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Heating and acceleration of coronal ions interacting with plasma waves through cyclotron and Landau resonance

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Abstract. On the basis of quasi-linear theory, the parallel and perpendicular wave heating and acceleration rates for gyrotropic particle velocity distribution functions are derived. These rates can be used in anisotropic multicomponent fluid equations, in order to describe the wave-particle interactions of ions with, for examples, kinetic Alfvén and electromagnetic or electrostatic ion cyclotron, respectively, magnetosonic waves propagating along or obliquely to the mean magnetic field. The waves of coronal origin propagating away from the Sun into the interplanetary medium can resonantly heat the solar wind ions and accelerate minor ions preferentially with respect to the protons. Such processes are required in order to explain and understand the measured characteristics of ion velocity distributions in the solar wind and to interpret the recent spectroscopic evidence obtained from EUV emission line measurements made by the Solar and Heliospheric Observatory (SOHO) spacecraft, which indicate cyclotron-resonance-related line broadenings and shifts.

1. Introduction

In this introduction we provide the main empirical motivation for the subsequent algabraic and formal derivations made in this essentially theoretical paper. Many observations in the fast solar wind have revealed clear evidence for interplanetary heating ($T_j > 10^6$ K) and preferential acceleration of heavy ions with respect to the protons [*Schmidt et al.*, 1980; *Marsch et al.*, 1982b, 1982c; *Bochsler et al.*, 1985; *von Steiger et al.*, 1995; *Hefti et al.*, 1998]. In the data there is a clear statistical trend for the temperature ratio, T_j/T_p, to be mostly proportional to the heavy ion mass ratio, m_j/m_p, and sometimes even more than mass proportional [*Collier et al.*, 1996; *Cohen et al.*, 1996]. Furthermore, their differential speeds, $(\mathbf{U}_j - \mathbf{U}_\alpha)$, with respect to He^{2+} are highly correlated [*Schmidt et al.*, 1980; *von Steiger et al.*, 1995] and about equal. Since in fast streams Helium ions travel by about the local Alfvén speed faster than protons [*Marsch et al.*, 1982b, 1982c; *Neugebauer et al.*, 1996], one can conclude that all heavy ions move faster than protons in fast solar wind. For reviews of these phenomena, see, for example, the work of *Marsch* [1991] and *von Steiger et al.* [1995].

Recent spectroscopic observations of the widths and shifts of extreme ultraviolet (EUV) emission lines of heavy ions, as obtained from measurements made on the Solar and Heliospheric Observatory (SOHO), also indicate that the minor ions [*Kohl et al.*, 1997, 1998; *Li et al.*, 1999; *Cranmer et al.*, 1999a, 1999b] have excessively high kinetic temperatures in coronal holes and stream differentially. In particular, O^{5+} is found to travel much faster than the protons in coronal holes [*Cranmer et al.*, 1999a]. Generally, the coronal minor ions coming in various ionization stages are rather hot [*Seely et al.*, 1997; *Kohl et al.*, 1997; *Wilhelm et al.*, 1998], particularly in the polar coronal holes, where the electrons are observed, on the contrary, to be rather cold [*David et al.*, 1998; *Wilhelm et al.*, 1998]. The ions show some ordering of their kinetic temperatures according to the local gyrofrequencies [*Tu et al.*, 1998].

Tu et al. [1998, 1999] found a trend for the minor ion temperatures in the lower corona to increase with the mass-per-charge number. This was interpreted as a possible indication for cyclotron resonant processes to influence the heavy ion thermal speeds. All these Ultraviolet Coronagraph Spectrometer (UVCS) and Solar Ultraviolet Measurement of Emitted Radiation (SUMER)

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observations suggest the relevance of wave-particle interactions in the corona for the ion heating and acceleration, which may be explained by high-frequency cyclotron wave resonance by analogy to the model proposed for the solar wind by *Marsch et al.* [1982a]. How the high-frequency waves are generated in the corona remains an open problem.

Many years ago, *Marsch et al.* [1982a] and *Isenberg and Hollweg* [1983] already modeled alpha particle and heavy ion temperatures and speeds in the near-Sun solar wind at distances beyond $10R_S$, thereby employing the quasi-linear rates. Recently, *Marsch* [1999] has again applied these model equations to show that minor ions, such as oxygen and iron, can be strongly accelerated by left- and right-handed waves near the ion gyrofrequency to the in situ observed differential speeds and temperatures. In the past years, *Hu et al.* [1997], *Li et al.* [1997] and *Czechowski et al.* [1998] have done anisotropic multifluid calculations, using ad hoc mass-proportional heating functions for the heavy ions in the corona and wind or relative heating functions, in which the heat is shared between the ions according to the quasi-linear rates with a fixed wave power spectrum density (PSD).

In this paper we derive from quasi-linear theory (QLT) the heating and acceleration rates for particles interacting with broadband waves either in Landau or cyclotron resonance. These rates are the relevant transport terms, which supplement and, in the collisionless solar corona and wind, dominate the Coulomb collision-related momentum and energy transfer terms used in standard MHD or multifluid equations. We establish for any particle's species the rates, which must be used in the anisotropic multifluid equations, in order to describe adequately the resonant interactions of ions with, for example, parallel Alfvén and ion cyclotron waves or with oblique fast magnetosonic waves in the solar wind and the Sun's corona. These waves are most likely generated by violent small-scale reconnection events [*Axford and McKenzie*, 1997] in the magnetic network of the Sun, either directly or through inhomogeneity effects, or may be assumed to be fed from MHD-range fluctuations by a turbulent cascade to the kinetic dissipation domain [*Tu et al.*, 1984; *Tu and Marsch*, 1995; *Marsch*, 1999; *Leamon et al.*, 1998a, 1998b). This paper addresses the microphysics of the dissipation domain of the solar corona and wind plasma.

The inclusion of Landau damping with acoustic or cyclotron resonance with right-handed fast waves is very important (Landau damping was already considered by *Barnes and Hung* [1973], not only because these waves are emitted from the solar corona as well as their left-handed counterparts [*Behannon*, 1976; *Goldstein et al.*, 1994] but also because they provide a limiting mechanism for the differential velocity $\mathbf{U}_j - \mathbf{U}_p$ [*Marsch et al.*, 1982a; *Marsch*, 1999]. They become increasingly more important when the tails of the minor ion distributions move into resonance with these waves. The prominent role of the fast mode in regulating the ion differential speed has long been recognized [*Montgomery et al.*, 1976; *Gary et al.*, 1976; *Schwartz*, 1980; *Dum et al.*, 1980]. The nearly nondispersive magnetosonic waves may actually lead to a "trapping" of the bulk velocity at about the wave phase speed [see, e.g., *Marsch*, 1998, 1999], an effect which explains naturally the observed close correlation of $\mathbf{U}_\alpha - \mathbf{U}_p$ with $\mathbf{V}_A$ and the radial decrease in the He^{2+} differential speed as observed by Helios [*Marsch et al.*, 1982b, 1982c] between 0.3 and 1 AU.

Gomberoff et al. [1996] and *Gomberoff and Astudillo* [1999] have suggested the idea that it is the self-consistent evolution of the dispersion relation that could essentially control the acceleration and heating processes. The preferential acceleration of minor ions is shown to change the topology of the dispersion relation in a way which favors species with a low mass-per-charge ratio and allows at last only the protons to resonate with the waves [*Tu and Marsch*, 1999]. *Isenberg and Hollweg* [1982] and *McKenzie* [1994] have also analyzed the dispersion relation from the multiion fluid point of view. *Hu et al.* [1997, 2000] generalized the concept of wave action conservation to a multifluid situation.

Self-consistency of the model wave PSD calculations is quite important. Namely, if one speaks about ion cyclotron wave heating and acceleration of minor ions, one has to consider simultaneously the drastic perpendicular heating or parallel cooling caused by these waves in the proton distribution. This heating leads to an erosion of the original wave power spectrum in the frequency range corresponding to proton resonant speeds of a few thermal speeds. The eroded power is then not available anymore for affecting the heavy ions. Therefore the evolution of the wave PSD is inextricably linked with the evolution of all parameters that characterize the energy and momentum state of solar wind and coronal ions (see the recent paper by Tu and Marsch, Cyclotron wave heating and acceleration of solar wind ions in the corona, submitted to *Journal of Geophysical Research*, 2000) (hereinafter referred to as Tu and Marsch, submitted manuscript, 2000).

Following *Tu et al.* [1984], the low-frequency-wave cascade was first considered as a viable mechanism of coronal heating and solar wind acceleration by *Hollweg* [1986] and *Hollweg and Johnson* [1988]. Recently, *Hu et al.* [1999] discussed the differences between the models using the Kolmogorov- or Kraichnan-type cascade for the corona heating. *Matthaeus et al.* [1999] pointed out that quasi two-dimensional (2-D) turbulence, which is perpendicular to the main field and implies 2-D magnetic reconnection, may help in cascading energy from the low-frequency to the high-frequency range. Such a mechanism would also require considering Landau damping for wave dissipation.

