The 3D MHD Simulation of Flux Tube Dynamics: Comparison with Thin Flux Tube Models

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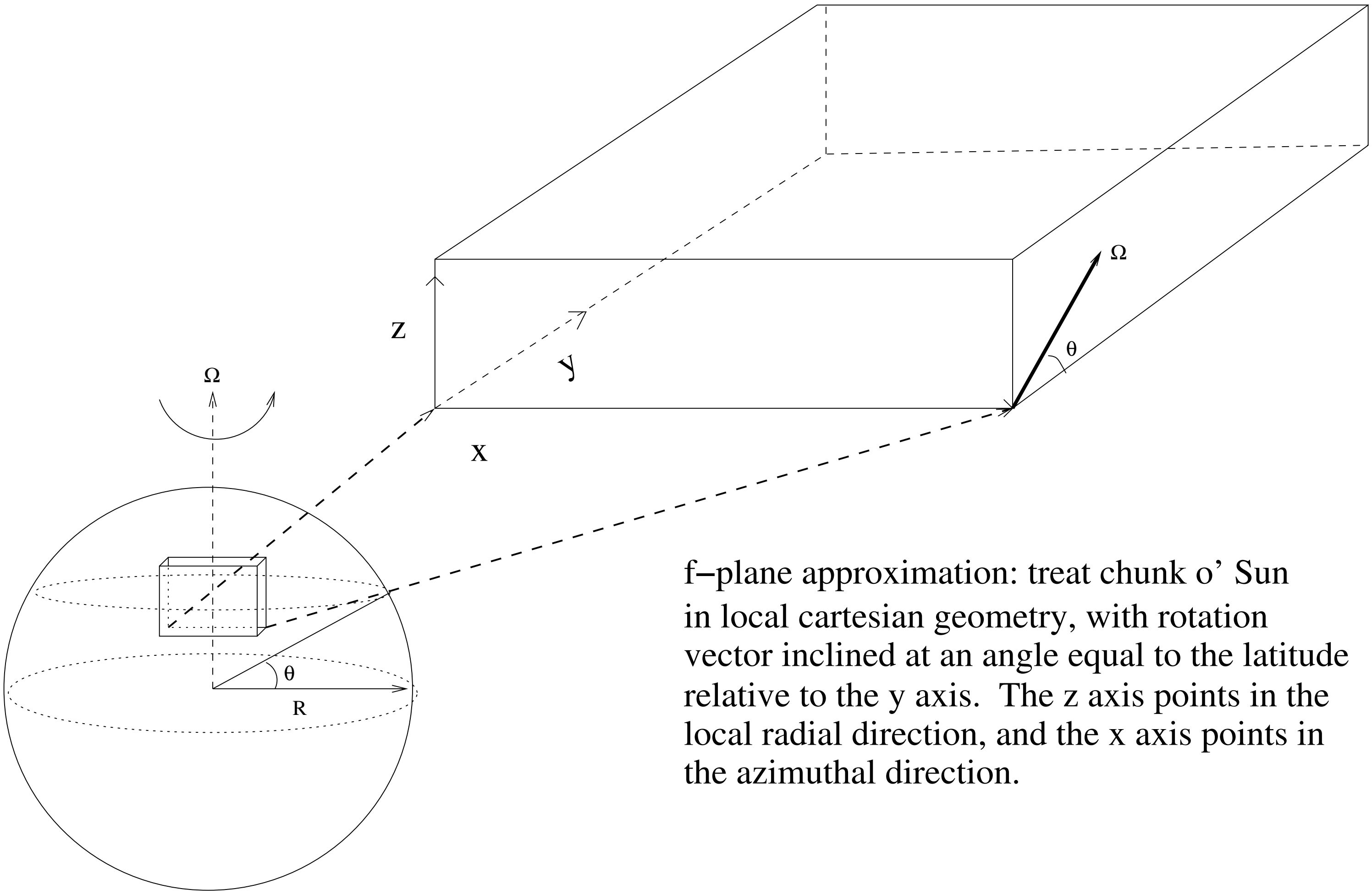
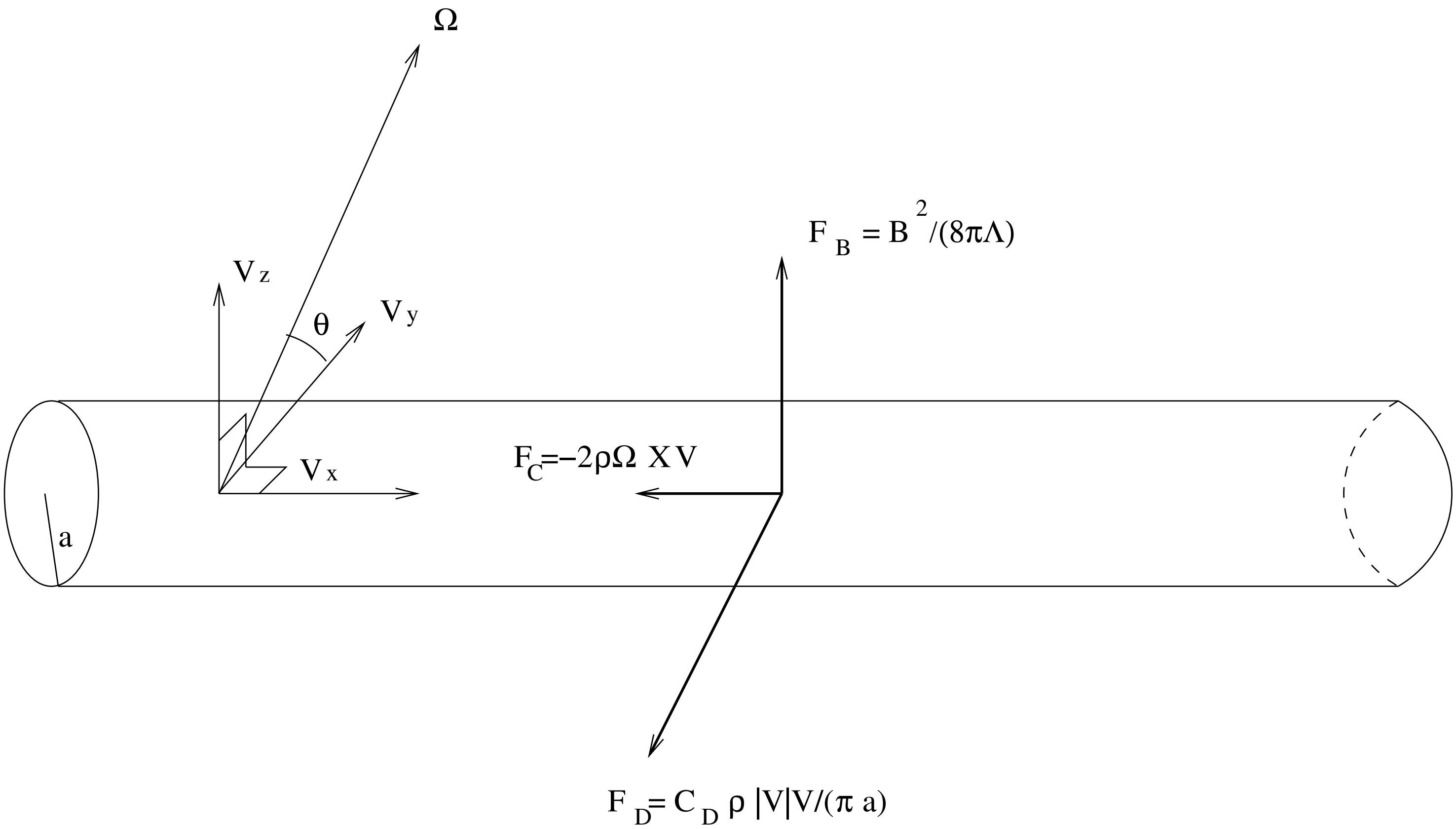
MHD simulations of buoyant magnetic flux tubes (see Bill Abbett's poster No. 1.36 next door) are done in Cartesian Geometry, with rotation included using the "f-plane" approximation. This diagram illustrates the relationship between the Cartesian box and the Sun:



f–plane approximation: treat chunk o' Sun in local cartesian geometry, with rotation vector inclined at an angle equal to the latitude relative to the y axis. The z axis points in the local radial direction, and the x axis points in the azimuthal direction.

