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Anisotropy of magnetosonic turbulence in the solar wind between 0.1 and 0.4 AU

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Abstract. The anisotropy of the density fluctuations in the solar wind is studied with the assumption that density fluctuations are connected to magnetosonic turbulence. It is shown that the anisotropy of density fluctuations does not coincide with that of the turbulence energy in the case of the fast magnetosonic waves while they are equal for the slow magnetosonic waves. The radial evolution of the two-dimensional (2-D) anisotropy ratio in the interplanetary scintillation (IPS) pattern plane due to the change of the ambient magnetic field direction is investigated. In the case of the Parker's spiral magnetic field the radial dependence of the 2-D anisotropy ratio is shown to be considerably stronger for the fast magnetosonic waves than for the slow magnetosonic waves. The comparison between theoretical estimations and the results of the Kashima high-frequency IPS measurements shows that the geometric effects are not sufficient to account for the observed radial dependence of the axial ratio, and that the angular distribution of turbulent energy evolves in the region of heliocentric distances between 0.1 and 0.4 AU toward the state enriched by the magnetosonic waves propagating at smaller angles relative to the ambient magnetic field. The nonlinear wave-wave interactions seem to be responsible for this evolution of the density turbulence spatial power spectrum.

1. Introduction

Radio occultation data and in situ measurements show that the solar wind plasma is nonsteady and irregular with the plasma parameters fluctuating at all spatial and temporal scales [*Tu and Marsch*, 1995]. One of the most important properties of the solar wind turbulence is the anisotropy caused evidently by sufficiently strong ambient magnetic field and/or motion of the plasma away from the Sun. The anisotropy of magnetic field fluctuations found by in situ measurements [*Coleman*, 1968; *Belcher and Davis*, 1971; *Solodina and Belcher*, 1976; *Bavassano et al.*, 1982; *Klein et al.*, 1991] (see also *Goldstein and Roberts* [1999] for a recent review) is consistent with the hypothesis that magnetic field turbulence is dominated by propagating Alfven waves. In situ measurements have been used to obtain information about the variances and one-dimensional power spectra in different components of magnetic fluctuations' vectors. If the magnetic fluctuations can be regarded as the plane Alfven wave, the maximum variance direction is perpendicular to both wave vector and ambient magnetic field vector. If we have the stochastic ensemble of Alfven waves with a broad angular distribution of wave vectors, then the minimum variance direction tends to coincide with the ambient magnetic field direction [*Daily*, 1973; *Barnes*, 1981]. However, the angular distribution of wave vectors is still unknown, as simultaneous measurements at several different spatial points are needed for its investigation. The small-scale density fluctuations including their spatial power spectra in the plane of the sky have been studied by several modifications of the radio occultation method. These observations show that the density fluctuations in the solar wind are anisotropic. According to the results of radio waves' scattering measurements, the anisotropy is very high in the solar wind acceleration region (at heliocentric distances $r < 10R_s$, where R_s is the solar radius), where density irregularities with scales 10–30 km are elongated along the radial direction with an axial ratio ~ 10 and even more [*Armstrong et al.*, 1990]. In the region outside 10 R_s the anisotropy becomes much weaker: the axial ratio is ~ 2 for irregularities with scales ≥ 1 km at heliocentric distances $r \approx 10R_s$ (low radio frequency scattering observations by *Bedevkin and Vitkevich* [1967]), and the same value of the axial ratio is typical also for the irregularities with scales 10–30 km in observations by *Armstrong et al.* [1990]. According to the results of interplanetary scintillation (IPS) observations, the axial ratio is distinctly less than 1.5 for

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irregularities with scales of order of 100 km at 0.5 AU $< r <$ 1 AU [*Coles and Maagoe*, 1972; *Kojima*, 1979]. Recently, the data on the anisotropy of solar wind density fluctuations were obtained within heliocentric distances $r = 10$–$70\ R_s$ using the analysis of IPS temporal power spectra measured at radio frequencies of 2 and 8 GHz by *Yamauchi et al.* [1996, 1998]. Their data show clearly that the elongation degree of irregularities with scales 10 km $< l <$ 100 km decreases with increasing heliocentric distance. Thus the radio occultation data show that the small-scale density fluctuations in the solar wind become more isotropic with increasing distance from the Sun.

Woo [1996] and *Woo and Habbal* [1997] ascribed the observed anisotropy of density irregularities to the finest quasi-static structures in the outer solar corona. Below we shall treat the density fluctuations' anisotropy assuming that the irregularities are caused by the waves propagating in the solar wind plasma. As the wavelength corresponding to the scales responsible for the scattering and scintillation effects is much larger than the ion cyclotron radius as well as the ion inertial scale, the density fluctuations in this spectral range can be caused by fast and slow magnetosonic waves [*Chashei and Shishov*, 1981, 1983; *Tu and Marsch*, 1995]. Using this assumption, we analyze the data on the anisotropy of density irregularities obtained from the Kashima IPS observations in 1992–1994 (for the details of analysis, see *Yamauchi et al.* [1998]). The aim of our study is to find some evidence of the evolutionary effects of the angular power spectrum of magnetosonic turbulence and to investigate the features typical of slow and fast magnetosonic waves. We restrict the study to the region of fully evolved solar wind, $r > 10R_s$, where the mean velocity does not depend on heliocentric distance [*Tokumaru et al.*, 1994] and the magnetosonic waves have the power law spectra which are in agreement with the hypothesis of their generation by Alfven waves in nonlinear cascading processes [*Chashei and Shishov*, 1981; *Tu and Marsch*, 1995].