2. Anisotropic Fluid Models Including Wave-Particle Interactions

The main idea of all the models mentioned so far is that preferential acceleration and heating of solar wind minor ions can be achieved via resonant interaction with ion cyclotron waves [*Hollweg and Turner*, 1978; *Dusenbery and Hollweg*, 1981; *McKenzie and Marsch*, 1982; *Marsch et al.*, 1982a] and also with magnetosonic waves [*Barnes and Hung*, 1973; *Marsch et al.*, 1982a]. *Dusenbery and Hollweg* [1981] did a comprehensive parameter study on the heating and acceleration of heavy ions by left-handed polarized waves for model wave power spectra prescribed according to observations [*Behannon*, 1976; *Denskat and Neubauer*, 1982]. These works assumed drifting bi-Maxwellians for the particle velocity distribution functions (VDFs) and found encouraging trends of the calculations in agreement with observed ion characteristics, showing $T_j/T_p \geq m_j/m_p$ and $U_j > U_p$. These ideas were further advanced by *Marsch et al.* [1982a]. Recently, *Marsch* [1999] and Tu and Marsch (submitted manuscript, 2000) have again considered a self-consistent fluid-type model, in which the radial evolution of wave spectra was calculated by taking local wave growth or damping into account within the framework of QLT as described in sections 3 and 4. We only quote here the stationary equations of motion for particles in a simple spherical geometry, where the distance from the Sun is denoted by r. The continuity equation reads

$$\frac{d}{dr}(r^2 \rho_j U_{j\|}) = 0 \,. \tag{1}$$

In the momentum equation, gravity and the electric field, stemming from the electron pressure gradient, need to be considered in the coronal hole and near-Sun solar wind, although the interplanetary electric field, $E_\|$, is usually smaller (since observations of SOHO and Helios indicate that $T_e \leq T_p \leq T_j$) than the partial ion pressure gradient. For spherical symmetry we have the following set:

$$\left(1 - \frac{V_{j\|}^2}{U_{j\|}^2}\right) U_{j\|} \frac{d}{dr} U_{j\|} - \frac{2V_{j\perp}^2}{r} + \frac{d}{dr} V_{j\|}^2 + \frac{G M_\odot}{r^2} - \frac{e_j}{m_j} E_\| = \frac{\partial}{\partial t} U_{j\|} \,. \tag{2}$$

The two energy equations, here in terms of the perpendicular and parallel thermal speeds squared, are

$$U_{j\|} \left(\frac{d}{dr} V_{j\perp}^2 + \frac{2V_{j\perp}^2}{r} \right) = \frac{\partial}{\partial t} V_{j\perp}^2 \,, \tag{3}$$

$$U_{j\|} \frac{d}{dr} V_{j\|}^2 + 2V_{j\|}^2 \frac{d}{dr} U_{j\|} = \frac{\partial}{\partial t} V_{j\|}^2 \,. \tag{4}$$

The right-hand sides are the big unknowns. In a collisionless medium these terms must be related to wave-particle interactions. These terms are generally nonlinear functions of the three moments, $U_{j\|}, V_{j\|}$, and $V_{j\perp}$, and functionals of the wave PSD. Even when assuming a rigid drifting bi-Maxwellian model velocity distribution function, the resulting set of equations is highly nonlinear. It evolves on multiple spatial scales, varying between the shortest scale, which is the wave length being typically of the order of $\lambda_A (= V_A/\Omega_p)$ or larger, and the largest scale, which is the solar radius, R_S, as the typical fluid scale in the corona and wind. These model equations comprise the standard double-adiabatic fluid equations but also include energy (temperature) and momentum (differential speed) transfer rates, which are obtained as integrals over the quasi-linear diffusion operator as shown in section 4.

It is necessary to emphasize the importance of the inhomogeneity of the expanding solar wind. An Alfvén wave originating in the corona with a frequency ω much less than the local gyrofrequency there, conserves its frequency propagating in the inertial frame away from the Sun and thus becomes gradually an ion cyclotron wave, which will at larger heliocentric distances, where $\omega \approx \Omega_p$, be damped and deliver its energy and momentum to the ions. This frequency sweeping mechanism has, in the context of recent solar wind and coronal funnel modeling, been shown to be capable of heating the protons [*Tu and Marsch*, 1997; *Marsch and Tu*, 1997] and also heavy ions [*Tu and Marsch*, 1999, submitted manuscript, 2000]. In contrast, *Matthaeus et al.* [1999] argued that it was not sufficient and some stronger turbulent cascade was needed.

3. Electric and Magnetic Field Fluctuations

Before we discuss the diffusion equation, we reiterate some of the basic equations and definitions of quasi-linear theory (QLT) needed subsequently. QLT has been described in many articles [*Kennel and Engelmann*, 1966] and textbooks. Here we refer mainly to the excellent books by *Melrose* [1986] and *Stix* [1992], and *Melrose and McPhedran* [1991]. QLT is quadratically nonlinear in the coupling terms between the fluctuations of the velocity distribution functions and the electromagnetic fields, but it is linear in the sense that these two types of fluctuations enter linearly in their product in the Vlasov equation. Hence the name QLT has been coined for this weak kinetic turbulence theory, in which only the reaction of the zeroth-order VDFs on the broadband wave spectrum is considered, while the wave-wave interactions and higher-order wave-particle interactions are neglected. The wave properties (such as dispersion and growth) are evaluated from linear dispersion theory with slowly time-varying VDFs and wave spectra, implying weak wave growth or dissipation.

In QLT it is assumed that the electromagnetic wave fields can generally be Fourier-decomposed in plane waves with the frequency, $\omega = \omega_M(\mathbf{k})$, and growth rate, $\gamma_M(\mathbf{k})$, for a particular wave mode (index M) and a given wave vector $\mathbf{k}$, which is assumed here to be directed arbitrarily with respect to the constant background field, $\mathbf{B}_0$. The background electric field is taken to be zero, and the background plasma may be multicomponent but is assumed to bear zero current and be quasi-neutral. The full dispersion equation for any linear plasma wave mode M in a multicomponent plasma can be found in the work of *Melrose* [1986]. *Mann et al.* [1997] have recently studied in detail the polarization properties of waves in a multicomponent plasma. QLT assumes the validity of the random-phase approximation, which ensures that no constructive interference occurs between the different waves modes, and thus these modes can be simply superposed linearly. Therefore we can write the Fourier-transformed electric field as

$$\tilde{\mathbf{E}}(\mathbf{k},\omega) = 2\pi \sum_M \delta\left[\omega - \omega_M(\mathbf{k})\right] \tilde{\mathbf{E}}_M(\mathbf{k}) . \tag{5}$$

The Fourier components of the electric field vector can be written in terms of the unimodular polarization vector, $\mathbf{e}_M(\mathbf{k})$, as follows:

$$\tilde{\mathbf{E}}_M(\mathbf{k}) = E_M(\mathbf{k})\mathbf{e}_M(\mathbf{k}) . \tag{6}$$

Similar expressions hold for the magnetic field, $\tilde{\mathbf{B}}_M(\mathbf{k})$, which is through the induction equation given by

$$\tilde{\mathbf{B}}_M(\mathbf{k}) = \frac{c}{\omega_M(\mathbf{k})} \mathbf{k} \times \tilde{\mathbf{E}}_M(\mathbf{k}) . \tag{7}$$

The spectral energy density of the electric field of mode M is given by $\mathcal{E}_M(\mathbf{k}) = \mid E_M(\mathbf{k}) \mid^2 / (8\pi)$ and evolves according to

$$\frac{\partial}{\partial t}\mathcal{E}_M(\mathbf{k}) = 2\gamma_M(\mathbf{k})\mathcal{E}_M(\mathbf{k}) , \tag{8}$$

which follows from the Fourier decomposition

$$\mathbf{E}_M(\mathbf{x},t) = \int d^3k \tilde{\mathbf{E}}_M(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} e^{-i\int_0^t dt' z_M(\mathbf{k},t')} , \tag{9}$$

where $\mathbf{x}$ is the spatial coordinate and t is the time. The growth rate, $\gamma_M(\mathbf{k})$, or damping rate if it is negative, together with the real frequency, $\omega_M(\mathbf{k})$, give the complex frequency, $z_M(\mathbf{k}) = \omega_M(\mathbf{k}) + i\gamma_M(\mathbf{k})$, whereby one has $\omega_M(\mathbf{k}) = -\omega_M(-\mathbf{k})$, $\gamma_M(\mathbf{k}) = +\gamma_M(-\mathbf{k})$, and thus $z_M^*(\mathbf{k}) = -z_M(-\mathbf{k})$. The asterisk indicates the complex conjugate number. Also, $\tilde{\mathbf{E}}_M^*(\mathbf{k}) = \tilde{\mathbf{E}}_M(-\mathbf{k})$, since the electric field in (9) must be real, so that $\mathcal{E}_M(\mathbf{k}) = \mathcal{E}_M(-\mathbf{k})$ by definition. It is often convenient to use the Doppler-shifted frequency denoted by a prime, $\omega'_M(\mathbf{k}) = \omega_M(\mathbf{k}) - k_\| U_{j\|}$, as measured in a frame of reference moving with the bulk speed component, $U_{j\|}$, of species j along $\mathbf{B}_0$.