Figure 2 in Bill Abbett's poster shows that: (1) Including the effects of solar rotation prevents the breakup of an initially untwisted flux tube, and (2) The rise speed of the buoyant magnetic flux tube is drastically reduced. **The objective of this poster is to address the 2nd of these two results and ask this question: Can we understand the reduced rise speed seen in these simulations by the use of simple thin flux tube concepts?**

Consider the forces on a chunk o' flux tube, oriented parallel to the azimuthal direction, and with little or no curvature...



The flux tube equation of motion in this case is:

$$\rho\frac{d\mathbf{v}}{dt} = B^2/(8\pi\Lambda)\hat{\mathbf{z}} - 2\rho\Omega\times\mathbf{v} - C_D\rho|\mathbf{v}_\perp|\mathbf{v}_\perp/(\pi a)\ . \qquad (1)$$

It is useful to re-write this in component form for the geometry we have chosen:

$$\frac{dv_x}{dt} = -2(\Omega cos\theta\ v_z - \Omega sin\theta\ v_y)\ , \tag{2}$$

$$\frac{dv_y}{dt} = -2\Omega sin\theta\ v_x - C_D v_y \sqrt{v_y^2 + v_z^2}/(\pi a)\ , \tag{3}$$

$$\frac{dv_z}{dt} = v_A^2/(2\Lambda) + 2\Omega cos\theta\ v_x - C_D v_y \sqrt{v_y^2 + v_z^2}/(\pi a)\ . \tag{4}$$

These equations have simple solutions in 3 limits:

(1) In the **Drag Limited, High Field Stength** limit, Coriolis forces are assumed to be weak compared to magnetic buoyancy, which is in the $\hat{\mathbf{z}}$ direction. Balancing Buoyancy and Drag, and treating the Coriolis forces as perturbations, results in these solutions:

$$v_z = \sqrt{\pi/(2C_D)}(a/\Lambda)^{1/2} v_A\ , \tag{5}$$

$$v_x = -2\Omega cos\theta\ v_z t\ , \tag{6}$$

$$v_y = 2\Omega^2 sin\theta\ cos\theta v_z t^2\ , \tag{7}$$

where to be consistent with our assumptions that $v_x, v_y << v_z$, we should have $a/v_z << t << 1/\Omega$.

(2) In the **Drag Limited, Low Field Strength** limit, Coriolis forces are assumed comparable in size to Buoyancy and Drag, but the inertial terms (lhs) of the equations are still neglected.
The solutions are then

$$v_z = (sin\theta/2)^{1/2}\sqrt{\pi/(2C_D)}(a/\Lambda)^{1/2}v_A \ , \tag{8}$$

$$v_y = cot\theta \ v_z \ , \tag{9}$$

$$v_x = -cot\theta \ csc\theta \ v_A^2/(2\Omega\Lambda) \ . \tag{10}$$

These solutions yield trajectories that are parallel to the rotation axis, have $v_y >> v_z$ at low latitudes, and strangely, have no dependence of v_y, v_z on the rotation rate Ω, even though the presence of rotation has a drastic effect on the value of the velocities. These solutions are valid for $t \gtrsim 1/\Omega$, and $t >> a/v_\perp$.

