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Notes on the radial dependence of solar luminosity measurements

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1 Introduction

Leif's "Friday Effect" memo showed two clear artifacts in the SORCE results: the Friday effect itself, and a small annual term. Figure 1 shows this to be of order 8 ppm (eyeball estimate, but once the Friday effect is figured out it would be nice to have an estimate and an error estimate for this component). The magnitudes of both of these effects are so small as to be heartening – it seems likely that not only will the data show these artifacts, but probably also low-level signals as well. That's a good motivation to figure out what is going on.

The "Refutation" section below has not been referred to any standard literature, e.g. to Chapter 1 of the Mihalas book on radiative transfer, so take it with a grain of salt. However my Glasgow colleague Graham Woan has confirmed the basic conclusion that the inverse-square law does not apply, since he was fooled by this same fallacy as an undergraduate.

2 Refutation of the "thermodynamic" derivation

One often hears the simple thermodynamic argument that the total irradiance must fall off with an inverse square law with distance from the Sun. The basis for this is the idea that a concentric integrating sphere, of area $4\pi\mathrm{R}^2$, would intercept all of the solar energy at any radius R. Hence the TSI should vary as $\mathrm{L}\odot/4\pi\mathrm{R}^2$, *q.e.d.*; this would make the solar radiation analogous to the electric field of a charged sphere or the gravitational field of a solid sphere, both rigorous inverse-square laws.

Unfortunately this simple argument is erroneous. This can be seen from a simple comparison for a Lambertian sphere with radiation intensity I. For a detector at infinity, we can write $S = I \times \Delta\Omega = I \times (\pi A_\odot)/(4\pi R^2)$ for the projected solar area $A_\odot$. As viewed from a great distance, $A_\odot = \pi R_\odot^2$, so there is an exact inverse-square dependence for $R >> R_\odot$. In this case (measurements made at an infinite distance) one infers a solar luminosity $L_\odot = \pi I R_\odot^2$. This is the correct result.

For an integrating sphere right at the solar surface, the integral for the total flux turns into

$$S = I/2 \int_0^\infty x/(1+x^2)^{3/2} dx,$$

while for the directed flux (the radial flow) one gets

$$S = I/2 \int_0^\infty x/(1+x^2)^2 dx.$$

The additional $\sqrt{1+x^2}$ term comes from the projection onto the radial direction. These relations just follow from the basic relationship $\Delta S = I \Delta\Omega$. The integrals have the values 2 and 1, respectively, so that without the extra projection cosine one gets the wrong solar luminosity by a factor of two.

3 Correction term, with limb darkening

Now we do the same thing and calculate the flux detected by a radiometer at a finite distance from the Sun, with the geometry given in Figure 2, for a specified limb-darkening law. At a finite distance we must calculate the flux incident upon the detector by integrating over the visible "hemisphere," as shown. The normal angle β to the solar surface no longer equals θ, the central angle at Sun center. The calculation also must include the finite normal angle α at the detector. We write

$$S = \int_0^{\theta_{max}} 2\pi sin(\theta) cos(\theta) cos(\alpha) \Lambda/(4\pi X^2) d\theta$$

where R is the distance from Sun center to the detector. If we set R$_\odot$ = 1, cos(θ_{max}) = 1/R. The distance from the observer to a point on the sphere is $X = \sqrt{D^2 + 1 - 2Rcos(\theta)}$ where R is the heliocentric distance (such that $X(0)+1 = R$). The variable Λ is the limb-darkening function, assumed here to be $(3cos(\beta)+2)/5$, which turns out not to matter very much.

The result of this integration (Figure 3) is shocking. This figure shows the difference between the assumption of a simple inverse-square law from Sun center is of order half a percent. The effective radial dependence of the flux at 1 AU is $\mathrm{R}^{-2.0048}$, which corresponds to a mean source location 0.52 $\mathrm{R}_\odot$ from Sun center, as expected. But this shocking result must be wrong! The predicted discrepancy over the Earth's orbit, relative to the inverse-square assumption, is 161 ppm (peak-to-peak), about an order of magnitude larger than the annual residual noted in Figure 1 from Leif's original Friday Effect memo. At present I don't understand this discrepancy.