2. The Spatial Power Spectra of Magnetosonic Density Fluctuations

In this section we discuss the spatial power spectra of density fluctuations caused by small-amplitude magnetosonic waves. As the small-scale turbulence is weak, we can represent the spatial-temporal plasma density fluctuations $\delta N(\mathbf{r}, t)$ with the superposition of the fast and slow magnetosonic waves:

$$\delta N(\mathbf{r}, t) = \Sigma_\sigma \int \delta N^\sigma_{\mathbf{k}} \delta(\omega - \omega^\sigma_{\mathbf{k}}) \exp(i\omega t - i\mathbf{kr}) d\omega d\mathbf{k}, \quad (1)$$

where Σ_σ denotes the sum over the magnetosonic modes, $\sigma = f, s$ for fast magnetosonic and for slow magnetosonic waves, respectively, $\delta N^\sigma_{\mathbf{k}}$ is a complex spectral amplitude, $\mathbf{k}$ is a wave vector, and $\omega^\sigma_{\mathbf{k}}$ is the dispersion relation in the reference frame moving with the plasma.

Using the continuity equation

$$\partial \delta N / \partial t + \nabla (N_0 \mathbf{v}) = 0, \quad (2)$$

where N_0 is mean plasma density and $\mathbf{v}$ is the speed of plasma motion in wave disturbances, we can find the spectral amplitudes:

$$\delta N^\sigma_{\mathbf{k}} = v^\sigma_{\mathbf{k}} N_0 (\mathbf{k} \mathbf{e}^\sigma_{\mathbf{k}}) / \omega, \quad (3)$$

where $v^\sigma_{\mathbf{k}}$ are the complex velocity amplitude and $\mathbf{e}^\sigma_{\mathbf{k}}$ are the unit vectors in the direction of velocity fluctuation (polarization vectors). Assuming that turbulence is quasi-steady and quasi-homogeneous and the normal modes are noncorrelated with each other, we have from (1) the relation for the density correlation function

$$\begin{aligned} K_N(\mathbf{d}, \tau) &= < \delta N(\mathbf{r}, t) \delta N(\mathbf{r} + \mathbf{d}, t + \tau) > \\ K_N(\mathbf{d}, \tau) &= \int \left[\Sigma_\sigma \Phi^\sigma_{\mathbf{k}} \delta(\omega - \omega^\sigma_{\mathbf{k}}) \right] \\ &\quad \times \exp(i\omega\tau - i\mathbf{kd}) d\omega d\mathbf{k}, \end{aligned} \quad (4)$$

where $\Phi^\sigma_{\mathbf{k}}$ are the spatial density fluctuations' spectra and

$$\begin{aligned} < \delta N^\sigma_{\mathbf{k}} \delta N^{\sigma *}_{\mathbf{q}} > &= \Phi^\sigma_{\mathbf{k}} \delta(\mathbf{k} - \mathbf{q}), \\ \Phi^\sigma_{\mathbf{k}} &= | \delta N^\sigma_{\mathbf{k}} |^2 . \end{aligned} \quad (5)$$

The relation between the density fluctuations' spectra $\Phi^\sigma_{\mathbf{k}}$ (equation (5)) and the spectra $W^\sigma_{\mathbf{k}}$ of wave energy density can be obtained from formula (3):

$$\begin{aligned} \Phi^\sigma_{\mathbf{k}} &= N_0 \left[(\mathbf{k} \mathbf{e}^\sigma_{\mathbf{k}})^2 / (\omega^\sigma_{\mathbf{k}})^2 \right] W^\sigma_{\mathbf{k}}, \\ W^\sigma_{\mathbf{k}} &= N_0 \mid v^\sigma_{\mathbf{k}} \mid^2 . \end{aligned} \quad (6)$$

The wave frequencies $\omega^\sigma_{\mathbf{k}}$ and polarization vectors $\mathbf{e}^\sigma_{\mathbf{k}}$ can be specified for fast and slow magnetosonic waves [*Akhiezer et al.*, 1975; *Tu and Marsch*, 1995] as

$$\begin{aligned} (\omega^\sigma_{\mathbf{k}})^2 &= k^2 C^2_{f,s}, \\ C^2_{f,s} &= 0.5 \left\{ (V_A^2 + V_S^2) \right. \\ &\quad \left. \pm \left[(V_A^2 + V_S^2)^2 - 4 V_A^2 V_S^2 \cos^2\theta \right]^{1/2} \right\}, \end{aligned} \quad (7)$$

$$\left. \begin{aligned} \mathbf{e}^f_{\mathbf{k}} &= \gamma \mathbf{k}_\perp / k_\perp + \alpha \mathbf{n}, \\ \mathbf{e}^s_{\mathbf{k}} &= \alpha \mathbf{k}_\perp / k_\perp - \gamma \mathbf{n}, \\ \mathbf{e}^f_{\mathbf{k}} &\perp \mathbf{e}^s_{\mathbf{k}}, \end{aligned} \right\}, \quad (8)$$

where $\mathbf{n}$ is the unit vector of ambient magnetic field $\mathbf{B}$, $\mathbf{n} = \mathbf{B}/B$, θ is the angle between $\mathbf{k}$ and $\mathbf{n}$, $\mathbf{k}_\perp$ is the $\mathbf{k}$ component perpendicular to $\mathbf{n}$, and $k_\perp = | \mathbf{k}_\perp |$. The numerical coefficients α and γ are defined by the relations

$$2\alpha^2 = 1 - Q, \quad 2\gamma^2 = 1 + Q, \quad \beta = V_S^2 / V_A^2, \quad (9)$$

$$Q^2 = \frac{[1 + \beta(\sin^2\theta - \cos^2\theta)]^2}{[1 + \beta(\sin^2\theta - \cos^2\theta)]^2 + 4(\beta \sin\theta \cos\theta)^2}.$$