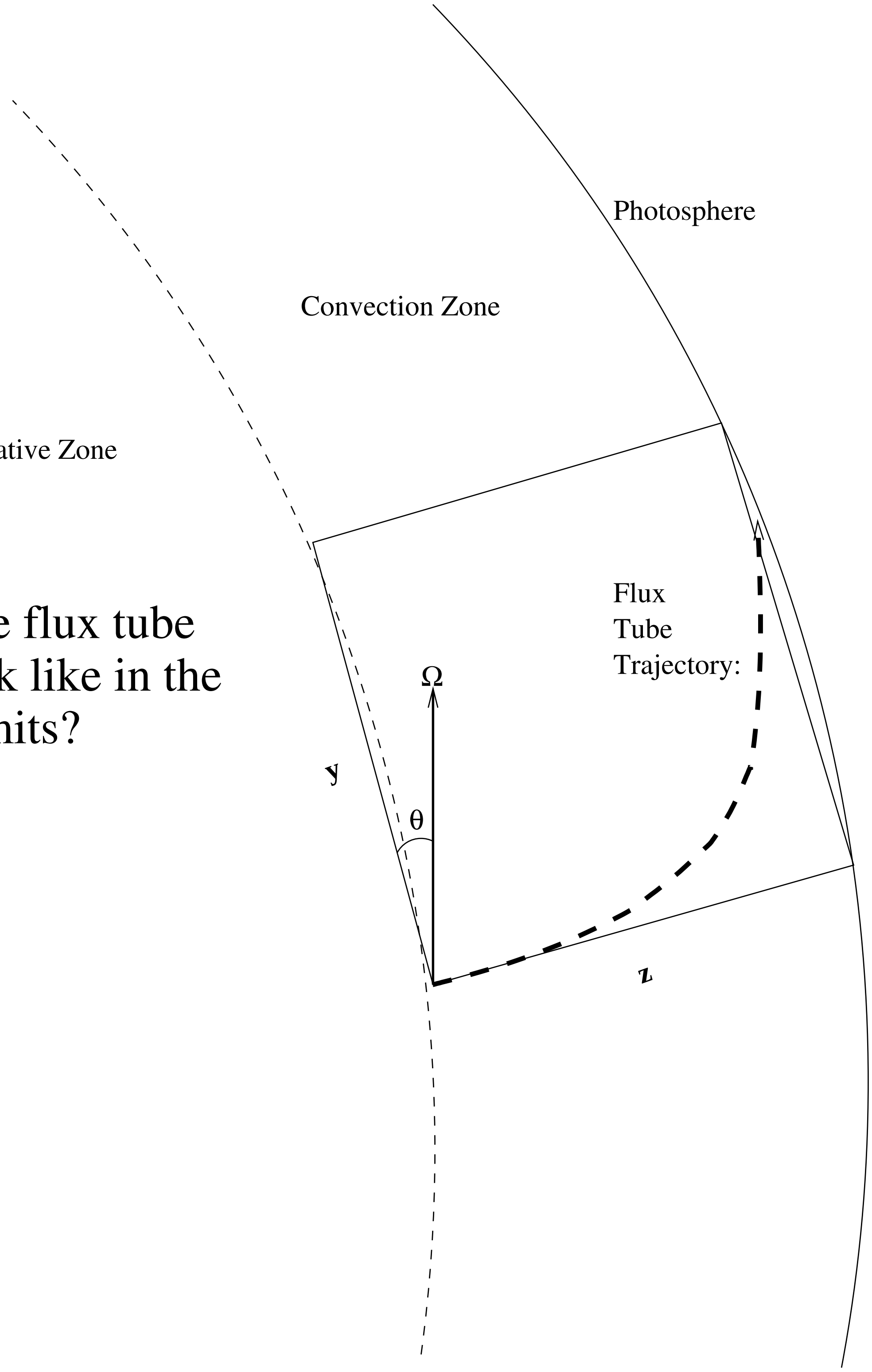
(3) In the **Inertial Oscillation** limit, the initial behavior of the flux tube is considered as it starts from rest under the driving force of magnetic buoyancy. Buoyancy and Coriolis forces are important, but the tube is not yet moving fast enough for Drag to be important. The solutions are:

$$v_z = v_A^2/(2\Lambda)(tsin^2\theta \ + cos^2\theta \ sin2\Omega t/(2\Omega)) \ , \tag{11}$$

$$v_y = v_A^2/(2\Lambda)sin\theta \ cos\theta \ (t - sin2\Omega t/(2\Omega)) \ , \tag{12}$$

