f_1, f_2)|^2}{< |X(f_1) X(f_1) X^*(f_1 + f_2)| >^2}$$

Such a normalization was used by *Kravtchenko-Berejnoi et al.* [1995]. The value of the bicoherence function depends upon the strength of the nonlinear interaction and varies between 0 and 1. The usual approach to the estimation of bicoherence consists of subdividing the data set into n ($n > 10$) subsets. Each of these subsets is treated as an ensemble. Increasing the number of ensembles leads to the decrease in the length of these subsets and so reduces the frequency resolution. These contradictory requirements, to provide statistically reliable averaging over a large number of ensembles and to provide a high frequency resolution, make the estimation of the bicoherence by the procedure described difficult for short data sets. A new method for such an estimation, which makes use of CWT, was proposed by *Dudok de Wit and Krasnosel'skikh*, [1995]. CWT of $X(t)$ at a scale a is defined as

$$X_{\mathrm{CWT}}(a,\tau) = \int_{t_1}^{t_2} X(t)\frac{1}{\sqrt{a}}h^*\left(\frac{t-\tau}{a}\right)dt$$

For a Morlet wavelet given by

$$h(t) = (1/\pi^{\frac{1}{4}})\ exp\ (-2\pi jt)\ exp\ (-t^2/2)$$

the scales can be directly related to an instantaneous frequency $f = 1/a$. The bispectrum estimate introduced by *Dudok de Wit and Krasnosel'skikh,* [1995] is

$$\mathcal{B}(f_1, f_2) = < X_{\mathrm{CWT}}(f_1) X_{\mathrm{CWT}}(f_2) X^*_{\mathrm{CWT}}(f_1+f_2) >$$

where the scales have been expressed in terms of frequencies. The statistical reliability of estimates based on CWT for short data sets is higher than for estimates based on fast Fourier transform (FFT) at the expense of reduced frequency resolution at higher frequencies.

3. IRM Shock on Day 364/1984

The magnetic field profile measured by IRM on day 364/1984 during a crossing of the Earth's bow shock is displayed in Figure 1. The sampling rate for this time interval was 32 Hz. The profile of the magnetic field possesses the prominent features of a supercritical quasiperpendicular shock, namely a well determined-ramp and overshoot. The ramp was crossed at 0429:47 UT. The position of the crossing was (3.2, -18.2, -0.9) R_e in GSE coordinates. The normal to the shock front, calculated using the model described in *Farris et al.,* [1991], was $\vec{n} \approx (0.7, -0.71, -0.03)$. The parameters of the upstream solar wind flow were the following: density $n_{\mathrm{up}} \approx 4\ cm^{-3}$, upstream solar wind velocity $V_{\mathrm{up}} \approx 640\ km/s$, the magnitude of the magnetic field $|\vec{B}_{\mathrm{up}}| \approx 8.5\ nT$. The shock is supercritical and quasiperpendicular because its Alfven Mach number $M_a \approx 4.5$ and the angle between the direction of the upstream magnetic field and the direction of the model shock normal $\vec{n}$ is $\theta_{Bn} \approx 50°$.

Oscillations in the magnetic field with periods about $0.3 - 0.5$ seconds persist for approximately 20 s upstream of the ramp. The amplitudes of these oscillations are higher in the y and z components than along the x