Below we consider a "low β" plasma; $\beta \leq 1$ for all the regions of the solar wind inside the Earth orbit with only one possible exception of comparatively narrow heliospheric current sheet, and $\beta \ll 1$ in the region considered below. In this case we find from (9) that

$Q \approx 1$, $\gamma \approx 1$, $\alpha \ll 1$, $C_f^2 \approx V_A^2$, and $C_s^2 \approx V_S^2 \cos^2\theta$, and in accordance with (8) the plasma motions are directed along $\mathbf{k}_\perp$ and along $\mathbf{n}$ for the fast and slow magnetosonic waves, respectively. So we have from (4)-(9) the following relation for the spatial power spectrum of the density fluctuations:

$$\begin{aligned} \Phi_{\mathbf{k}} &= \Phi_{\mathbf{k}}^f + \Phi_{\mathbf{k}}^s, \\ \Phi_{\mathbf{k}} &= N_0 \left[(k_\perp^2/k^2)(W_{\mathbf{k}}^f/V_A^2) + (W_{\mathbf{k}}^s/V_S^2) \right]. \end{aligned} \quad (10)$$

The factor $(k_\perp^2/k^2)$ in the first term of (10) corresponds to the known property that density fluctuations can be caused by fast magnetosonic waves only if these waves are propagating at angles $\theta \neq 0$ with respect to the ambient magnetic field.

3. Anisotropy of the Density Fluctuations Power Spectrum

We consider now the anisotropy of the density fluctuations' power spectrum $\Phi_{\mathbf{k}}$ (equation (10)). Three-dimensional quasi-steady power spectra $W_{\mathbf{k}}^{f,s}$ are generated in the fully evolved solar wind through the nonlinear processes which are responsible for the spectral cascading of the turbulent energy. The probabilities of nonlinear processes (matrix elements in the kinetic equations for wave energy) depend on the angles between wave vectors and ambient magnetic field [*Akhiezer et al.*, 1975; *Tu et al.*, 1984], so one can expect that the power spectra of turbulent energy are nonisotropic for the excited magnetosonic waves as well as for the driving Alfven waves. However, the exact solutions of the three-dimensional nonlinear kinetic equations describing three-mode MHD turbulence in the solar wind [*Chashei and Shishov*, 1985] are not known at present. Instead, the scaling approach is usually used. In the frame of scaling theory the one-dimensional power spectrum of turbulence is considered using the Kraichnan and the Kolmogorov cascading functions [*Tu and Marsch*, 1995]. The smooth radial gradients of solar wind parameters result in a radial dependence of the turbulence level and of the turbulence outer scale [*Chashei and Shishov*, 1981, 1985; *Tu et al.*, 1984; *Tu and Marsch*, 1995]. Additionally, we can introduce the axial ratios ξ_σ, which characterize the anisotropy of turbulence spectra $W_{\mathbf{k}}^\sigma$ with one of the principal elongation axes coinciding with the direction of the ambient magnetic field,

$$W_{\mathbf{k}}^{f,s} = D^{f,s}(\xi_{f,s}^2 k_\parallel^2 + k_\perp^2)^{n/2}, \quad (11)$$

where $D^{f,s}$ are the structure constants and n is the power exponent; $n = 7/2$ for the Kraichnan turbulence, and $n = 11/3$ for the Kolmogorov turbulence [*Tu and Marsch*, 1995]. The anisotropy parameters ξ_σ depend on their initial levels in the inner solar wind, where the turbulence regime becomes cascade dominated, and on the heliocentric distance. The latter dependence describes a possible radial evolution of the angular spectrum of the wave energy density.

From (6) the anisotropy ratio μ_s of the density fluctuations' spectrum $\Phi_{\mathbf{k}}^s$ is exactly equal to that of the power spectrum of the turbulent energy $W_{\mathbf{k}}^s$, i.e., $\mu_s = \xi_s$. However, the anisotropy ratios of the density fluctuations' spectrum $\Phi_{\mathbf{k}}^f$ and the turbulent energy spectrum $W_{\mathbf{k}}^f$ do not coincide, i.e., $\mu_f \neq \xi_f$. We estimate now the two-dimensional anisotropy ratio for the spectrum $\Phi_{\mathbf{k}}^f$ (equation (10)) using the condition that the correlation $K(x, y, z; 0)$ (equation (4)) is equal at spatial lags x_0 and y_0 in the directions along and perpendicular to the ambient magnetic field, respectively,

$$\begin{aligned} &\int_{-\infty}^{\infty} K^f(x = x_0, y = 0, z, \tau = 0)dz \\ &\quad = \int_{-\infty}^{\infty} K^f(x = 0, y = y_0, z, \tau = 0)dz, \\ \mu_f &= x_0/y_0. \end{aligned} \quad (12)$$