4. Quasi-linear Diffusion Operator

The quasi-linear diffusion equation describes the evolution of the velocity distribution function of any particle species in an inertial frame of reference, in which the particles and waves are supposed to propagate. For magnetic-field-aligned waves it has been given, for example, by *Davidson* [1972] and applied by *Marsch et al.* [1982a] to solar wind ions. The general diffusion equation for any type of waves in a magnetized plasma and for gyrotropic background velocity distribution functions, $f_j(v_\|, v_\perp)$, has been derived originally by *Kennel and Engelmann* [1966] and is derived explicitly in the book of *Stix* [1992] or by *Melrose* [1986], who uses an elegant semiclassical treatment of radiation processes in plasma. We will throughout the paper assume that the VDF is normalized to a density of unity. The diffusion equation reads

$$\begin{aligned}&\frac{\partial}{\partial t} f_j(v_\perp, v_\|) \\ &= \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left\{v_\perp\left(D_{\perp\perp}\frac{\partial}{\partial v_\perp} + D_{\perp\|}\frac{\partial}{\partial v_\|}\right) f_j(v_\perp, v_\|)\right\} \\ &+\frac{\partial}{\partial w_\|}\left\{\left(D_{\|\perp}\frac{\partial}{\partial v_\perp} + D_{\|\|}\frac{\partial}{\partial v_\|}\right) f_j(v_\perp, v_\|)\right\} .\end{aligned} \tag{10}$$

In the diffusion tensor elements the species index j has been suppressed to ease the notation. It is evident that they depend on j. The velocities in the proper frame of species j are obtained by replacing the inertial frame coordinates as follows: $v_\perp \to w_\perp$, and $v_\| \to v_\| - U_{j\|} = w_\|$. Equation (10) is quoted here without derivation as the starting point of our paper.

Before we give the details of the diffusion tensor, we will define several quantities. Note that the Cartesian components of the wave vector (with gyrophase angle ψ) are $\mathbf{k} = (k_\perp \cos(\psi), k_\perp \sin(\psi), k_\|)$, and the velocity vector is $\mathbf{v} = (v_\perp \cos(\phi), v_\perp \sin(\phi), v_\|)$ (with gyrophase angle ϕ). When calculating the Fourier transform of the current density, one needs to consider the following expressions, which are obtained by using the generating function for Bessel functions:

$$\mathbf{v} e^{i(k_\perp v_\perp/\Omega_j)\sin(\phi-\psi)} = \sum_{s=-\infty}^{+\infty} e^{is(\phi-\psi)}\mathbf{V}(\mathbf{k},\mathbf{v},s) . \tag{11}$$

The new vector introduced here is defined as

$$\begin{aligned}\mathbf{V}(\mathbf{k},\mathbf{v},s) = \Big[&\frac{1}{2}v_\perp(e^{i\psi}J_{s-1} + e^{-i\psi}J_{s+1}), \\ &\frac{1}{2i}v_\perp(e^{i\psi}J_{s-1} - e^{-i\psi}J_{s+1}), v_\| J_s\Big] .\end{aligned} \tag{12}$$

The Bessel functions depend on the argument $x = (k_\perp v_\perp)/\Omega_j$, i.e., $J_s = J_s(x)$, and they obey the symmetry relation $J_s(-x) = (-1)^s J_s(x)$. The following definitions hold: The ion charge is e_j, mass is m_j, density is n_j, and the plasma frequency is $\omega_j^2 = (4\pi e_j^2 n_j)/m_j$.

The ion gyrofrequency, which by the definition carries the sign of the charge, reads as follows: $\Omega_j = (e_j B_0)/(m_j c)$. The fractional mass density of species j is defined as $\hat{\rho}_j = n_j m_j/\rho$, with the total mass density $\rho = \sum_j n_j m_j$. The Alfvén velocity is $V_A^2 = B_0^2/(4\pi\rho)$.

These definitions allow one to express concisely the dielectric as well as the diffusion tensor [*Melrose*, 1986], which can be generally written as

$$\begin{pmatrix} D_{\parallel\,\parallel} & D_{\parallel\,\perp} \\ D_{\perp\,\parallel} & D_{\perp\,\perp} \end{pmatrix}$$
$$= 8\pi^2 (\frac{e_j}{m_j})^2 \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} d^3k \sum_M \mathcal{E}_M(\mathbf{k}) \frac{1}{\omega_M^2(\mathbf{k})}$$
$$\times \sum_{s=-\infty}^{+\infty} \delta\left[\omega_M(\mathbf{k}) - s\Omega_j - k_\parallel v_\parallel\right] \quad (13)$$
$$\times \mid \mathbf{e}_M^*(\mathbf{k}) \cdot \mathbf{V}_j(\mathbf{k};\mathbf{v};s) \mid^2 \begin{pmatrix} k_\parallel^2 & k_\parallel \frac{s\Omega_j}{v_\perp} \\ k_\parallel \frac{s\Omega_j}{v_\perp} & (\frac{s\Omega_j}{v_\perp})^2 \end{pmatrix} .$$

5. Heating and Acceleration Rates as Velocity Moments

We can now take velocity moments of $\partial/\partial t f_j$ as given by the diffusion equation (10). The zeroth moment expresses the conservation of particle number density n_j. The first moment gives the bulk acceleration. The mean speed along $\mathbf{B}_0$ is given by $U_{j\parallel} =< v_\parallel >$. The heating rates are defined by the second parallel and perpendicular moments. We recall that the mean thermal speeds parallel and perpendicular to the field are defined by the second moments, $V_{j\parallel}^2 =< w_\parallel^2 >$ and $V_{j\perp}^2 =< w_\perp^2/2 >$, where the brackets stand for the full velocity space integration over the respective VDF of species j. Therefore we have

$$< \frac{\partial}{\partial t} f_j >= 2\pi \int_0^\infty dw_\perp w_\perp \int_{-\infty}^{\infty} dw_\parallel \frac{\partial}{\partial t} f_j = 0 \, , \quad (14)$$

expressing conservation of n_j, or of normalization to unity in our case. Note that the distribution function vanishes at infinity, which implies that $f_j(w_\perp, \pm\infty) = 0$ and $f_j(\infty, w_\parallel) = 0$. The first moment of (10) gives after a partial integration the bulk acceleration:

$$\frac{\partial}{\partial t} U_{j\parallel} =< v_\parallel \frac{\partial}{\partial t} f_j >=$$
$$- < \left(D_{\parallel\,\perp} \frac{\partial}{\partial v_\perp} + D_{\parallel\,\parallel} \frac{\partial}{\partial v_\parallel} \right) f_j(v_\perp, v_\parallel) > \, . \quad (15)$$

The heating rates are, after a partial integration, given by the following moments:

$$\frac{\partial}{\partial t} V_{j\parallel}^2 =< w_\parallel^2 \frac{\partial f_j}{\partial t} >= -2 < (v_\parallel - U_{j\parallel}) \left(D_{\parallel\,\perp} \frac{\partial}{\partial v_\perp} \right.$$
$$\left. + D_{\parallel\,\parallel} \frac{\partial}{\partial v_\parallel} \right) f_j(v_\perp, v_\parallel) > \, , \quad (16)$$

$$\frac{\partial}{\partial t} V_{j\perp}^2 =< \frac{w_\perp^2}{2} \frac{\partial f_j}{\partial t} >= - < v_\perp \left(D_{\perp\,\perp} \frac{\partial}{\partial v_\perp} \right.$$
$$\left. + D_{\perp\,\parallel} \frac{\partial}{\partial v_\parallel} \right) f_j(v_\perp, v_\parallel) > \, . \quad (17)$$

Note that the combination of diffusion coefficients and partial derivatives appearing in (16) and (17) yields terms transforming into the pitch angle gradient:

$$D_{\perp\,\perp} \frac{\partial}{\partial v_\perp} + D_{\perp\,\parallel} \frac{\partial}{\partial v_\parallel} \quad \mapsto \quad \left(v_\perp \frac{\partial}{\partial v_\parallel} + \frac{s\Omega_j}{k_\parallel} \frac{\partial}{\partial v_\perp} \right) . \quad (18)$$

The diffusion coefficients involve a sum over the wave modes, the wave vectors, and the spectrum. They involve a delta function describing the cyclotron or Landau resonance condition and a matrix element of the current density vector projected onto the polarization vector of the wave mode considered. These factors correspond, in the quantum language, to a transition probability for the wave-particle interaction [*Melrose*, 1986], and they express energy conservation in this microprocess.