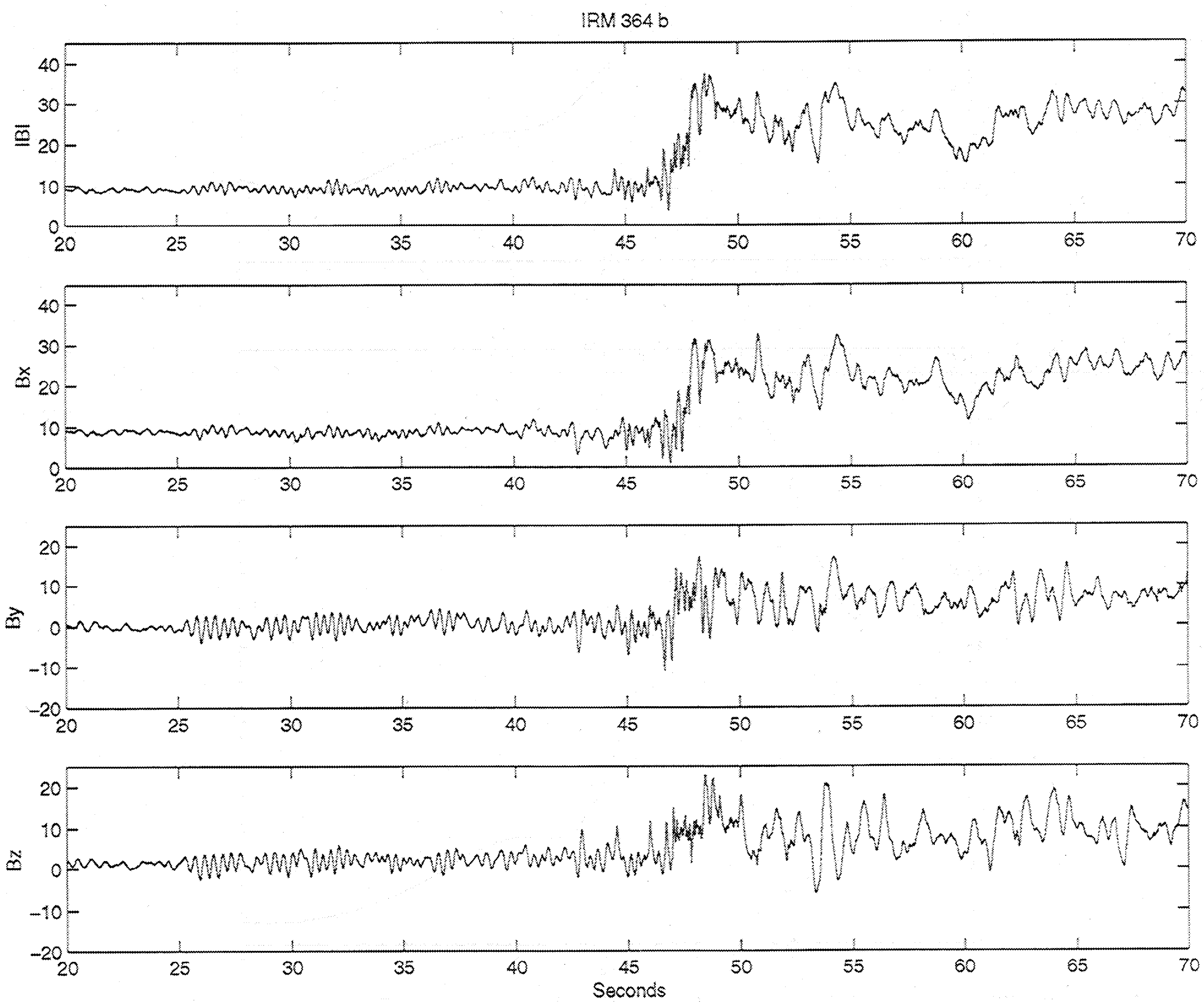

Figure 1. Absolute value and three components of magnetic field as measured by IRM during bow shock crossing which occurred at about 0429:48 UT on the day 364/1984. Time scale is in seconds after 0429:00 UT. The moment of the ramp crossing occurs ≈ 0429:48 UT.

axis. The oscillations have considerable amplitude, δB, which sometimes exceeds $3.5nT$, which is more than 40% of the magnitude of the upstream magnetic field. Therefore it is likely that nonlinear effects should play a considerable role in the evolution of these fluctuations. The oscillations disappear downstream of ramp.

The spectra of fluctuations, calculated making use of the CWT (Morlet) decomposition, during the time interval 0429:21.5-0429:32.5 UT for the three components of the magnetic field are displayed in Figure 2. Spectra are displayed on a logarithmic scale in arbitrary units. The frequency $f \approx 2.5$ Hz corresponds to the main maximum. As a result of the direction of propagation of the corresponding waves, which is almost along the GSE x axis, this maximum is most prominent in the components B_y and B_z. In addition to the main maximum, other local maxima are found at $f \approx 5$ Hz, $f \approx 1.0$ Hz and at the high frequency edge of the spectra at $f \approx 8.0$