For simplicity, we substitute the factor $(k_\perp^2/k^2)$ in (10) by the factor $k_\perp^2/(\xi_f^2 k_\parallel^2 + k_\perp^2)$, which will slightly shift the estimated value of μ_f from unity if $\xi_f \neq 1$. Carrying out the integration over $\mathbf{k}$ and z in (4) with the spatial spectrum (10) and (11), we find the following estimate of density fluctuation anisotropy ratio:

$$\mu_f = \xi_f (I_1/I_2)^{1/(n-2)}, \quad (13)$$

where [*Abramovits and Stegun*, 1970]

$$\begin{aligned} I_1 &= B(1/2; (n+1)/2), \\ I_2 &= B(3/2; (n-1)/2), \end{aligned} \quad (14)$$

n is the power exponent of the spectrum $W_{\mathbf{k}}^f$ (equation (11)), and $B(a;b)$ is the B function. From (13), using the known representation of B function, $B(a;b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$, where $\Gamma(a)$ is the Γ function [*Abramovits and Stegun*, 1970], we have

$$\mu_f = \xi_f \mu_1, \quad \mu_1 = (n-1)^{1/(n-2)}. \quad (15)$$

The relation (15) shows that the fast magnetosonic density fluctuations are anisotropic even if the power spectrum of turbulent energy is isotropic. In this particular case, $\xi_f = 1$, and the anisotropy ratio of the density fluctuations (equation 15)) depends only on the power exponent n, $\mu_f = \mu_1$. For the typical range of possible values of the power exponent n, $3 < n < 4$, we have from (15) that $2 > \mu_1 > \sqrt{3}$.

In the IPS observations the anisotropy ratio is measured in the plane of the sky perpendicular to a line of sight. Neglecting the integration effect, which is sufficiently small [*Yamauchi et al.*, 1998] for the observational data used below, we can consider that the measured anisotropy ratio ζ is related to the region in the vicinity of the proximate point to the Sun on a line of sight. Since at the proximate point the magnetic field is not in the plane perpendicular to the line of sight while the radial vector is in it, the value of ζ will not be equal to $\mu_s = \xi_s$ nor $\mu_f = \mu_1 \xi_f$. Instead, we can find that

$$\zeta_s^2 = (\xi_s^2 - 1)\cos^2\theta + 1 \quad (16)$$

in the case of the slow magnetosonic waves [*Yamauchi et al.*, 1998], and

$$\zeta_f^{n-2} = \frac{I_1(\xi_f')^{n-2} + I_2 \sin^2\theta}{I_2 + I_1(\xi_f')^{n-2}\sin^2\theta}, \quad (17)$$
$$\xi_f' = (\xi_f^2 \cos^2\theta + \sin^2\theta)^{1/2},$$

for the case of the fast magnetosonic waves, where θ is the angle between the radial direction and the direction of the local magnetic field at the line of sight proximate point. We assume below Parker's spiral model for the ambient interplanetary magnetic field,

$$B_\phi = B_r(\Omega r \cos\Lambda / V), \quad \tan\theta = r\cos\Lambda, \quad (18)$$

where B_ϕ and B_r are the azimuthal and the radial components of the magnetic field, Ω is the Sun's rotation angular speed, V is the solar wind speed, Λ is the heliolatitude, and r is the heliocentric distance in AU. Also, we neglect below the latitudinal effects assuming $\cos\Lambda \approx 1$, because the data of *Yamauchi et al.* [1996, 1998] are obtained mainly at low and middle latitudes. Using (18), we find the following expressions for the radial dependence of the anisotropy ratio observed by IPS method:

$$\zeta_s^2 = (\xi_s^2 + r^2)/(1+r^2), \quad (19)$$
$$\zeta_f^{n-2} = \left[(n-1)(1+r^2)(\xi_f^2+r^2)^{(n-2)/2} + r^2(1+r^2)^{(n-2)/2}\right] \Big/ \left[(1+r^2)^{n/2} + (n-1)r^2(\xi_f^2+r^2)^{(n-2)/2}\right], \quad (20)$$

where ξ_s and ξ_f are the anisotropy ratios for the slow magnetosonic and fast magnetosonic waves power spectra. Evidently, (19) and (20) lead to $\zeta_s = \xi_s$ and $\zeta_f = \mu_1 \xi_f$ at $r = 0$ and to $\zeta_{s,f} \to 1$ at $r \to \infty$.

We described above the anisotropy of the turbulence power spectra using the spectral representation (equation (11)) which is valid if the nonlinear processes in the $k_\parallel$ and $k_\perp$ space are mutually connected. In the presence of very strong ambient magnetic field, the rates of nonlinear processes changing $k_\parallel$ and $k_\perp$ can be very different; numerical simulations of *Shebalin et al.* [1983] show that in the presence of very strong ambient magnetic field, the rates of nonlinear processes changing $k_\parallel$ and $k_\perp$ can be strongly different. In this extreme case another representation of the anisotropic power spectra may be used,

$$W_{\mathbf{k}}^\sigma = D^\sigma k_\parallel^{-m} k_\perp^{-l}, \quad (21)$$

with two power exponents, m and l, $m \neq l$. We will not consider the possible representation (equation (21)) in this paper though it can be done easily using the approach similar to the one above. However, it should be noted that in this case the values μ_σ defined by (12) will be dependent on the correlation level at which the anisotropy ratio is measured.