Upon inserting (13) into (15), we can finally write the acceleration rate in the concise form

$$\rho_j \frac{\partial}{\partial t} U_{j\parallel} = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} d^3k \sum_M \mathcal{E}_M(\mathbf{k}) \left(\frac{\omega_j}{\omega_M(\mathbf{k})} \right)^2$$
$$\times \sum_{s=-\infty}^{+\infty} \int_{-\infty}^{+\infty} d^3v \frac{1}{v_\perp} T_M(\mathbf{k}, \mathbf{v}, s) \, , \quad (19)$$

where we have introduced the specific "transition probability", T_M (per velocity space volume unit). Note that by considering the defining equation, T_M has indeed the correct dimension (s^{-1}) and refers to the cyclotron ($s \neq 0$) and Landau ($s = 0$) resonant wave-particle interactions. We obtain

$$T_M(\mathbf{k}, \mathbf{v}, s) =$$
$$-2\pi\delta\left[\omega_M(\mathbf{k}) - s\Omega_j - k_\parallel v_\parallel\right] \mid \mathbf{e}_M^*(\mathbf{k}) \cdot \mathbf{V}_j(\mathbf{k};\mathbf{v};s) \mid^2$$
$$\times k_\parallel^2 \left[v_\perp \frac{\partial}{\partial v_\parallel} + \left(\frac{\omega_M(\mathbf{k})}{k_\parallel} - v_\parallel \right) \frac{\partial}{\partial v_\perp} \right] f_j(v_\perp, v_\parallel) \, . \quad (20)$$

The velocity distribution function is normalized to unity, whereby the density is absorbed in the plasma frequency ω_j of species j. With T_M the heating functions can similarly be written as

$$\rho_j \frac{\partial}{\partial t} V_{j\parallel}^2 = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} d^3k \sum_M \mathcal{E}_M(\mathbf{k}) \left(\frac{\omega_j}{\omega_M(\mathbf{k})} \right)^2$$
$$\times \sum_{s=-\infty}^{+\infty} \int_{-\infty}^{+\infty} d^3v \frac{2v_\parallel}{v_\perp} T_M(\mathbf{k}, \mathbf{v}, s) \, , \quad (21)$$

$$\rho_j \frac{\partial}{\partial t} V_{j\perp}^2 = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} d^3k \sum_M \mathcal{E}_M(\mathbf{k}) \left(\frac{\omega_j}{\omega_M(\mathbf{k})}\right)^2 \times \sum_{s=-\infty}^{+\infty} \int_{-\infty}^{+\infty} d^3v \frac{s\Omega_j}{k_\parallel v_\perp} T_M(\mathbf{k}, \mathbf{v}, s) . \quad (22)$$

Because of the delta function, the parallel intergration in the velocity space integrals can be carried out, whereby we introduce the s-order parallel resonant speed in the bulk frame of species j through

$$w_j(\mathbf{k}, s) = \frac{\omega'_M(\mathbf{k}) - s\Omega_j}{k_\parallel} . \quad (23)$$

It is convenient to introduce an s-order contribution to the antihermitian part (see again, *Melrose* [1986]) of the dielectric tensor (DT) via

$$\varepsilon_{j,s}^A = -i2\pi^2 \left(\frac{\omega_j}{\omega_M(\mathbf{k})}\right)^2 \frac{k_\parallel}{\mid k_\parallel \mid} \times \int_0^\infty dw_\perp \mathbf{V}_j(\mathbf{k}; \mathbf{v}; s) \mathbf{V}_j^*(\mathbf{k}; \mathbf{v}; s) \times \left[w_\perp \frac{\partial}{\partial w_\parallel} + \left(\frac{\omega'_M(\mathbf{k})}{k_\parallel} - w_\parallel\right) \frac{\partial}{\partial w_\perp} \right] \times f_j(w_\perp, w_\parallel) \mid_{w_\parallel = w_j(\mathbf{k}, s)} , \quad (24)$$

which appears in the integrands of (19), (21), and (22). It is well known that the antihermitian part of the DT determines the wave absorption and thus the wave energy dissipation (see again the book by *Melrose* [1986], or the classical text by *Stix* [1992]). What matters in the heating and acceleration rates are indeed the components of $\varepsilon_{j,s}^A$, which represent the absorption coefficients. This tensor when being contracted with the polarization matrix, $\mathbf{e}_M(\mathbf{k})\mathbf{e}_M^*(\mathbf{k})$, of wave mode M defines the strength of the absorption. We introduce the absorption coefficient or resonance function of species j through

$$\mathbf{e}_M^*(\mathbf{k}) \cdot \mathrm{Im}(\varepsilon_{j,s}^A) \cdot \mathbf{e}_M(\mathbf{k}) = \left(\frac{\omega_j}{\omega_M(\mathbf{k})}\right)^2 \mathcal{R}_j(\mathbf{k}, s) , \quad (25)$$

which only depends on $\mathbf{k}$ and, as a functional, on the VDF. Note that summation over all indices s and over species j gives the full antihermitian part of the dielectric tensor: $\sum_{s,j} \varepsilon_{j,s}^A = \varepsilon^A[\omega_M(\mathbf{k}), \mathbf{k}]$. The resonance function is by definition dimensionless and reads

$$\mathcal{R}_j(\mathbf{k}, s) = -(2\pi)^2 \frac{k_\parallel}{\mid k_\parallel \mid} \int_0^\infty dw_\perp \times \left[\mid \mathbf{e}_M^*(\mathbf{k}) \cdot \mathbf{V}_j(\mathbf{k}; \mathbf{v}; s) \mid^2 \left(w_\perp \frac{\partial}{\partial w_\parallel} + \frac{s\Omega_j}{k_\parallel} \frac{\partial}{\partial w_\perp} \right) \times f_j(w_\perp, w_\parallel) \right]_{w_\parallel = w_j(\mathbf{k}, s)} . \quad (26)$$

Finally, one can write the rates as follows:

$$\begin{pmatrix} \rho_j \frac{\partial}{\partial t} U_{j\parallel} \\ \rho_j \frac{\partial}{\partial t} V_{j\parallel}^2 \\ \rho_j \frac{\partial}{\partial t} V_{j\perp}^2 \end{pmatrix} = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} d^3k \sum_M \mathcal{E}_M(\mathbf{k}) \left(\frac{\omega_j}{\omega_M(\mathbf{k})}\right)^2 \times \sum_{s=-\infty}^{+\infty} \mathcal{R}_j(\mathbf{k}, s) \begin{pmatrix} k_\parallel \\ 2k_\parallel w_j(\mathbf{k}, s) \\ s\Omega_j \end{pmatrix} . \quad (27)$$

To calculate these rates explicitly, one must know the VDF and the wave dispersion relation and spectral intensity. From a strictly theoretical point of view, it is clear that assuming $f_j(\mathbf{w})$ and $\mathcal{E}_M(\mathbf{k})$ to be given is not a good approach and not self-consistent, since the electric PSD and particle VDF are expected to evolve substantially within a linear growth time, which is given by $1/\gamma_M(\mathbf{k})$. Yet this assumption has been made in most of the applications to the solar corona and wind. Note that, in general, the wave heating of the particles will be anisotropic, depending on the wave dispersion and intensity in dependence on the parallel and perpendicular wave vector components.

6. Energy Considerations and Dielectric Properties

By summing up (19), (21), and (22) we obtain the total rate of change of the thermal and kinetic energy for the particles of species j. This relation can be further summed up over all particle species and, by using the dispersion relation, modified to obtain the total energy conservation law within QLT for a multicomponent plasma [see, e.g., *Davidson*, 1972]. We sum up the components in the columns of the rates in (27) to obtain the change in total kinetic energy density of species j. By exploiting (23) and (25) this gives

$$K_j = \frac{1}{2} \rho_j \frac{\partial}{\partial t} (U_{j\parallel}^2 + V_{j\parallel}^2 + 2V_{j\perp}^2) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} d^3k \sum_M \mathcal{E}_M(\mathbf{k}) \times 2\,\mathrm{Im} \left\{ \mathbf{e}_M^*(\mathbf{k}) \cdot \varepsilon_j^A [\omega_M(\mathbf{k}), \mathbf{k}] \cdot \mathbf{e}_M(\mathbf{k}) \right\} . \quad (28)$$

This relation can also be directly obtained by calculating the work done by the electric field on the current density associated with species j:

$$K_j = \langle \mathbf{J}_j(\mathbf{x}, t) \cdot \mathbf{E}(\mathbf{x}, t) \rangle = \lim_{T,V \to \infty} \frac{1}{(2\pi)^3} \int \frac{d\omega}{T} \int \frac{d^3k}{V} \left[\tilde{\mathbf{J}}_j(\mathbf{k}, \omega) \cdot \tilde{\mathbf{E}}^*(\mathbf{k}, \omega) \right] . \quad (29)$$