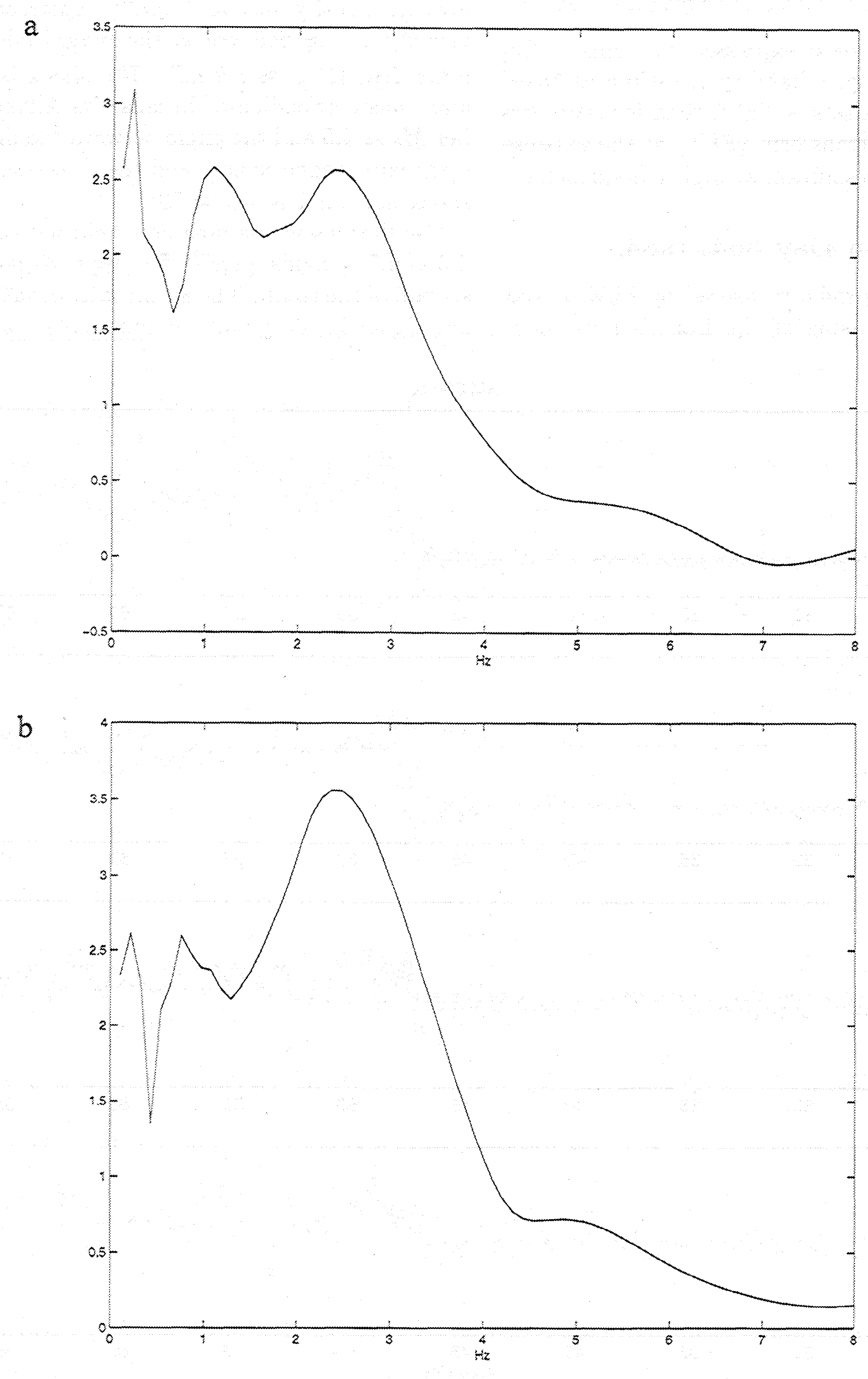

Figure 2. The spectra of fluctuations for (a) B_x, (b) B_y and (c) B_z components of the magnetic field as measured by IRM on day 364/1984 during the time interval 0429:21.5-0429:32.5 UT. Spectra have been calculated using CWT (Morlet) decomposition and are displayed on a logarithmic scale in arbitrary units.

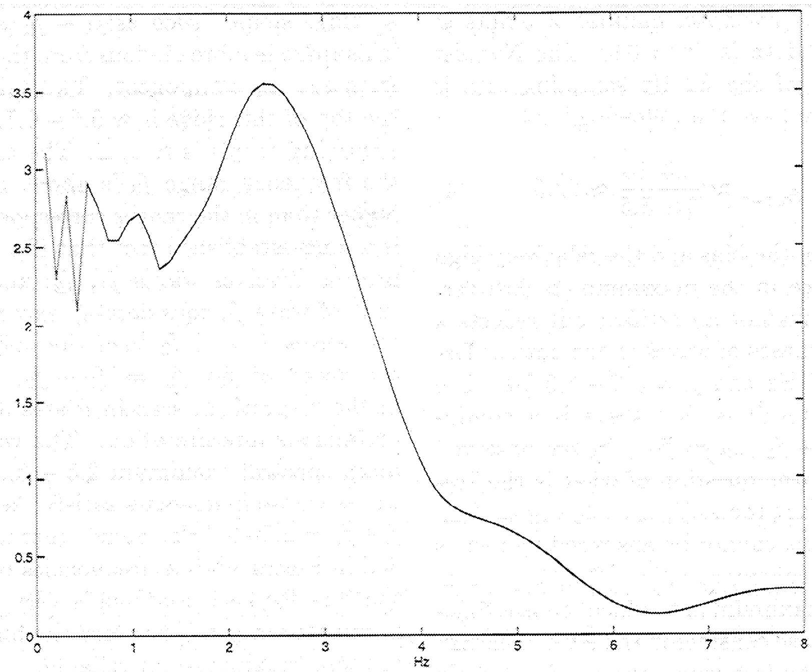

Figure 2. (continued)

Hz. The latter maximum can be seen only in the B_z component.

The evolution of the dynamic spectra of low-frequency turbulence as the satellite crosses the shock front is displayed in Plate 1. This spectrogram has been calculated making use of CWT (Morlet) decomposition. In the upstream region the most prominent maximum in the spectrum is around a frequency ≈ 2.5 Hz. This maximum is always related to the oscillations described above. Less prominent maxima can be seen at lower frequencies. For example, the B_y component secondary maxima can be seen in frequency ranges $1 - 1.5$ Hz and $0.6 - 0.8$ Hz during a time interval of 5-10 seconds (0429:27-0429:32 UT). It has been shown that the fluctuations in the main maximum are plasma waves propagating in the whistler mode almost along the GSE x axis [*Balikhin et al.,* 1997]. Some wave activity can be seen also in frequencies above $f = 3.5$ Hz. As mentioned above, the higher temporal resolution of CWT methods is achieved at the expense of reduced frequency resolution at higher frequencies. Therefore the spectral features of waves with frequencies above 4 Hz are more obvious from the FFT representation published in Figure 3 of *Balikhin et al.*,[1997].