4. Comparison With Observational Results

In Figures 1 and 2 the radial dependence of the anisotropy ratios ζ_s (equation (19)) and ζ_f (equation

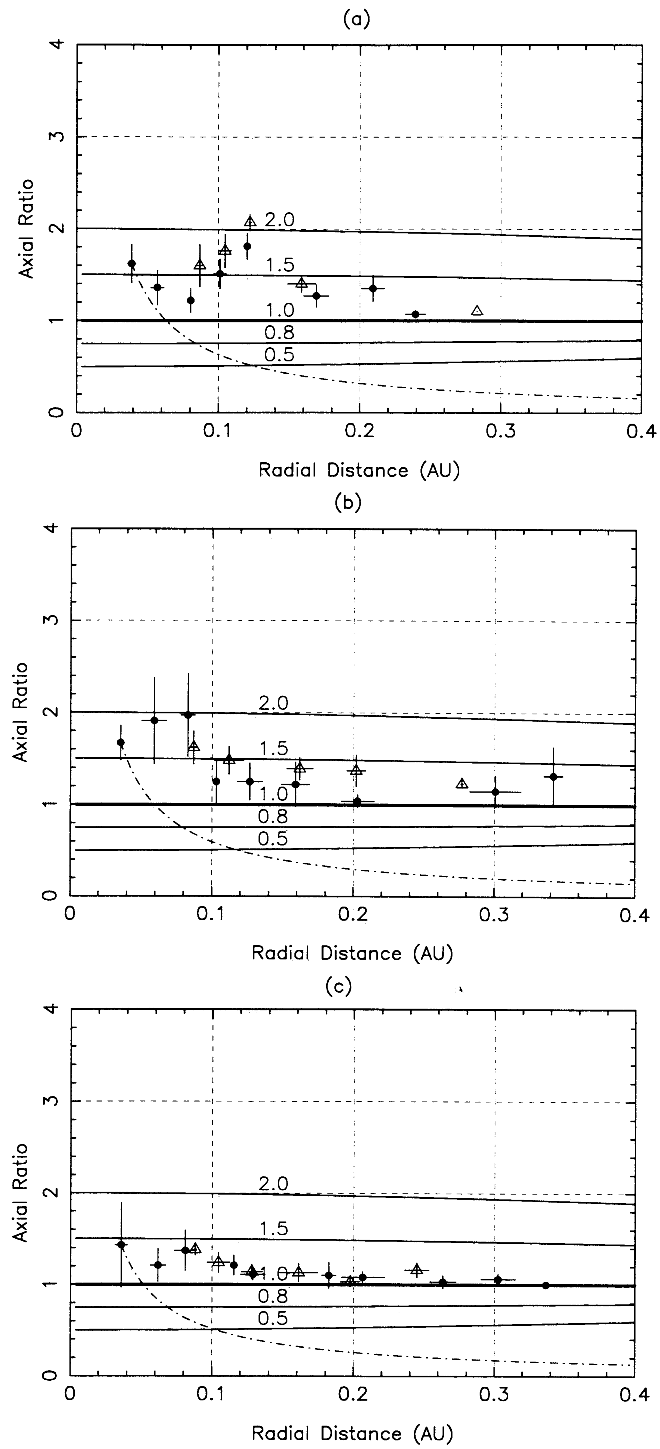

Figure 1. The radial dependence of the anisotropy ratio of density fluctuations for slow magnetosonic waves (equation (19)), represented by the solid lines. Numbers labeled on lines (0.5, 0.8, 1.0, 1.5, and 2.0) are anisotropy ratio ξ_s. The dependence of the isotropic turbulence spectrum for $\xi_s = 1.0$ is distinguished by a thicker line. A dash-dotted line corresponds to the radial dependence of the anisotropy ratio in the case of linear propagation. Axial ratios deduced from the Kashima interplanetary scintillation (IPS) observations are shown with solid circles (slow solar wind) and open triangles (fast solar wind) for observations of (a)1992, (b)1993, and (c)1994. Vertical bars denote the estimation error, and horizontal bars denote the heliocentric distance bins for data average.