Here the angle brackets mean space-time averaging in the limit of infinite volume V and time period T. Since the current density is related to the electric field through the conductivity tensor and hence the dielectric tensor (see, e.g., the general sections by *Melrose* [1986], or any other plasma physics textbook such as that by *Stix* [1992]), we can write the last equation as

$$K_j = \lim_{T,V\to\infty} \frac{1}{(2\pi)^3} \int \frac{d\omega}{T} \int \frac{d^3k}{V} \frac{1}{8\pi} \times 2\,\mathrm{Im}\left[\omega\, \tilde{\mathbf{E}}^*(\mathbf{k},\omega) \cdot \varepsilon_j^A(\mathbf{k},\omega) \cdot \tilde{\mathbf{E}}(\mathbf{k},\omega)\right] . \quad (30)$$

By making use of (5) and (6), the formal relation $[2\pi\delta(\omega)]^2 = T2\pi\delta(\omega)$, and the symmetry properties of the Fourier amplitudes, we retain by inserting all these relations the previous result (equation (28)) . Similarly, one obtains the volumetric acceleration rate, R_j, of species j from the equation

$$m_j\mathbf{R}_j =< \rho_j^e(\mathbf{x},t)\mathbf{E}(\mathbf{x},t) > + \frac{1}{c} < \mathbf{J}_j(\mathbf{x},t) \times \mathbf{B}(\mathbf{x},t) > = \lim_{T,V\to\infty} \frac{1}{(2\pi)^3} \int \frac{d\omega}{T} \int \frac{d^3k}{V} \mathbf{k}\, \tilde{\mathbf{E}}^*(\mathbf{k},\omega) \cdot \tilde{\mathbf{J}}_j(\mathbf{k},\omega), \quad (31)$$

whereby the charge density is denoted as $\rho_j^e (= e_j n_j)$ and the continuity equation for the charge conservation has been used. It is thus obvious that the parallel acceleration $R_{j\|} = \mathbf{R}_j \cdot \hat{\mathbf{B}}_0$ and the mean heating rate $Q_j = K_j - R_{j\|}U_{j\|}$ can be expressed by the dielectric tensor, yet this is not true anymore for the parallel and perpendicular heating rates individually, $Q_{j\|}$ and $Q_{j\perp}$, which require not just the components of the current density but also higher-order moments, related to the pressure perturbations, to be calculated.

7. Rates in Terms of the Magnetic Field Spectrum

It is, for plasma waves at frequencies near the ion gyrofrequency Ω_j and below, more convenient to work with the magnetic field spectral density instead of electric field. Using (7), we find

$$\mathcal{B}_M(\mathbf{k}) = \left(\frac{kc}{\omega_M(\mathbf{k})}\right)^2 (1 - \mid \hat{\mathbf{k}} \cdot e_M(\mathbf{k}) \mid^2)\mathcal{E}_M(\mathbf{k}) . \quad (32)$$

We can also make use of the relation $\hat{\rho}_j\Omega_j^2 = \omega_j^2 V_A^2/c^2$ and then rewrite the rates as

$$\begin{pmatrix} \frac{\partial}{\partial t}U_{j\|} \\ \frac{\partial}{\partial t}V_{j\|}^2 \\ \frac{\partial}{\partial t}V_{j\perp}^2 \end{pmatrix} = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} d^3k \sum_M \hat{\mathcal{B}}_M(\mathbf{k}) (\frac{\Omega_j}{k})^2 \frac{1}{1 - \mid \hat{\mathbf{k}} \cdot e_M(\mathbf{k}) \mid^2} \times \sum_{s=-\infty}^{+\infty} \mathcal{R}_j(\mathbf{k},s) \begin{pmatrix} k_\| \\ 2k_\| w_j(\mathbf{k},s) \\ s\Omega_j \end{pmatrix} , \quad (33)$$

where we have introduced the spectrum $\hat{\mathcal{B}}_M(\mathbf{k})$, which is normalized to the background magnetic field density, $B_0^2/8\pi$. Equation (33) is the main result of this paper. It expresses the general parallel acceleration rate and perpendicular and parallel heating rates in terms of a weighted spectral average and the sums over the mode number M and resonance-order number s for resonant wave-particle interactions and gyrotropic velocity distribution functions. These rates can be evaluated once the VDF, $f_j(w_\perp, w_\|)$, and the PSD, $\hat{\mathcal{B}}_M(k_\perp, k_\|)$, are known explicitly. In the solar wind literature, often a simple power law form of the spectrum was assumed, such as in the early studies by *Isenberg and Hollweg* [1983] or recently by *Cranmer et al.* [1999a, 1999b] for coronal hole heating. However, as already *Marsch et al.* [1982a] and recently *Marsch* [1999] and *Tu and Marsch* [1999; submitted manuscript, 2000], have shown, this approximation is not sufficient, and therefore they used the quasi-linear equation (8) to calculate the spectrum self-consistently.

8. Resonance Function for a Drifting bi-Maxwellian

Observationally, it is found in interplanetary space [*Marsch et al.*, 1982b, 1982c] that the prominent features of the ion velocity distribution functions are a core temperature anisotropy and a secondary proton beam or heavy ion component, i.e., an ion beam streaming along $\mathbf{B}_0$ at a speed of 1-1.5 times the local Alfvén speed. These features can be modeled by a VDF which is composed of one (or several) relatively drifting bi-Maxwellian, which is given by

$$f_j(\mathbf{v}) = \frac{1}{(2\pi)^{3/2} V_{j\|} V_{j\perp}^2} \exp\left(-\frac{(v_\| - U_{j\|})^2}{2V_{j\|}^2} - \frac{(v_\perp)^2}{2V_{j\perp}^2}\right) . \quad (34)$$

With this VDF the heating and acceleration rates can be calculated largely analytically from (19), (21), and (22). We do not restrict the waves to any propagation direction but include oblique propagation as well (leading, e.g., to electrostatic ion cyclotron waves, kinetic Alfvén waves, which have recently been revisted by *Hollweg* [1999a], or magnetosonic and ion acoustic waves). In coronal holes the waves propagate mainly away from the solar surface into interplanetary space. The corresponding normalized magnetic field spectra are denoted by the symbol $\hat{\mathcal{B}}_M$. The rates are given in section 7.

For the model VDF of a drifting bi-Maxwellian (equation (34)) the resonance function can be evaluated analytically and reads

$$\mathcal{R}_j(\mathbf{k},s) = \frac{2}{V_{j\perp}^4} \int_0^\infty dw_\perp w_\perp \exp\left[-\frac{1}{2}(\frac{w_\perp}{V_{j\perp}})^2\right] \times \mid \mathbf{e}_M^*(\mathbf{k}) \cdot \mathbf{V}_j(\mathbf{k};\mathbf{v};s) \mid^2_{v_\|=w_j(\mathbf{k},s)+U_{j\|}} \times \sqrt{\frac{\pi}{2}} \frac{k_\|}{\mid k_\| \mid} \exp\left[-\frac{1}{2}\xi_j^2(\mathbf{k},s)\right] \times \left(\xi_j(\mathbf{k},s)\,\frac{T_{j\perp}}{T_{j\|}} + s\frac{\Omega_j}{k_\| V_{j\|}}\right) , \quad (35)$$

with the normalized real part of the resonant speed being defined by $\xi_j(\mathbf{k},s) = w_j(\mathbf{k},s)/V_{j\|}$. The perpendicular integration can also be carried out, if the matrix element is further evaluated. This requires some straightforward but lengthy calculations. We use the defining equation (12) and introduce the new polarization vectors

$$\mathbf{e}_M^{\pm}(\mathbf{k}) = e^{\mp i\psi}\left[\mathbf{e}_{Mx}(\mathbf{k}) \pm i\mathbf{e}_{My}(\mathbf{k})\right] , \tag{36}$$

in which the gyrophase angle ψ of $\mathbf{k}$ has been absorbed in the exponential phase factor, and we can then simply write the squared matrix element as

$$\mid \mathbf{e}_M^* \cdot \mathbf{V}_j \mid^2 = \mid \frac{v_\perp}{2}(J_{s-1}e_M^+ + J_{s+1}e_M^-) + v_\| J_s e_{Mz} \mid^2 . \tag{37}$$

Here we omitted, to ease the notation, the arguments for a moment. For $k_\perp = 0$ one has $J_s(0) = \delta_{s,0}$, and the vector $e_M^{\pm}$ corresponds to left- and right-hand polarization. Then the matrix element factorizes into the three independent contributions from ion cyclotron waves, magnetosonic waves, and the field-aligned longitudinal slow-mode wave. Otherwise, there are also mixed terms involving all products of the three components of $e_M(\mathbf{k})$. Using the Bessel function relation,

$$J_{s\pm 1}(x) = \frac{s}{x}J_s(x) \mp J_s'(x) , \tag{38}$$

we can also write the matrix element, evaluated at the resonance, $w_\| = w_j(\mathbf{k},s)$, as follows:

$$\mid \mathbf{e}_M^*(\mathbf{k}) \cdot \mathbf{V}_j(\mathbf{k};\mathbf{v};s) \mid^2_{v_\| = w_j(\mathbf{k},s)+U_{j\|}} = \left| \frac{s\Omega_j}{k_\perp}\left[J_s(x)e_{Mx}^* - \frac{x}{s}J_s'(x)\, ie_{My}^*\right] + \frac{\omega_M(\mathbf{k}) - s\Omega_j}{k_\|}J_s(x)e_{Mz}^* \right|^2 . \tag{39}$$