There are two possible scenarios which can explain the generation of the waves observed outside the main maximum. The first is that a number of generation mechanisms operate independently in the shock front. The most efficient one is responsible for the generation of waves which correspond to the most prominent maximum of the spectrum. Other mechanisms lead to secondary maxima in the spectrum. In such a scenario of a number of independently acting mechanisms the phases of waves observed should also be independent. The second scenario is that the nonlinear evolution of the waves, which correspond to the main spectral maximum, leads to generation of waves observed as secondary features of the dynamical spectrum. In the latter case, some dependency between the phases of the observed waves should become evident.

The time interval 0429:21.5-0429:32.5 UT was chosen to perform bispectral analysis. Bicoherence was calculated making use of the CWT approach for all three components of the magnetic field. The results of this calculation are displayed in Plate 2. The level of bicoherence in B_x is generally lower than in the B_y and B_z components as a result of the fact that the observed waves are propagating at small angles with respect to the x axis. The bicoherence for the B_y component exhibits a maximum ($b^2 > 0.6$) in the frequency range $f_1 \approx f_2 \approx (2.5 - 3.0)$ Hz. The existence of phase coherence between waves whose frequencies are f_1, f_2 and $f_3 = f_1 + f_2 \approx 2(2.5 - 3)\text{Hz} = (5 - 6)\text{Hz}$ arises from such a maximum. The duration of the time interval which was chosen to calculate bicoherence, corresponds to about 25 periods of waves with frequency 2.5 Hz and to an even larger number of periods for higher frequencies. Therefore the existence of this maximum is statistically reliable. It is possible to "translate" the reliability of the position of this maximum into numbers making use of an estimate of the bias Bi^2 given by *Dudok de Wit and Krasnosel'skikh* [1995]:

$$Bi^2 \approx \frac{6.5}{N} \frac{f_{\text{Nyq}}}{f_1 + f_2} \tag{3}$$

Here N is the total number of samples and f_{Nyq} is the Nyquist frequency. The total number of points in the chosen interval of data is $N = 340$. The Nyquist frequency in the case of the 32 Hz sampling rate is $f_{\mathrm{Nyq}} = 16$ Hz. Thus we have the following bias:

$$Bi^2_{f_{2.5-3}+f_{2.5-3}\Leftrightarrow f_{5-6}} \approx \frac{6.5}{340}\frac{16}{5.5} \approx 0.05 \qquad (4)$$

The low value of 0.05 of the bias and the relatively high value of the bicoherence in the maximum (> 0.6) verify that this maximum is not an artifact but reflects a real relation between phases of waves in the data at frequencies $f = 2.5 - 3.0$ Hz and $f = 5.0 - 6.0$ Hz. The interpretation of this result is that there is a nonlinear interaction $f_{2.5-3} + f_{2.5-3} \Leftrightarrow f_{5-6}$ in the observed turbulence. Generally, the question of what is the "direction" of this nonlinear process $f_{2.5-3}+f_{2.5-3} \Leftarrow f_{5-6}$ or $f_{2.5-3}+f_{2.5-3} \Rightarrow f_{5-6}$ cannot be answered by means of the magnitude of bicoherence alone. Both processes will lead to the same maximum in the bicoherence function. However, in the case considered the level of energy of waves observed in the frequency range $2.5 - 3.0$ Hz is about 3 orders of magnitude higher than in the frequency range $5.0 - 6.0$ Hz. Therefore it is reasonable to suggest that the energy is transfered from waves observed in the primary maximum $2.5 - 3.0$ Hz to the waves observed at $5.0 - 6.0$ Hz:

$$f_{2.5-3} + f_{2.5-3} \Rightarrow f_{5-6} \qquad (5)$$

Process (5) is generation of a second harmonic, which usually is a part of a steepening of nonlinear wave. The evidence for process (5) is most obvious from the bispectrum (or bicoherence) calculated for the B_y component (see Plates 2b and 2e) but can also be seen in the other two components. Contrary to this, the maximum in the bicoherence with an abscissa $f_1 \approx 2.5 - 3.0$ Hz and an ordinate $f_2 \approx 5.0 - 5.5$ Hz can be observed only in the bicoherence calculated for the B_z component. The value of this maximum is $b^2_{f_{2.5-3}+f_{5-5.5}} \approx 0.58$. The bias for this maximum is

$$Bi^2_{f_{2.5-3}+f_{5.-5.5}\Leftrightarrow f_{7.5-8}} \approx \frac{6.5}{340}\frac{16}{7.5} \approx 0.04$$

The interpretation of this result is that a process of nonlinear coupling is taking place which involves one wave from the primary maximum and its second harmonic. This nonlinear coupling leads to the generation of waves observed at frequencies $7.5 - 8.0$ Hz. The CWT approach can only be used to study frequencies $\leq \frac{1}{2} f_{\mathrm{Nyq}}$. Thus $f_1 + f_2 = 8$ Hz is an upper limit for the resulting frequency which can be studied by this method in that particular case. However, in reality a whole range of waves is generated as a second harmonic of a primary maximum:

$$f_{2.5-3.0} + f_{5.0-6.0} \Leftrightarrow f_{7.5-9.0}$$

Plates 3a-3f show the low-frequency portion of Plates-2a-2f. The first results that emerge from these figures are bispectrum maxima in Plates 3b and 3c which have a "ridge shape" such as: $f_1 + f_2 = f_3 \approx 2.5 - 3.0$ Hz. This ridge is more obvious from the B_y component than from the B_z component. The value of bicoherence at the top of this ridge is $\approx 0.6 - 0.7$. The bias calculated according to (3) is ≈ 0.12. The amplitude of waves in the frequency range f_3 is about 1 order of magnitude higher than in the ranges corresponding to f_1 and f_2. It is a long-established fact that if a nonlinear three-wave process involves waves f_1, f_2, and f_3 and the amplitude of wave f_3 considerably exceeds the amplitudes of the waves f_1 and f_2 then the only possible process is the decay of f_3: $f_3 \Rightarrow f_1 + f_2$. Therefore the ridge in the bispectrum seen in Plates 3b and 3c has an unambiguous interpretation. The waves observed in the main spectral maximum $2.5 - 3.0$ Hz decay into two waves whose frequencies satisfy the resonance condition $f_1+f_2 = 2.5-3.0$ Hz. Some other maxima can be identified in Plates 3d-3f at frequencies below 1 Hz. However, the bias for such maxima is very large due to the denominator in (3). Therefore the physical interpretation of these maxima is not reliable.

To eliminate the possibility that detected nonlinear maxima resulted from some instrumental nonlinearity, the data from Interball Tail Probe obtained in the vicinity of the quasiperpendicular part of Earth's bow shock were also subjected to the bispectral analysis.

4. Interball Tail Probe Shock on Day 65/1998

The B_z component of the magnetic field as measured by Interball Tail Probe on day 65/1998 during the crossing of the Earth's bow shock is displayed in Figure 3. The sampling rate was 16 Hz. The ramp was crossed at about 0554:15 UT. The coordinates of the crossing were (6.0, 18.1, 9.4) R_e in GSE coordinates. The normal to the shock front was $\vec{n} \approx (0.75, 0.58, 0.31)$. This shock is also supercritical and quasiperpendicular (almost perpendicular), with $M_a \approx 6$ (A. Fedorov, private communication, 1998) and $\theta_{Bn} \approx 85°$. Low-frequency oscillations in the magnetic field persist for approximately 40 seconds upstream of the ramp. The main energy of turbulence lies in the frequency range $0.8 - 1Hz$. The bicoherence estimated for the B_z component on the time interval 0554:25-0554:45 is plotted in Plate 4. The most prominent maximum in Plate 4 corresponds to $f_1 \approx f_2 \approx 0.8 - 1.0$. That assumes the nonlinear process $f_{0.8-1} + f_{0.8-1} \Leftrightarrow f_{1.6-2}$ of frequency doubling, is taking place. This process is similar to that which was detected in IRM data $f_{2.5-3}+f_{2.5-3} \Leftrightarrow f_{5-6}$. There are two other maxima with $f_2 \approx 0.8 - 1$ Hz. The first, with $f_1 \approx 1.5 - 2Hz$, corresponds to the process $f_{0.8-1} + f_{1.5-2} \Leftrightarrow f_{2.3-3}$. The second corresponds to $f_{0.8-1} + f_{2.3-3} \Leftrightarrow f_{3.1-4}$.

The maxima in the bottom left corner of the bicoherence triangle in Plate 4 correspond to the processes which involved only frequencies less than 0.5 Hz. The temporal interval of 20 s on which bicoherence was cal-

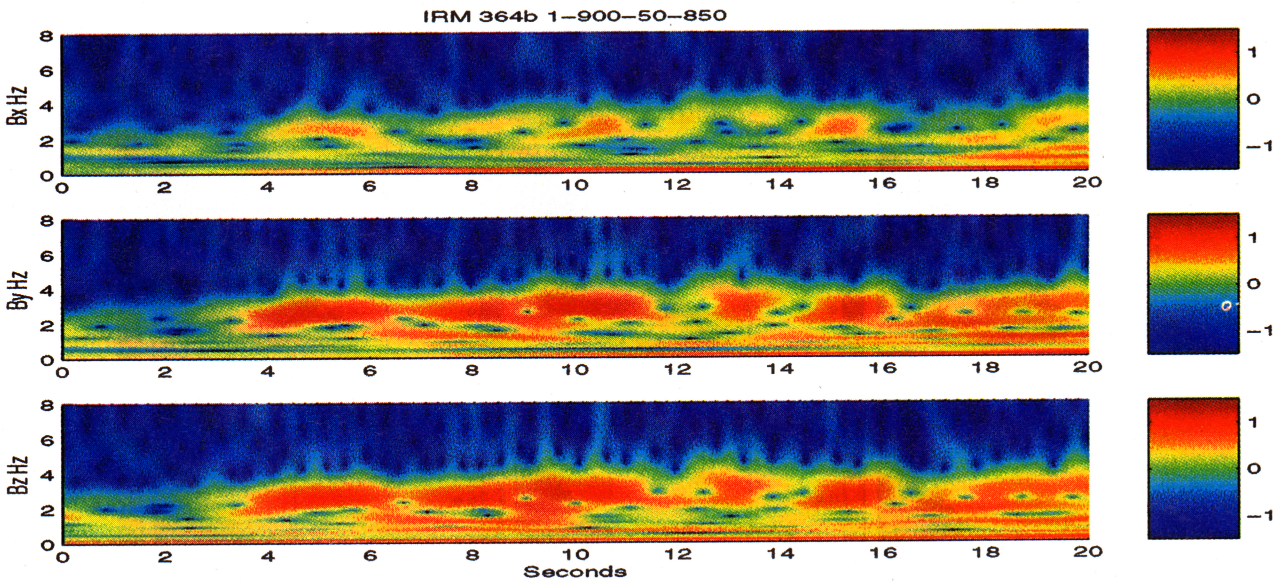

Plate 1. The dynamic spectra of the fluctuations of the three components of magnetic field as measured by IRM upstream of the ramp of the Earth's bow shock on day 364/1984. Time scale is in seconds after 0429:22 UT. The spectra are displayed on a logarithmic scale with arbitrary units.