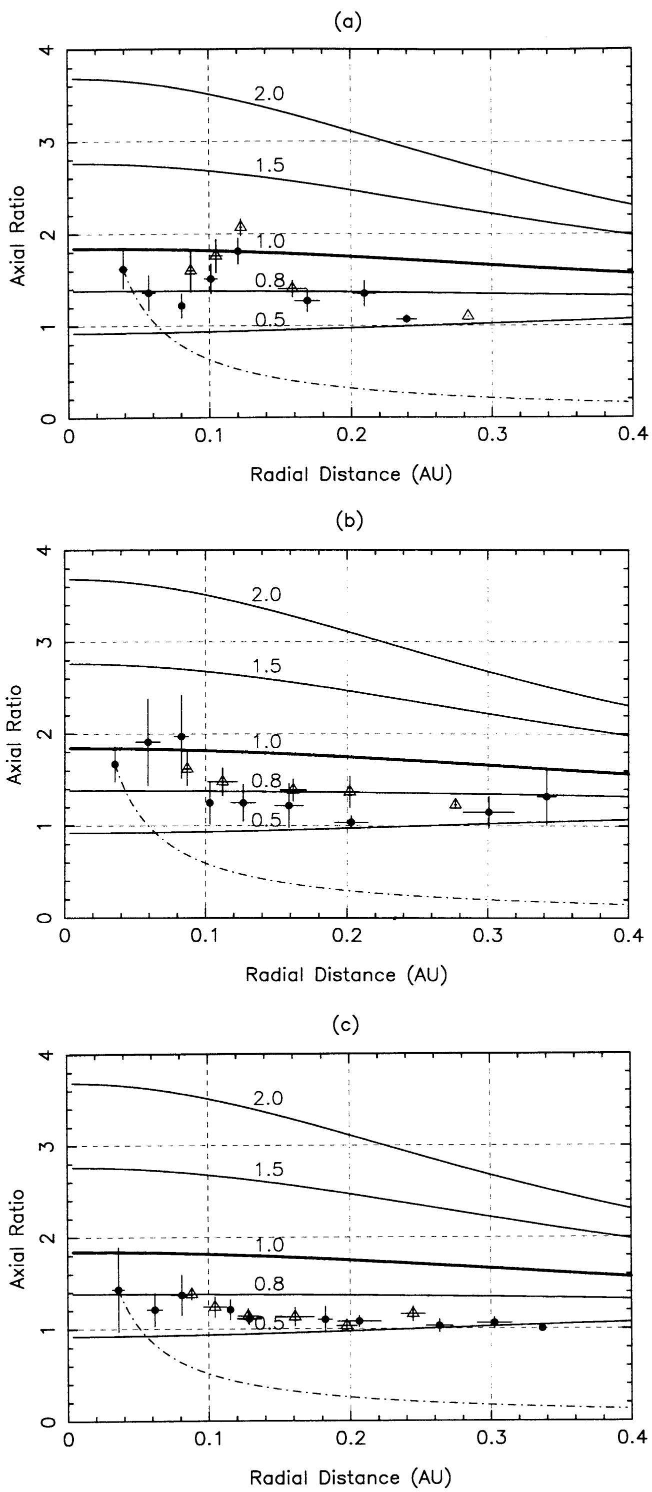

Figure 2. Same as Figure 1 but for radial dependence of anisotropy ratio of density fluctuations for fast magnetosonic waves ($n = 3.5$) (equation (20)): (a)1992, (b)1993, and (c)1994. Data shown with solid circles and open triangles are the same ones as those in Figure 1.

(20)) are calculated for the Kraichnan turbulence with a power exponent $n = 3.5$. The calculations of the anisotropy ratios were made for the anisotropy ratios of $\xi_{s,f} = 0.5$, 0.8, 1.0, 1.5, and 2.0. Our estimations show that the anisotropy ratios $\zeta_{s,f}$ calculated for Kolmogorov turbulence with $n = 11/3$ differ insignificantly from that for the Kraichnan turbulence. We can see from Figure 2 that ζ_f decreases with radial distance if $\xi_f > 0.5$ and that ζ_f increases if $\xi_f \leq 0.5$. The radial dependence of ξ_f (Figure 2) is rather weak at $0.5 \leq \xi_f < 1.0$ because the anisotropy is partially or completely compensated by the polarization factor $k_\perp^2/k^2$ in the density fluctuations' power spectrum (equation (10)). The comparison between Figures 1 and 2 shows that the radial dependence of ζ_f for $\xi_f \geq 1$ is sufficiently stronger than that of ζ_s for $\xi_s \geq 1$. Also, we present in Figures 1 and 2 for comparison the radial dependence of the anisotropy ratios corresponding to the propagation of waves in the linear regime by dash-dotted lines. The linear dependence of the components of wave vectors was found by *Toptygin* [1973] and *Hollweg* [1975] for Alfven waves propagating in a spherically symmetric solar wind moving with radial super-Alfvenic speed. In the case of linear propagation the wave vectors are approaching the radial direction with increasing distance from the Sun, namely, $k_r \approx$const and $k_{\phi,\theta} \sim 1/r$ (k_r is the radial component of $\mathbf{k}$, and $k_{\phi,\theta}$ are the components perpendicular to radial directions). The same dependence of the $\mathbf{k}$ components will be also valid for the fast and slow magnetosonic waves propagating in the linear regime in the super-Alfvenic and supersonic solar wind, so $\zeta_{s,f} \sim 1/r$: one of the principal axes of the irregularities elongation ellipsoid is in the radial direction but not in the direction of the magnetic field.

The measured values of the anisotropy ratios obtained by the analysis of the Kashima IPS observations [*Yamauchi et al.*, 1996, 1998] are presented in Figures 1 and 2 by the solid dots (slow solar wind) and by the open triangles (fast solar wind) together with their error limits. It is easily seen that for both slow (Figure 1) and fast (Figure 2) magnetosonic waves, we can not find a curve with $\xi_s =$const or $\xi_f =$const which would correspond to the observational data. This means that the anisotropy of the waves' power spectrum changes with heliocentric distance, namely, the average value of ξ_s decreases from $\xi_s \approx 1.5$ to $\xi_s \approx 1.0$, and the average value of ξ_f decreases from $\xi_f \approx 0.8$ to $\xi_f \approx 0.5$ as the heliocentric distance increases from 0.05 to 0.2 AU. The observational data reveal some tendency of $\xi_{s,f} \approx$const at $r > 0.2$ AU. We do not see any systematic difference in the values and in the radial dependence between the Kashima data for the slow and for the fast solar wind. The measured dependence $\zeta(r)$ is evidently incompatible with the expected radial dependence of anisotropy ratio for the linear regime, represented by dash-dotted lines in Figures 1 and 2.

We can conclude from the above comparison that no single curve with constant anisotropy ratio matches the observational data. That means that the geometric effects are too weak to explain the observed radial dependence of anisotropy of the irregularities responsible for IPS. This fact can be considered as evidence that the angular distribution of the magnetosonic turbulence evolves with heliocentric distance in such a way that the spatial power spectrum is enriched by waves propagating at smaller angles relative to the ambient magnetic field. The possible reason for this evolution is connected with the local nonlinear wave-wave interactions.

5. Discussion

We discuss now possible physical reasons for the anisotropy in the turbulence power spectra observed by radio occultation methods. The most probable source of magnetosonic waves and, consequently, density fluctuations is their local nonlinear excitation by Alfven waves propagating away from the Sun [*Hollweg*, 1975; *Chashei and Shishov*, 1981, 1983; *Zank and Matthaeus*, 1992; *Tu and Marsch*, 1995]. Indeed, the recent results of numerical simulations show that propagating Alfven disturbances, in the form of solitary waves or in the presence of drivers, generate density fluctuations [*Buti and Nocera*, 1999; *Velli et al.*, 1999; *Buti et al.*, 1999].