Note that for $x \ll 1$ the expansion of the Bessel function gives $J_s(x) = (x/s)^s/s!$, and then the normalized matrix element can be written as

$$\mid \mathbf{e}_M^* \cdot \mathbf{V}_j \mid^2 \frac{1}{V_{j\perp}^2} \approx J_s(x)^2 \left| \mathbf{e}_M^* \cdot \left(\frac{s\Omega_j}{k_\perp V_{j\perp}}, -i\frac{s\Omega_j}{k_\perp V_{j\perp}}, \frac{\omega_M(\mathbf{k}) - s\Omega_j}{k_\| V_{j\perp}}\right)\right|^2 . \tag{40}$$

This is a concise form but has the disadvantage that the limit of vanishing $k_\perp$ is not evident but requires an expansion of the Bessel function. To calculate (35) further without any expansion of the Bessel functions, one has to write the squared matrix element as a quadratic form in the components of $\mathbf{e}_M(\mathbf{k})$ and then carry out the integration in (35) over the perpendicular velocity component. This procedure leads to three standard integrals over Bessel functions [see, e.g., *Stix*, 1992] defined as follows:

$$\int_0^\infty d\left(\frac{w_\perp^2}{2V_{j\perp}^2}\right)\exp\left(-\frac{w_\perp^2}{2V_{j\perp}^2}\right)\begin{pmatrix} J_s^2(x) \\ x\,J_s(x)\,J_s'(x) \\ [x\,J_s'(x)]^2 \end{pmatrix} = \begin{pmatrix} I_s(q_j) \\ q_j\,[I_s'(q_j) - I_s(q_j)] \\ s^2 I_s(q_j) - 2q_j^2\,[I_s'(q_j) - I_s(q_j)] \end{pmatrix}\exp(-q_j), \tag{41}$$

where we defined the new variable, $q_j = (k_\perp V_{j\perp})^2/\Omega_j^2$, and made use of the modified Bessel function, $I_s(q_j)$. It is convenient to abbreviate the factor involving the number of resonant particles and the pitch angle anisotropy in (35) as $N_j(\mathbf{k},s)$; that is, we define

$$N_j(\mathbf{k},s) = \sqrt{\frac{\pi}{2}}\frac{k_\|}{\mid k_\| \mid}\exp\left[-\frac{1}{2}\xi_j^2(\mathbf{k},s)\right] \times\{\xi_j(\mathbf{k},s)\frac{T_{j\perp}}{T_{j\|}} + s\frac{\Omega_j}{k_\| V_{j\|}}\} . \tag{42}$$

The remaining perpendicular integration can by help of (41) be performed. After some lengthy but straightforward algebra we obtain the resonance function, or the wave absorption (damping) or emission (growth) coefficient, for any species j in the compact form

$$\mathcal{R}_j(\mathbf{k},s) = N_j(\mathbf{k},s)\,2\exp(-q_j)\{a_- I_{s-1}(q_j) + a_+ I_{s+1}(q_j) + a_0 I_s(q_j)\} . \tag{43}$$

Such a separation is only possible for a gyrotropic distribution, such as a bi-Maxwellian, in which the parallel and perpendicular velocity distributions factorize into two independent Gaussians. The coefficients $a_\pm$ and a_0 contain essentially the polarization vector components and read as follows:

$$\begin{aligned} a_0(\mathbf{k},s) &= \left(\frac{\omega_M(\mathbf{k}) - s\Omega_j}{k_\| V_{j\perp}}\right)^2 \mid e_{Mz} \mid^2 \\ &- 2\,\mathrm{Im}\{e_{My}^* e_{Mz}\}\frac{k_\perp}{k_\|}\frac{\omega_M(\mathbf{k}) - s\Omega_j}{\Omega_j} \\ &+ 2\left(\frac{k_\perp V_{j\perp}}{\Omega_j}\right)^2 \mid e_{My} \mid^2 , \end{aligned} \tag{44}$$

$$\begin{aligned} a_\mp(\mathbf{k},s) &= \pm\frac{s}{2} \mid e_M^{\pm} \mid^2 \\ &\pm \mathrm{Re}\{e_{Mz}^* e_M^{\pm}\}\frac{k_\perp}{k_\|}\frac{\omega_M(\mathbf{k}) - s\Omega_j}{\Omega_j} \\ &\mp \left(\frac{k_\perp V_{j\perp}}{\Omega_j}\right)^2 \mathrm{Im}\{e_{My}^* e_M^{\pm}\} . \end{aligned} \tag{45}$$

Note that these coefficients are well defined for both cases, $k_\perp = 0$ and $s = 0$, and that they attain for parallel propagation a simple form. Parallel propagation leads to

$$\mathcal{R}_j(0, k_{\|}, s) = N_j(0, k_{\|}, s)\{\delta_{s,1} \mid e_M^+ \mid^2 + \delta_{s,-1} \mid e_M^- \mid^2 + 2\delta_{s,0} \mid e_{Mz} \mid^2 (\frac{\omega_M(k_{\|})}{k_{\|} V_{j\perp}})^2\} . \quad (46)$$

At this point one needs the dispersion relation and the polarization vectors explicitly in order to calculate the detailed form of the resonance function. We do not intend to present the general case here, because this is not the purpose of our paper. A lucid treatment of the full cold plasma wave polarization can be found in the work of *Melrose* [1986, pp. 167 - 168]. The validity of the MHD modes is restricted to small **k**, as compared to the proton inertial length. Then the coefficients depend only on $k_\perp$ and $k_{\|}$.

9. Summary and Conclusions

In this paper we have derived the acceleration and heating rates (see equations (27) and (33)) for any particle species (electrons, protons, and heavier ions) according to QLT in the limit of weak wave damping or growth and under the weak turbulence assumption that the waves can be adequately described as a random-phased superposition of a broadband spectrum of the linear normal modes existing in a multicomponent plasma. These rates are functionals of the background particle VDF and wave PSD, which both are assumed to be gyrotropic. The rate equations derived here and the related more restricted or special equations decribed earlier by *Marsch* [1998, 1999] can be incorporated in multifluid models of the solar corona and wind, thus complementing and refining the models put forward recently for the solar wind by *Li et al.* [1997, 1999], *Czechowski et al.* [1998], *Hu et al.* [1999], *Hu and Habbal* [1999], and *Cranmer et al.* [1999a, 1999b].

The rates depend essentially on what was called here the resonance function, $\mathcal{R}_j$, which describes the wave absorption of a species j and is proportional to the number of resonant particles through the pitch angle gradient of $f_j(\mathbf{w})$ at the resonant speed for either cyclotron or Landau resonance. The wave dispersion and polarization play an important role. A detailed study of the resonance function, in terms of a surface over the wave vector plane, for arbitrary wave propagation angle with respect to the background magnetic field should be carried out in the future in order to evaluate quantitatively the effects of wave polarization and dispersion on the wave dissipation or absorption. For a proton-alpha-particle plasma *Li and Habbal* [2000a, 2000b] have recently performed an instructive parameter study on the growth rates for the parallel ion cyclotron and fast-magnetocoustic waves and their corresponding heating rates, in particular, as a function of the ion differential speed. Their papers also summarize the relevant literature within the solar wind context.

The influence of resonant wave-particle interactions on the dynamics of minor heavy ions in the outer solar corona has been investigated recently again by *Marsch* [1999] and Tu and Marsch [submitted manuscript, 2000], whose papers describe successfully some basic characteristics of observed proton and α-particle distributions [*Marsch et al.*, 1982b, 1982c] near 0.3 AU. It has been shown that ion cyclotron waves propagating away from the Sun along $\mathbf{B_0}$ are capable of accelerating heavy ions through the proton bulk speed. These waves preferentially heat the minor ions perpendicular to the field and raise in this way their average temperature until T_j amounts to a considerable multiple, more than A_j times, of the proton temperature. *Hollweg* [1999b, 1999c, 1999d] has recently approached this problem from a different point of view, using coronal ions moving as test particles in a potential well (arising from gravity, electric field, and time-varying magnetic field), and he comes to similar conclusions. Also, *Cranmer* [2000] has calculated the summed effect of more than 2000 heavy ion species in the corona on wave damping. Since the requirements for the validity of QLT are fulfilled in the corona and solar wind, we do not see any reason why this theory should not be applied to all kinds of ions with very low abundances.