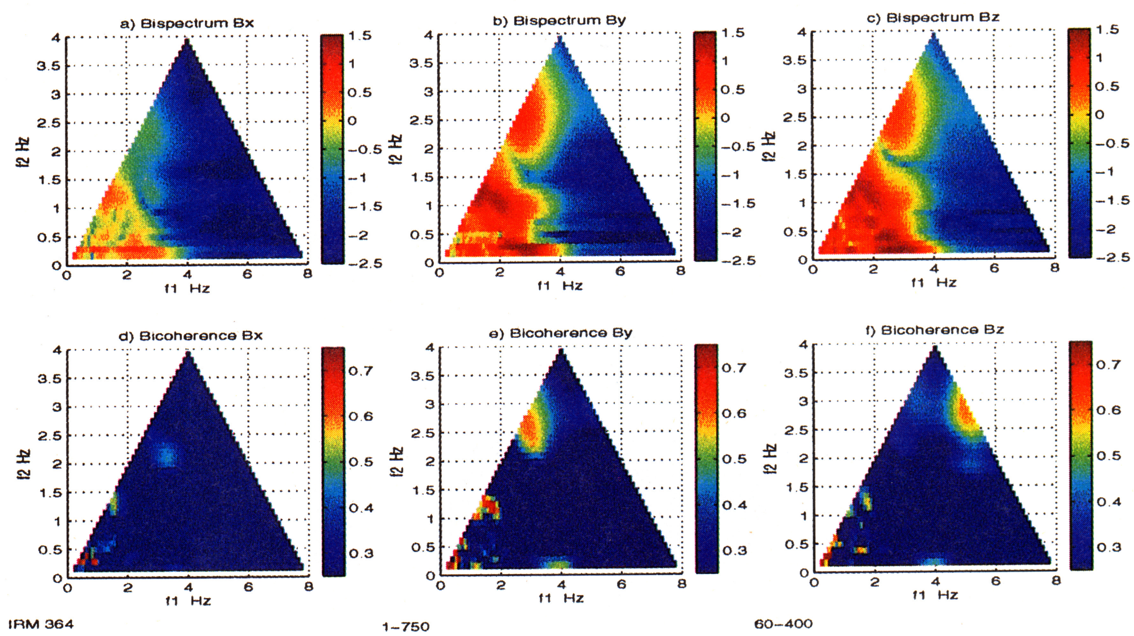

Plate 2. The bispectra and bicoherence calculated for the three components of the magnetic field as measured by IRM on day 364/1984 during the time interval 0429:21.5-0429:32.5 UT. Calculations have been made with the use of CWT (Morlet) decomposition.

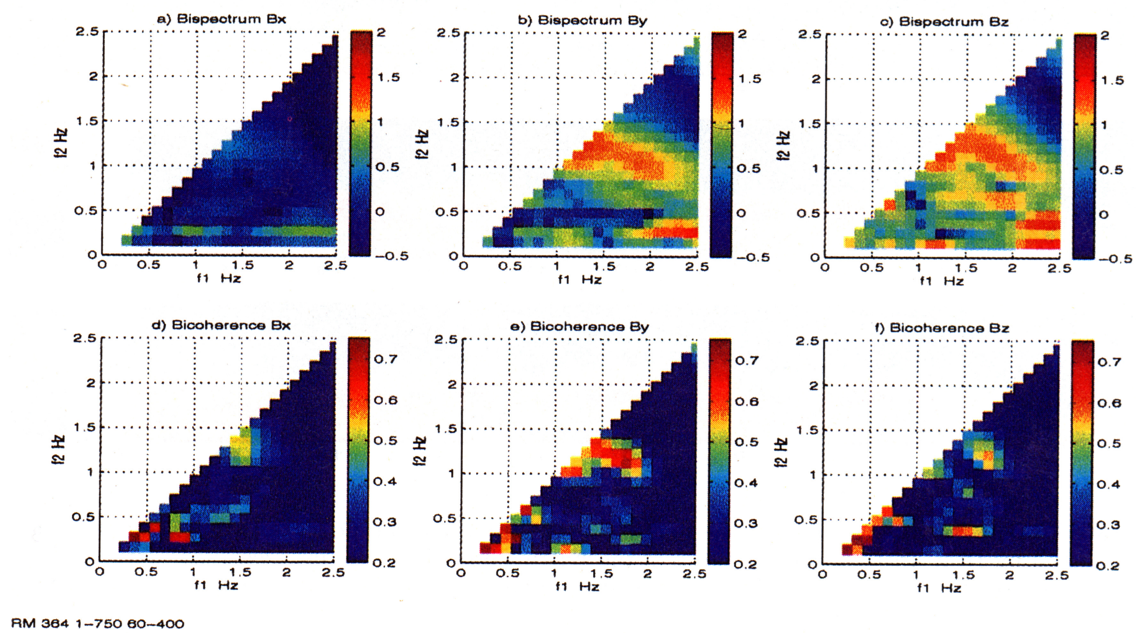

Plate 3. The same as in Plate 2 but for the low frequency parts only.