Assuming that the initial source of the Alfven disturbances is located at the coronal base, we follow the evolution of the wave vectors with distance from the Sun in an inhomogeneous expanding plasma. As mentioned in section 4, the linear effects of the evolution of Alfven wave vectors can be described by the relations $\omega = \mathbf{k}(\mathbf{V} + \mathbf{n}V_A)$ =const, $k_{\phi,\theta} \sim 1/r$. Because of the sharp gradient of the Alfven speed V_A in the chromosphere-corona transition layer, the angular spectrum of Alfven waves entering the corona and propagating out will be anisotropic with the predominance of perpendicular wave vector component. Another effect of the strong V_A gradient in the transition layer is that the turbulence in the corona becomes weak even if the initial turbulence is strong. If the initial power spectrum of Alfven waves is nearly isotropic and the nonlinear processes in the corona are negligibly weak, the anisotropy ratio ξ_{ac} for $\mathbf{k}$ vectors of Alfven waves propagating beyond the corona can be estimated as

$$\xi_{ac} \approx k_\perp/k_\| \approx (V_{Ac}/V_{A*})(\mathrm{R}_s/r_c), \tag{22}$$

where V_{Ac} and V_{A*} are the values of the Alfven speed at the outer boundary of the corona ($r = r_c$) and at the base of the transition layer ($r = r_* \approx R_s$). For the known values $V_{Ac} \sim 5 \times 10^7$ cm s^{-1}, $V_{A*} \sim 10^6$ cm s^{-1}, and $r_c \approx 1.5R_s$ [*Chashei*, 1999], we estimate from (22) the axial ratio $\xi_{ac} \approx 30$ for the Alfven waves entering the solar wind acceleration region from the corona. This means that the propagating magnetic field disturbances can be assumed to be radial at heliocentric distances greater than 2–3 R_s.

In the solar wind acceleration region located at heliocentric distances approximately between 1.5 and 10 R_s, we should take into account the linear effects, resulting in the evolution of the $\mathbf{k}$ vectors, in the relative increase of the turbulence level, and nonlinear processes. Among the possible nonlinear processes in the frequency range responsible for the scattering and scintillation effects, the most important one is the decay of an Alfven wave to the slow magnetosonic wave and the oppositely propagating Alfven wave [*Chashei and Shishov*, 1983]. For this reason the density fluctuations in the acceleration region are dominated by the slow magnetosonic waves, and cascading of the turbulent energy to the higher frequencies does not play a considerable role [*Chashei and Shishov*, 1983]. The decay process results quickly in the equipartition between the levels of the Alfven waves propagating in opposite directions and produces $k_\|^{-1}$ one-dimensional power spectra of magnetic field and density fluctuations [*Chashei*, 1999]. At the same time, the nonlinear evolution of $k_\perp$ distribution is comparatively slow, as its rate is proportional to the relative level of magnetosonic turbulence [*Chashei*, 1999]. Thus, in the acceleration region we have a specific regime for the small-scale turbulence, when the level of turbulence and frequency power spectra are defined by the nonlinear processes, and when the angular distribution of wave energy is controlled by the slow linear evolution of wave vectors (equation (22)). As a result, the elongation of density fluctuations along the magnetic field will be strong, with the axial ratio $\xi_s \approx \xi_a \approx 10$ at r = (3–5) R_s. This estimate is in good agreement with the radio scattering measurements made using the Very Large Array (VLA) [*Armstrong et al.*, 1990].

The changes of $k_\perp$ in nonlinear decay processes may result in isotropization of the turbulence power spectra. In the noncascading regime this effect becomes important at $r \geq$ (6–7) R_s for the wavelength corresponding to irregularity scales $\sim$ 20 km [*Chashei*, 1999]. Another reason for the isotropization of the spectra is connected with switching the cascading of turbulent energy from low frequencies to high frequencies. The regime of the solar wind turbulence becomes cascade dominated at heliocentric distances $r > 10R_s$, i.e., in the region of the fully evolved solar wind [*Chashei and Shishov*, 1983]. Simultaneously, the electron Landau damping of the fast magnetosonic waves is suppressed owing to wave-induced distortions of the electron velocity distribution function [*Chashei and Shishov*, 1983]. Thus one can expect that the anisotropy ratio of the turbulence power spectra decreases strongly at $r > 10R_s$ compared with its value at $r < 6R_s$.

Comparing the observational Kashima data, presented in Figures 1 and 2, with the results obtained by *Armstrong et al.* [1990], we can see that the anisotropy ratio of density fluctuations decreases strongly for $6R_s < r < 10R_s$. This is in good agreement with the qualitative model described above. The evolution of the anisotropy of density fluctuations continues also in the region of the fully evolved solar wind up to the heliocentric distance $r \approx 40R_s$. The axial ratio does not change considerably for $40R_s < r < 70R_s$, and its approximate constancy is confirmed also by the measurements of *Coles and Maagoe* [1972], who did not find any marked anisotropy at $r > 0.5$ AU. The strong difference between the measured radial dependence and the dependence expected from linear propagation means that the angular distribution of waves in the fully evolved solar wind is controlled by nonlinear processes. That the anisotropy ratio approaches a constant value implies that the initial anisotropy introduced by the conditions in the lower corona is lost in the distant regions owing to the nonlinear evolution of magnetosonic turbulence.