The inhomogeneity of the expanding corona and wind (note that V_A and Ω_p decrease with increasing solar distance) has the effect that the fastest heavy ions in the tails of the distributions move into increasingly stronger resonance with right-handed polarized magnetosonic waves. These waves now further accelerate the heavy ions until their differential speed is about V_A. Also, the waves can preferentially heat the particles parallel to the field to the effect that finally their temperature anisotropy shows the signature of fast-wave heating, with $T_{j\|} > T_{j\perp}$ (a signature which can also arise as the outcome of magnetic moment conservation for ions expanding in a magnetic mirror) and that $V_{j\|} \geq V_{p\|}$ is accomplished. This relation between ion thermal speeds is one of the striking observed characteristics of heavy solar wind ions [*Schmidt et al.*, 1980; *Marsch et al.*, 1982b, 1982c; *von Steiger et al.*, 1995; *Collier et al.*, 1996; *Cohen et al.*, 1996]. So far, only the papers by *Marsch et al.* [1982a] and *Marsch* [1999] have explained this as wave-friction-induced trapping of heavy ions at the Alfvén speed. The present work provides the rates in (27) and (33), which enable us to include in fluid models oblique wave propagation and to incorporate the Landau damping of obliquely propagating compressive waves, effects that will become important for the perpendicular energy cascade of 2-D turbulence [*Leamon et al.*, 1998a, 1998b; *Matthaeus et al.*, 1999].

The wave absorption coefficient described here by the resonance function $\mathcal{R}_j$ depends crucially upon the shape of the VDF at the resonance. Commonly, a drifting bi-Maxwellian VDF is assumed, for which we derived $\mathcal{R}_j$

in (43). Indeed, the in situ observed VDF of protons [*Marsch et al.*, 1982b; *Marsch and Goldstein*, 1983] have mostly a Gaussian shape as a function of $v_\perp$ but deviate considerably from it with a sizable skewness as a function of $v_\parallel$. It is easy to generalize our rates to such a situation, where we describe the VDF in terms of a product of two reduced distribution functions [see, e.g., *Dum et al.*, 1980; *Marsch*, 1998], in order to adjust the rates for a larger class of VDFs. Yet it would be better to evaluate and model the VDF self-consistently. First steps in this direction were done by *Tam and Chang* [1999] and P.A. Isenberg et al. (A kinetic model of coronal heating and acceleration by ion-cyclotron waves: Preliminary results, submitted to *Solar Physics*, 2000).

Self-consistency of the PSD and VDFs is very important. The wave spectrum cannot be fixed, but its radial evolution must be calculated as well, because proton and ion damping or wave excitation can lead to a dramatic reshaping of the originally injected wave spectrum, or even to a complete erosion of the wave power [*Marsch et al.*, 1982a; *Marsch*, 1999; Tu and Marsch (submitted manuscript, 2000]. Also *Cranmer* [2000] deals specifically with the issue that damping has to be considered for a specific ion species. Some ions with extremely low abundance may not affect the wave spectrum at all, while the most abundant ones will lead to severe wave dissipation. Consequently, the self-consistent relaxation times toward a dynamic equilibrium between heavy ions and waves can be orders of magnitudes larger than the ones estimated on the basis of rigid wave spectra. On the other hand, an exact evaluation of the wave spectra requires integrations to be carried out on the kinetic scales of the ions, which is not what one wants in an average fluid-like description of the expanding corona and wind (see again the equations (1) – (4)). As a compromise or for the sake of simplicity, one may rely on fixed wave spectra, such as *Cranmer et al.* [1999a, 1999b], *Hu et al.* [1999, 2000] and others did recently in their coronal hole models; yet one must be aware of the serious limitations and shortcomings of such an approach.

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References

Axford, W. I., and J. F. McKenzie, The solar wind, in *Cosmic Winds and The Heliosphere*, edited by J. R. Jokipii, C. P. Sonett, and M. S. Giampapa, pp. 31–66, Univ. of Ariz. Press, Tucson, 1997.

Barnes, A., and R. J. Hung, On the kinetic temperature of He in the solar wind, *Cosmic Electrodyn.*, *3*, 416, 1973.

Behannon, K. W., Observations of the Interplanetary Magnetic Field Between 0.46 and 1 AU by the Mariner 10 Spacecraft, *GSFC X Doc.*, *692-76-2*, NASA, Goddard Space Flight Center, Greenbelt, Maryland, USA, 1976.

Bochsler, P., J. Geiss, R. Joos, Kinetic temperatures of heavy ions in the solar wind, *J. Geophys. Res.*, *90*, 10779, 1985.

Cohen, C.M.S., M.R. Collier, D.C. Hamilton, G. Gloeckler, R.B. Sheldon, R. von Steiger, and B. Wilken, Kinetic temperature ratios of O^{6+} and He^{2+}: Observations from Wind/MASS and Ulysses/SWICS, *Geophys. Res. Lett.*, *23*, 1187, 1996.

Collier, M.R., D.C. Hamilton, G. Gloeckler, P. Bochsler, and R.B. Sheldon, Neon-20, Oxygen-16, and helium-4 densities, temperatures, and suprathermal tails in the solar wind determined with WIND/MASS, *Geophys. Res. Lett.*, *23*, 1191, 1996.

Cranmer, S. R., Ion cyclotron wave dissipation in the solar corona: The summed effect of more than 2000 ion species, *Astrophys. J.*, *532*, 1197, 2000.

Cranmer, S. R., et al., An empirical model of a polar coronal hole at solar minimum, *Astrophys. J.*, *511*, 481, 1999a.

Cranmer, S. R., G. B. Field, and J. L. Kohl, Spectroscopic constraints on models of ion cyclotron resonance heating in the polar solar corona and high-speed solar wind, *Astrophys. J.*, *518*, 937, 1999b.

Czechowski, A., R. Ratkiewicz, J. F. McKenzie, and W. I. Axford, Heating and acceleration of minor ions in the solar wind, *Astron. Astrophys.*, *335*, 303, 1998.

David, C., et al., Measurement of the electron temperature gradient in a solar coronal hole, *Astron. Astrophys.*, *336*, L90, 1998.

Davidson, R. C., *Methods in Nonlinear Plasma Theory*, Academic, San Diego, Calif., 1972.

Denskat, K. U., and F. M. Neubauer, Statistical properties of low-frequency magnetic field fluctuations in the solar wind from 0.29 to 1 AU during solar minimum conditions: Helios-1 and Helios-2, *J. Geophys. Res.*, *87*, 2215, 1982.

Dum, C. T., E. Marsch, and W. G. Pilipp, Determination of wave growth from measured distribution functions and transport theory, *J. Plasma Phys.*, *23*, 91, 1980.

Dusenbery, P. B., and J. V. Hollweg, Ion-cyclotron heating and acceleration of solar wind minor ions, *J. Geophys. Res.*, *86*, 153, 1981.

Gary, S. P., M. D. Montgomery, W. C. Feldman, and D. W. Forslund, Proton temperature anisotropy instabilities in the solar wind, *J. Geophys. Res.*, *81*, 1241, 1976.

Goldstein, M., D. A. Roberts, and C. A. Fitch, Properties of the fluctuating magnetic helicity in the inertial and dissipation ranges of solar wind turbulence, *J. Geophys. Res.*, *99*, 11,519, 1994.

Gomberoff, L., and H. F. Astudillo, Ion heating and acceleration in the solar wind, in: *Solar Wind Nine*, edited by S. R. Habbal et al., *AIP Conf. Proc.*, *471*, 461, 1999.

Gomberoff, L., F. T. Gratton, and G. Gnavi, Acceleration and heating of heavy ions by circularly polarized Alfvén waves, *J. Geophys. Res.*, *101*, 15,661, 1996.

Hefti, S., et al., Kinetic properties of solar wind minor ions and protons measured with SOHO/CELIAS, *J. Geophys. Res.*, *103*, 29,697, 1998.

Hollweg, J. V., Transition region, corona, and solar wind in coronal holes, *J. Geophys. Res.*, *91*, 4111, 1986.

Hollweg, J. V., Kinetic Alfvén wave revisited, *J. Geophys. Res.*, *104*, 14,811, 1999a.

Hollweg, J. V., Cyclotron resonance in coronal holes, 1. Heating and acceleration of protons, O^{5+}, and Mg^{9+}, *J. Geophys. Res.*, *104*, 24,781, 1999b.

Hollweg, J. V., Cyclotron resonance in coronal holes, 2. A two-proton description, *J. Geophys. Res.*, *104*, 24,793, 1999c.

Hollweg, J. V., Potential wells, cyclotron resonance, and ion heating in coronal holes, *J. Geophys. Res.*, *104*, 505, 1999d.

Hollweg, J. V., and W. Johnson, Transition region, corona, and solar wind in coronal holes: Some two-fluid models, *J. Geophys. Res.*, *93*, 9547, 1988.