$$v_x = v_A^2/(4\Lambda\Omega)(cos2\Omega t - 1) \ . \tag{13}$$

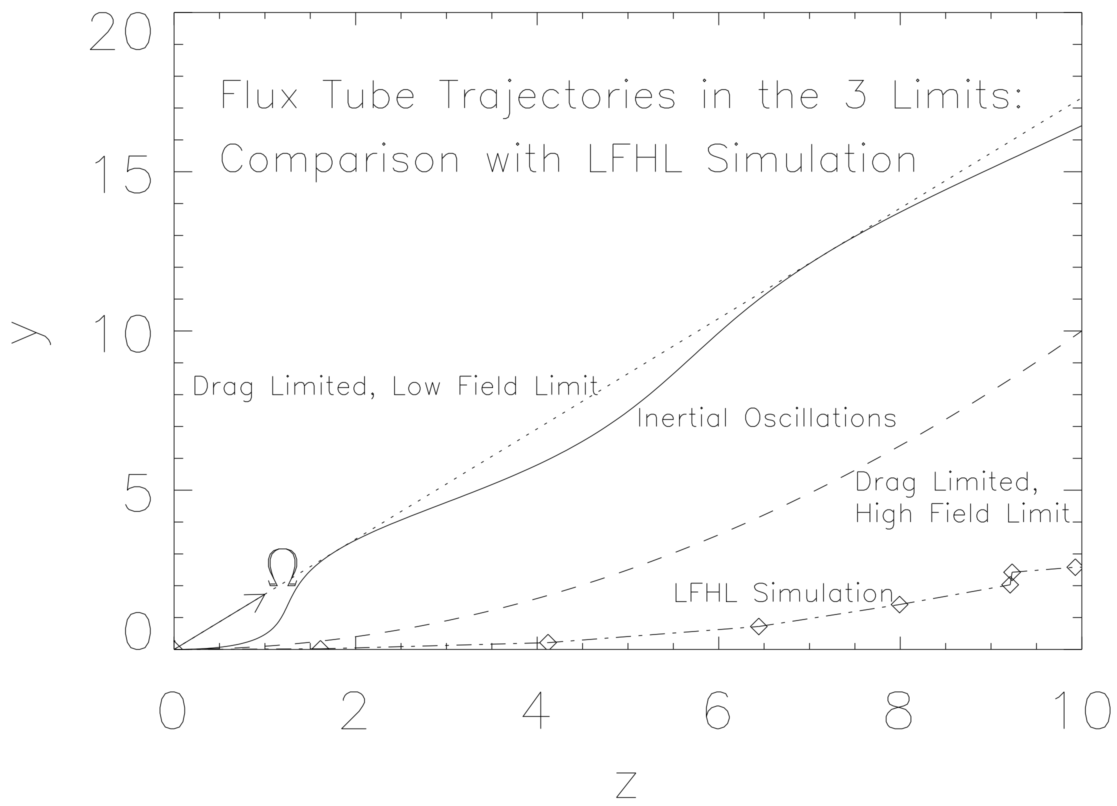
These solutions apply when $t << a/v_\perp$, or when $v_\perp$ is much smaller than either of the drag limited velocities identified above.



What does the flux tube trajectory look like in the 3 different limits?