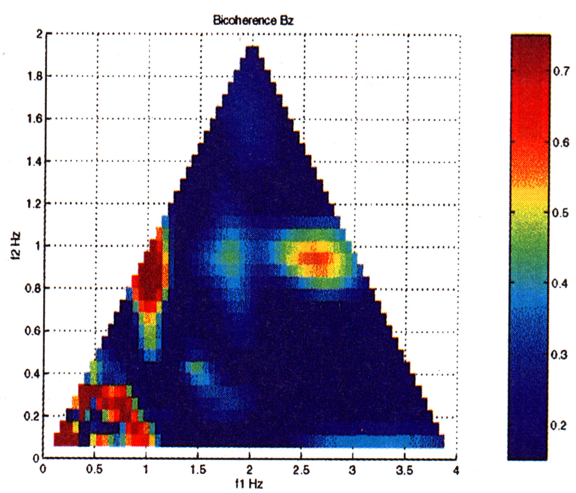

Plate 4. The bicoherence calculated for the B_z component as measured by Interball on the day 65/1998 during the time interval about 0554:25-0554:45 UT.

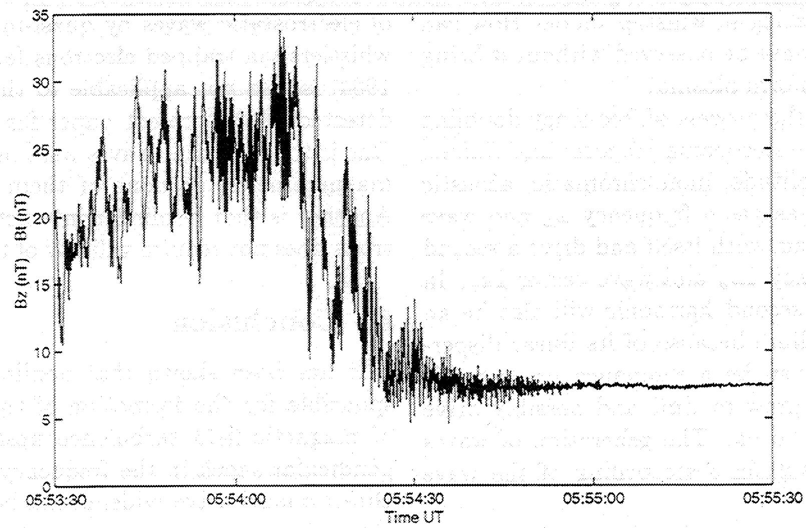

Figure 3. B_z as measured by Interball during bow shock crossing which occurred at about 0554:15 UT on the day 65/1998.

culated was too short to provide reliable results and low bias for such small frequencies. However, the ridge-like maximum can be seen adjacent to this low-frequency area. This "ridge" is similar to the one observed in IRM data and discussed above. It corresponds to the decay of waves in the main maximum $0.8-1$ Hz into pairs of waves which correspond to resonance condition $f_1+f_2=0.8-1$ Hz.

5. Discussion

It is worth noting that although only two cases are presented above, bispectral analysis was performed on other magnetic field data sets obtained by AMPTE IRM. The results were similar to those presented.

As mentioned in the introduction, theoretical models for the generation of low-frequency whistler waves adjacent to the ramp of a quasiperpendicular shock that are in agreement with experimental data can only explain a single narrow maximum in the spectrum of the upstream turbulence. The existence of additional maxima in the spectra of the turbulence was explained as the result of activity of other generation mechanisms, particularly other plasma instabilities. However, estimates of the bicoherence function based upon IRM and Interball magnetometer data imply that a strong correlation exists between the phases of the waves which correspond to the main spectral maximum and other maxima in the spectra. The only possible interpretation of this is that the main source of energy for the generation of turbulence observed in the secondary maxima is energy stored in the waves corresponding to the main spectral maximum. Processes like second harmonic generation or the decay of the main wave into two other waves with lower frequencies and lower wave vectors are responsible for the flow of the energy from the main maximum to other parts of the spectrum. The secondary waves can also, in turn, be involved in the nonlinear processes, which lead to the generation of tertiary waves, etc. Processes such as $f_{2.5-3.0}+f_{5.0-6.0} \Leftrightarrow f_{7.5-9.0}$ (IRM) and $f_{0.8-1}+f_{1.5-2} \Leftrightarrow f_{2.3-3}$ (Interball) identified above are examples of nonlinear processes which involve secondary waves and transmit their energy to tertiary waves. All of these nonlinear processes together form a complicated cascade which provides the dissipation of turbulent energy from the main spectral maximum and results in the effective widening of the turbulent spectrum.