Hollweg, J. V., and J. M. Turner, Acceleration of solar wind He^{++}, 3. Effects of resonant and nonresonant interactions with transverse waves, *J. Geophys. Res.*, *83*, 97, 1978.

Hu, Y.-Q., R. Esser, and S. R. Habbal, A fast solar wind model with anisotropic temperature, *J. Geophys. Res.*, *102*, 14,661, 1997.

Hu, Y.-Q., and S. R. Habbal, Resonant acceleration and heating of solar wind ions by dispersive ion cyclotron waves, *J. Geophys. Res.*, *104*, 17,045, 1999.

Hu, Y.-Q., S. R. Habbal, and X. Li, On the cascade processes of Alfvén waves in the fast solar wind, *J. Geophys. Res.*, *104*, 24,819, 1999.

Hu, Y.-Q., R. Esser, and S. R. Habbal, A four-fluid turbulence-driven solar wind model for preferential acceleration and heating of heavy ions, *J. Geophys. Res.*, *105*, 5093, 2000.

Isenberg, P. A., and J. V. Hollweg, Finite amplitude Alfvén waves in a multi-ion plasma: Propagation, acceleration and heating, *J. Geophys. Res.*, *87*, 5023, 1982.

Isenberg, P. A., and J. V. Hollweg, On the preferential acceleration and heating of solar wind heavy ions, *J. Geophys. Res.*, *88*, 3923, 1983.

Kennel, C. F., and F. Engelmann, Velocity space diffusion from weak plasma turbulence in a magnetic field, *Phys. Fluids*, *9*, 2377, 1966.

Kohl, J. L., et al., First results from the SOHO UltraViolet Coronagraph Spectrometer, *Sol. Phys.*, *1750*, 613, 1997.

Kohl, J. L., et al., UVCS/SOHO empirical determinations of anisotropic velocity distributions in the solar corona, *Astrophys. J.*, *501*, L127, 1998.

Leamon, R. J., W. H. Matthaeus, C. W. Smith, and H. K. Wong, Contribution of cyclotron-resonant damping to kinetic dissipation of interplanetary turbulence, *Astrophys. J. Lett.*, *507*, L181, 1998a.

Leamon, R. J., C. W. Smith, N. F. Ness, W. H. Matthaeus, and H. K. Wong, Dissipation range dynamics: Kinetic Alfvén waves and the importance of β_e, *J. Geophys. Res.*, *103*, 4775, 1998b.

Li, X., and S. R. Habbal, Proton/alpha magnetosonic instability in the fast solar wind, *J. Geophys. Res.*, *105*, 7483, 2000a.

Li, X., and S. R. Habbal, Ion cyclotron waves, instabilities and solar wind heating, *Sol. Phys.*, *190*, 485, 2000b.

Li, X., R. Esser, S. R. Habbal, and Y.-Q. Hu, Influence of heavy ions on the high-speed solar wind, *J. Geophys. Res.*, *102*, 17,419, 1997.

Li, X., S. R. Habbal, J. V. Hollweg, and R. Esser, Heating and cooling of protons by turbulence-driven ion cyclotron waves in the fast solar wind, *J. Geophys. Res.*, *104*, 2521, 1999.

Mann, G., P. Hackenberg, and E. Marsch, Linear mode analysis in multi-ion plasmas, *J. Plasma Phys.*, *58*, 205, 1997.

Marsch, E., Kinetic physics of the solar wind, in *Physics of the Inner Heliosphere*, edited by R. Schwenn and E. Marsch, vol. 2, pp. 45–133, Springer-Verlag, New York, 1991.

Marsch, E., Closure of multifluid and kinetic equations for cyclotron-resonant interactions of solar wind ions with Alfvén waves, *Nonlinear Processes Geophys.*, *5*, 111, 1998.

Marsch, E., Dissipation and wave-ion interaction in the solar wind: Links between fluid and kinetic theory, *Nonlinear Processes Geophys.*, *6*, 149, 1999.

Marsch, E., and H. Goldstein, The effects of Coulomb collisions on solar wind ion velocity distributions, *J. Geophys. Res.*, *88*, 9933, 1983.

Marsch, E., and C.-Y. Tu, Solar wind and chromospheric network, *Sol. Phys.*, *176*, 87, 1997.

Marsch, E., C. K. Goertz, and K. Richter, Wave heating and acceleration of solar wind ions by cyclotron resonance, *J. Geophys. Res.*, *87*, 5030, 1982a.

Marsch, E., K. H. Mühlhäuser, R. Schwenn, H. Rosenbauer, W. Pilipp, and F. Neubauer, Solar wind protons: Three-dimensional velocity distributions and derived plasma parameters measured between 0.3 and 1 AU, *J. Geophys. Res.*, *87*, 52, 1982b.

Marsch, E., K. H. Mühlhäuser, H. Rosenbauer, R. Schwenn, and F. Neubauer, Solar wind helium ions: Observations of the Helios solar probes between 0.3 and 1 AU, *J. Geophys. Res.*, *87*, 35, 1982c.

Matthaeus, W. H., G. P. Zank, S. Oughton, D. J. Mullan, and P. Dmitruk, Coronal heating by magnetohydrodynamic turbulence driven by reflected low-frequency waves, *Astophys. J.*, *523*, L93, 1999.

McKenzie, J. F., Interaction between Alfvén waves and a multicomponent plasma with differential ion streaming, *J. Geophys. Res.*, *99*, 4193, 1994.

McKenzie, J. F., and E. Marsch, Resonant wave acceleration of minor ions in the solar wind, *Astrophys. Space Sci.*, *81*, 295, 1982.

Melrose, D. B., *Instabilities in Space and Laboratory Plasmas*, Cambridge Univ. Press, New York, 1986.

Melrose, D. B., and R. C. McPhedran, *Electromagnetic Processes in Dispersive Media*, Cambridge Univ. Press, New York, 1991.

Montgomery, M. D., S. P. Gary, W. C. Feldman, and D. W. Forslund, Electromagnetic instabilities driven by unequal proton beams in the solar wind, *J. Geophys. Res.*, *81*, 2743, 1976.

Neugebauer, M., B. E. Goldstein, E. J. Smith, and W. C. Feldman, Ulysses observations of differential alpha-proton streaming in the solar wind, *J. Geophys. Res.*, *101*, 17,047, 1996.

Schmidt, W. K. H., H. Rosenbauer, E. G. Shelley, and J. Geiss, On temperature and speed of He^{2+} and O^{6+} ions in the solar wind, *Geophys. Res. Lett.*, *7*, 697, 1980.

Schwartz, S., Plasma instabilities in the solar wind: A theoretical review, *Rev. Geophys.*, *18*, 313, 1980.

Seely, J. F., U. Feldman, U. Schühle, K. Wilhelm, W. Curdt, and P. Lemaire, Turbulent velocities and ion temperatures in the solar corona obtained from SUMER line widths, *Astrophys. J.*, *484*, L87, 1997.

Stix, T. H., *Waves in Plasmas*, Am. Inst. of Phys., New York, 1992.

Tam, S. W. Y., and T. Chang, Kinetic evolution and acceleration of the solar wind, *Geophys. Res. Lett.*, *26*, 3189, 1999.

Tu, C.-Y., and E. Marsch, MHD structures, waves and turbulence in the solar wind: Observations and theories, *Space Sci. Rev.*, *73*, 1, 1995.

Tu, C.-Y., and E. Marsch, A two-fluid model for heating of the solar corona and acceleration of the solar wind by high-frequency Alfvén waves, *Sol. Phys.*, *171*, 363, 1997

Tu, C.-Y., and E. Marsch, Study of the heating mechanism of solar wind ions in coronal holes, in *Solar Wind Nine*, edited by S.R. Habbal et al., *AIP Conf. Proc.*, *471*, 373, 1999.

Tu, C.-Y., Z. Y. Pu, and F.-S. Wei, The power spectrum of interplanetary Alfvénic fluctuations: Derivation of the governing equation and its solution, *J. Geophys. Res.*, *89*, 9695, 1984.

Tu, C.-Y., E. Marsch, K. Wilhelm, and W. Curdt, Ion temperatures in a solar polar coronal hole observed by SUMER on SOHO, *Astrophys. J.*, *500*, 475, 1998.

Tu, C.-Y., E. Marsch, and K. Wilhelm, Ion temperatures as observed in a solar coronal hole, *Space Sci. Rev.*, *87*, 331, 1999.

von Steiger, R., J. Geiss, G. Gloeckler, and A. B. Galvin, Kinetic properties of heavy ions in the solar wind from SWICS/Ulysses, *Space Sci. Rev.*, *72*, 71, 1995.

Wilhelm, K., E. Marsch, B. N. Dwivedi, D. M. Hassler, P. Lemaire, A. Gabriel, and M. C. E. Huber, The solar corona above polar coronal holes as seen by SUMER on SOHO, *Astrophys. J.*, *500*, 1023, 1998.

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