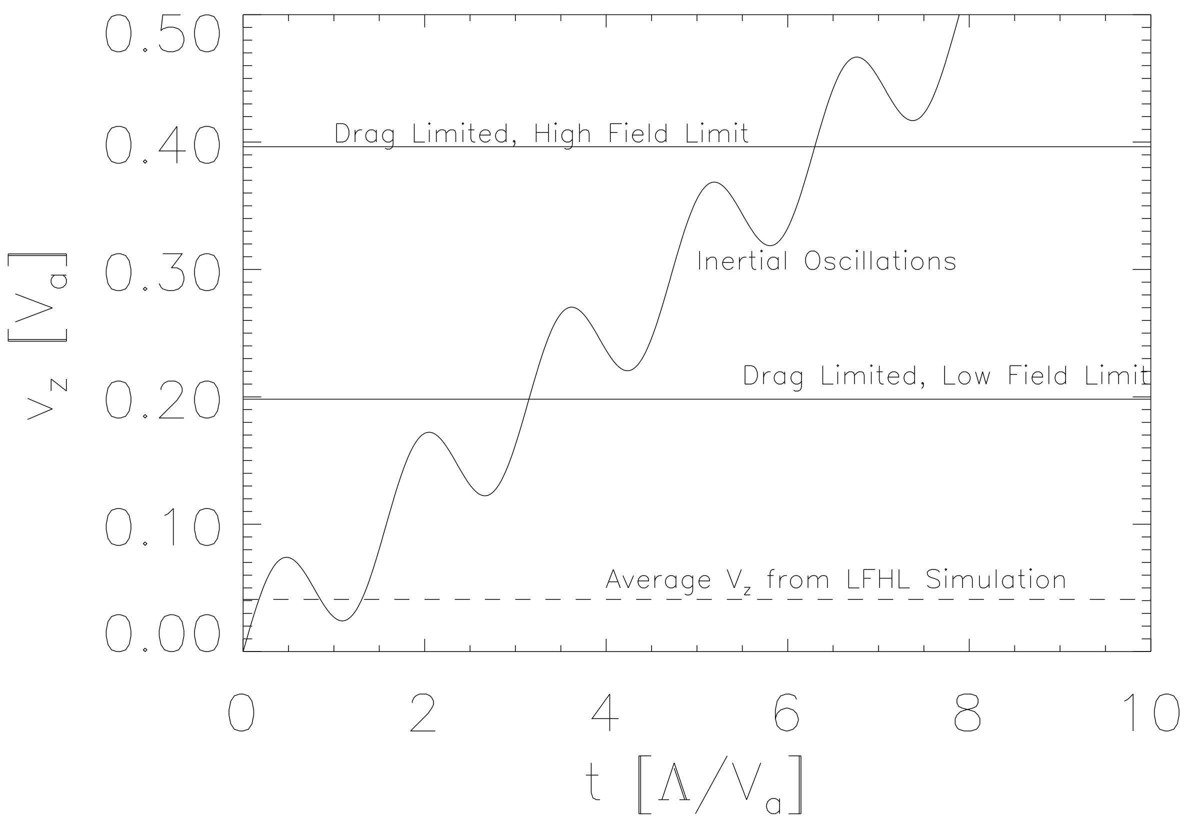
$dy/dz = v_y / v_z$

We can use these 3 limiting solutions to the flux tube dynamics equations to explore the predicted trajectories in the $z - y$ plane, and compare this with the results from Bill Abbett's full 3D MHD simulations of the LFHL (Low Field, High Latitude) case, in which we compare to the evolution of the magnetic field weighted position of the flux tube apex. All of the results are shown below:



The simple thin flux tube solutions all predict trajectories which are nearly parallel to the rotation axis, because of the strong influence of the Coriolis force. The 3D MHD simulation, while it does exhibit a notable deflection of the apex in the $\hat{\mathbf{y}}$ direction, shows a dramatically smaller effect.

In addition to the flux tube trajectories, we can also compare the velocity amplitudes predicted by the 3 different flux tube limits to the rise speed found from the motion of the apex of the simulation of Bill Abbett's LFHL flux tube. These velocities are shown in the Figure below:



We have plotted the inertial oscillations solution beyond its range of validity (*i.e.* when the velocity exceeds the drag limits) just to illustrate its behavior. Note that the rise speed in the LFHL simulation is dramatically lower than either of the drag limits identified here. While the drag limited behavior is consistent with a roughly constant rise speed seen during the later part of the simulation, the amplitude of the rise speed is substantially lower than the thin flux tube models predict.

Conclusions:

- Simple thin flux tube concepts lead to predictions of poleward trajectories and an asymptotic rise speed, both of which are observed in Bill Abbett's 3D MHD simulations.
- The rise speed is reduced substantially below the limits predicted by the thin flux tube models.
- There is much less poleward deflection seen in the simulations than is predicted by the thin flux tube models.
- The phenomeonon of aerodynamic drag, which is employed in many thin flux tube calculations (including those in this poster!) may be dramatically altered by the presence of rotation. Bill Abbett's Simulations have shown that Coriolis forces dramatically change the flow field around a rising flux tube compared to that from a non-rotating model.
- Using 2D tools and concepts to study 3D MHD can be dangerous and misleading.
- It is not a good idea to wait until the last day to make your poster for the SPD meeting