The second harmonic generation corresponds to the process $f_{2.5-3.0}+f_{2.5-3.0} \Leftrightarrow f_{5.0-6.0}$ in the IRM data set and $f_{0.8-1}+f_{0.8-1} \Leftrightarrow f_{1.6-2}$ in Interball data. As already mentioned, the waves observed by IRM in the primary maximum $2.5-3.0$ Hz are whistler waves [*Balikhin et al.*, 1997]. The two waves from the left side of (5) propagate in the same direction. Thus the resulting waves observed in the range $5-6$ Hz should propagate in the parallel direction. This was confirmed by the application of minimum variance analysis to the waves in the frequency range $5-6$ Hz (The estimated angle between $k_{5.0-6.0}$ and $k_{2.5-3.0}$ to be a few degrees). The dispersion of whistler waves is well known:

$$\omega = \Omega_{ce} \cos(\theta^a_{Bk}) \frac{k^2c^2}{\omega^2_{pe}},$$

where Ω_{ce} is the electron cyclotron frequency, ω_{pe} is the electron plasma frequency and θ^a_{Bk} is the angle of the direction of wave propagation with respect to the ambient magnetic field. If the two waves on the left side of (1) satisfy this quadratic dispersion relation, a third wave which propagates in the same direction as these two cannot satisfy both the resonance conditions and the quadratic dispersion relation. Thus the waves observed by IRM in the frequency range $5.0-6.0$ Hz are not waves propagating in whistler mode. How can the second harmonic wave be observed without it being an eigenmode of a uniform plasma?

In an ordinary gas, the process of frequency doubling occurs as a part of wave steepening [*Sagdeev and Galeev,* 1969]. If a large-amplitude, monochromatic, acoustic wave in an ordinary gas has a frequency ω_a and wave vector k_a, it will interact with itself and drive a second harmonic with frequency $2\omega_a$ and wave vector $2k_a$. In an ordinary gas, this second harmonic will also be an eigenmode of the medium because of its linear dispersion. Thus being driven by a resonance process this second harmonic will grow in time and possibly drive higher harmonics and so on. The generation of waves with higher k will result in a steepening of the wave front.

For a whistler wave in a plasma, harmonics are not eigenmodes themselves because of the nonlinear dispersion. In this case, the change in amplitude of the second harmonic wave can be described by:

$$i\frac{\partial A_{2\omega}}{\partial t} = C_{2\omega,\omega,\omega}A_\omega^2 - \gamma A_{2\omega}$$

where A_ω and $A_{2\omega}$ are amplitudes of a large-amplitude (initial) wave and its second harmonic, respectively, γ is the damping rate of the second harmonic due to its interaction with the plasma (since it is not an eigenmode inclusion of this damping is necessary), and $C_{2\omega,\omega,\omega}$ is some factor which depends upon ω. In the stationary case, the second harmonic should have an amplitude

$$A_{2\omega} = \frac{C_{2\omega,\omega,\omega}A_\omega^2}{\gamma}$$

The second harmonic propagates, not in a uniform plasma, but in a system of a uniform plasma with a large amplitude wave propagating through it. Though the amplitude $A_{2\omega}$ usually remains small, it can nevertheless be observed [*Sagdeev and Galeev,* 1969]. This is in accordance with the observed low level of waves in the frequency range $5.0-6.0$ Hz in the spectrum.

The interaction of the second harmonic with the thermal plasma should result in strong damping. Thus in spite of their small amplitude they could provide an efficient means for the dissipation of energy from the main spectral component.

As noted, nonlinear processes which involve whistler turbulence were detected in the subauroral ionosphere by means of bicoherence [*Tanaka et al.,* 1987; *Ohnami et al.,* 1993]. Numerous theoretical models were developed for such an interaction in the subauroral ionosphere. However, these models cannot be directly applied to the whistler waves observed at the terrestrial bow shock. The analytical description of the nonlinear interaction between whistler waves and ion cyclotron harmonic waves was developed by *Trakhtengerts and Hayakawa,* [1993] under the condition $\Omega_{ce} \gg \omega_{pe}$. This condition is not valid for the terrestrial bow shock. The excitation of electrostatic waves by quasi-monochromatic oblique whistlers via trapped electrons [e.g., *Matsumoto et al.,* 1984] is also not applicable to the nonlinear processes detected in the present paper for a number of reasons. One is that all three waves were measured by a fluxgate magnetometer, so none of them can be electrostatic. Another is that nonlinear interaction via trapped electrons does not require validity of the phase relation (3).

6. Conclusion

It has been shown that nonlinear interaction is responsible for the formation of the observed spectrum of magnetic field turbulence upstream of a quasiperpendicular shock in the frequency range $10^0 - 10^1$ Hz. Direct quantitative evidence has been given of a cascade of nonlinear processes such as the generation of second harmonic and decay instability. These processes are responsible for the transformation of energy from waves which correspond to the primary maximum in the spectrum to other scales. This leads to an effective widening of the turbulent spectrum. Thus the broad spectrum of waves observed in the frequency range $10^0 - 10^1$ Hz upstream of the ramp of a quasiperpendicular shock is not the result of various competing generation mechanisms but is the result of the nonlinear evolution of a quasi-monochromatic whistler mode wave with a frequency which corresponds to the main maximum in the spectrum.

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