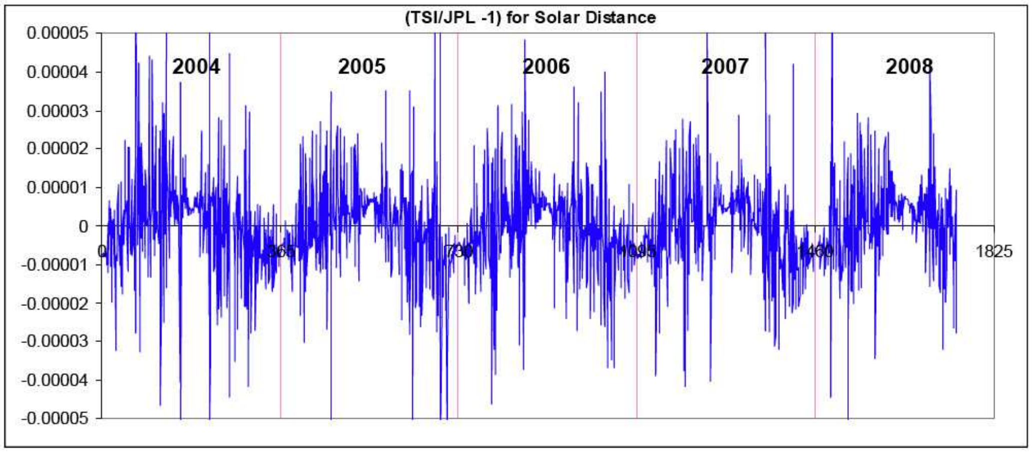

Figure 1: Leif's Figure 1: residuals of the SORCE daily averages against an independent ephemeris calculation. Note the excursions (and lower noise levels) near aphelion and perihelion times (January and July). The spikes are the now-famous "Friday Effect," which turn out to be mainly identifiable with Fridays. Is this what Yahweh intended?

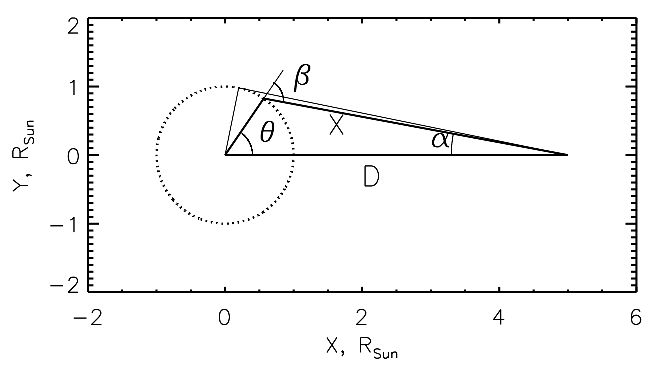

Figure 2: Geometry

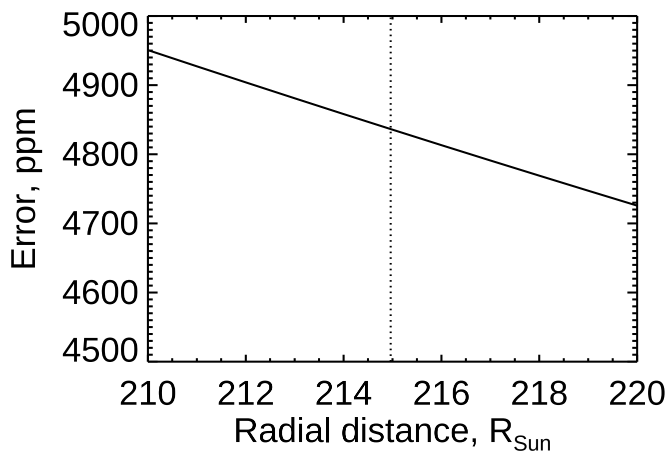

Figure 3: Correction term to be applied to TSI for a finite radial distance, expressed as a fraction of the luminosity as observed at infinity (true minus apparent).