In the region of the fully evolved solar wind the Landau damping of slow magnetosonic waves on ions remains strong [*Hung and Barnes*, 1973; *Akhiezer et al.*, 1975]; for fast magnetosonic waves the Landau damping on ions is weak in low β plasma [*Akhiezer et al.*, 1975], and the Landau damping of electrons is suppressed by the distortions of the electron distribution function [*Chashei and Shishov*, 1983]. In this region the ratio V_S^2/V_A^2 (equation (10)) is not sufficiently small to compensate for the strong difference in the rates of linear damping, and the density fluctuations are caused mainly by the fast magnetosonic waves [*Chashei and Shishov*, 1985]. Another argument in favor of the fast magnetosonic waves is the observed radial dependence of the level of density fluctuations. According to the observational data [*Asai et al.*, 1998], the fractional density fluctuations are almost constant or depend very weakly on the heliocentric distance in the fully evolved solar wind. This is consistent with the first term in (10) and is inconsistent with the second term, if the ratio between magnetosonic waves' energy and Alfven wave energy does not increase strongly with heliocentric distance. Consequently, we can assume $\Phi^s_{\mathbf{k}} \ll \Phi^f_{\mathbf{k}}$ in (10). Then, according to Figure 2, the power spectrum of the magnetosonic turbulence passes the isotropic state $\xi_f = 1$ inside $r = 10R_s$, and $\xi_f < 1$ in the fully evolved solar wind. This means that the wave vectors of fast magnetosonic waves are typically orientated at angles less than 45° with respect to the ambient magnetic field. The probability of the nonlinear decay process f+Alfven wave $\rightarrow f$ from the participation of an Alfven wave and two fast magnetosonic waves approaches zero if $k_\perp \rightarrow 0$ [*Akhiezer et al.*, 1975], because in this case the fast magnetosonic wave turns into the nondecaying Alfven wave. For this reason the local "lifetime" of the magnetosonic waves with $k_\perp \ll k$ is greater than the local "lifetime" of magnetosonic waves with $k_\perp \approx k$, which explains some concentration of the wave vectors in the direction of the local magnetic field. We can consider the value $\xi_f \approx 0.5$, to which the anisotropy ratio approaches at large heliocentric distances, as the anisotropy characteristic of the fast magnetosonic wave power spectrum at the final stage of its quasi-steady evolution. In theoretical modeling of the solar wind compressible turbulence, the stationary solution of the set of kinetic equations for the multimode turbulence spectra should correspond to this state, which does not depend on initial conditions. Similar conclusions can be made, in principle, if the density fluctuations are dominated by the slow magnetosonic waves with the only difference that the power spectrum evolves to the nearly isotropic state. However, the interpretation with a dominant contribution of slow magnetosonic waves, as mentioned above, is much less probable for the fully evolved solar wind.

As mentioned in section 4, the Kashima data do not show any distinct difference in the anisotropy ratio between the solar winds at high and low latitudes. However, in situ measurements clearly show that in the period near solar activity minimum the properties of high-latitude fast solar wind and of low-latitude slow solar wind are different. The similarity between the high-latitude and the low-latitude Kashima data may indicate that there is no strong difference in the anisotropy of small-scale density irregularities in the low-speed and in the high-speed streams, because the main evolution of the spatial turbulence structure takes place in the acceleration region for both types of the solar wind. Another explanation is that the two-stream solar wind structure was not clearly pronounced during the Kashima observation series at the descending phase of the solar activity cycle in 1992-1994 or/and that the real difference was masked because some mixture of high-speed and low-speed streams was existing along the line of sight in the observations at high heliolatitudes. More refined analysis involving richer observational data as well as additional information about the global structure of the solar wind for each IPS observation would be desirable for the final conclusion about the similarity or difference between the anisotropy in the slow-speed and in the high-speed solar wind.

6. Conclusions

The following conclusions can be obtained from the above analysis. The anisotropy of the magnetosonic waves' angular distribution can be found from the axial ratio of the small-scale density fluctuations studied by radio occultation methods. For the fast magnetosonic waves the anisotropy of density fluctuations is caused by the anisotropic angular distribution of wave energy as well as by the polarization effects. The anisotropy ratio observed by IPS method is sensitive to the angle between the radial direction of the line of sight proximate point and the direction of the local magnetic field, and this sensitivity is larger for the fast magnetosonic density fluctuations than for the slow magnetosonic density fluctuations. We have presented the results of the density fluctuations' anisotropy measurements carried out using the Kashima IPS observations. These results show clearly the decrease of the anisotropy ratio for heliocentric distances between 10 and 40 R_s and the lack of distinct dependence beyond 40 R_s. We argued that the density fluctuations are caused by the fast magnetosonic waves in the considered range of solar wind. In the frame of this interpretation the observed radial dependence of the density fluctuations' anisotropy corresponds to the radial evolution of the fast magnetosonic waves angular distribution; the wave vector components of the fast magnetosonic waves evolve to be predominant parallel to the ambient magnetic field; that is, the spatial energy distribution is perpendicular to the magnetic field. However, the density irregularities measured using IPS observations at $r \geq 10R_s$ are slightly elongated parallel to the magnetic field, which is perpendicular to the above expected direction. This can be explained by the polarization effect owing to which the density fluctuations' anisotropy ratio is almost twice higher than the actual value. Combining our results with the observational data of *Armstrong et al.* [1990], we note the importance of detailed IPS observations in the region of heliocentric distances from 5 to (10–15) R_s. The density fluctuations' characteristics were observed to change at $r \sim 10R_s$. This change can be connected to the contribution change from mainly slow magnetosonic to mainly fast magnetosonic. Simultaneously, we can expect here the strong increase of the role of nonlinear processes in the evolution of the anisotropy of waves' angular distribution.

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