What is the minimum Earth detectable by photometry of a Sun-like star?

The photometric signature of an exoplanet would be a dip in its flux as the transit happened. Since the radius of Earth is about 1% $R_\odot$, the dip would be about 0.01%, or 100 μmag on the arcane magnitude scale.

But this is not the question. Clearly one could not detect 0.01% very easily if the background variability of the star were much larger, and a further complication is that an Earth, by definition, would only occult its star once each year, and so the signal would have to be quite robust. To answer this question fully one needs to estimate the stellar background variability on the time scale of the occultation. Since the angular diameter of the Sun is about half a degree, that time scale is about 1/720 of year or about half a day; thus we need to estimate the stellar variability at a frequency of $1/43200 \approx\ 20\mu$Hz. From the observed power spectrum in Fröhlich & Lean (2004), we can read off values for the power at that frequency at 30 (ppm^2)/μHz, hence $\Delta S/S \approx \sqrt{20 \times 30} \approx 25$ ppm. Finally, our SNR estimate for an Earth transiting an exo-Sun would be about 4σ. This sounds very weak, but if it could be checked for a few years, maybe one could believe that the Earth itself is the minimum detectable Earth-like exoplanet!

How round is the Sun?

The Sun rotates at about 2 km/s at the surface (corresonding to a roughly one-month rotation period). That means the centrifugal acceleration $v^2/R_\odot \approx 0.6$ cm/s^2. Solar gravity $g_\odot \approx 2.7 \times 10^4$ cm/s^2, so the oblateness from rotation should be about 20 ppm.

This is a reasonable estimate, but not a really good one. The Sun is a ball of gas and a test particle at its surface is controlled not just by two-body kinematics, but by fluid effects in the stellar structure. It turns out that the actual oblateness is about half of this estimate. The Sun is pretty round, but other faster-rotating stars can be very elliptical indeed.

How big does a solar optical telescope need to be?

This is a very simple question. A photon has a mean free path, and that subtends an angle at the Earth. In the meanwhile the solar atmosphere is infinite, in the sense that there is no solid surface. Hence we just equate the subtense of scale height H/AU to the Rayleigh criterion for optical angular resolution. My estimate for the subtense is 1.2×10^{-6} radians, which for 5000 Å matches the Airy resolution criterion, at 5000 Å, for an aperture diameter of 50 cm. But this is not good enough really; to be sure about an observation we would want many pixels per resolution element. Thus a DKIST with 4-m aperture would be desirable (and now exists).

But again this is far from the story. The initial DKIST images, and other high-resolution telescopes, reveal structure much finer than the scale height. This results from magnetic structuring on scales dictated by wholly different physics.

How deep can a Wilson Depression be?

At the most basic level, this is really an easy question to answer: a sunspot can sometimes reach a field strength of 6000 G, for a magnetic pressure $P_B \approx 1.4 \times 10^6$ dyne/cm^2. This is about 20 times the gas pressure, hence the Wilson depression could be about 3 scale heights deep ($e^3 \approx 20$) – about 0.5 Mm.

In reality this is of course a more complicated and quite interesting question; a Wilson depression is defined observationally in terms of opacity, not geometry. And considering the Lorentz force, how could it balance gas pressure if the field (as observed) were vertical? Finally, the sunspot's presence alone distorts the convective energy transport from the solar interior, and this surely plays an important role in defining the structure.