Nonadiabatic Electron Heating at High–Mach-Number Perpendicular Shocks


Inertial Fusion and Plasma Theory, Group X-1, Applied Theoretical Physics Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 28 October 1985)

Fully kinetic simulations of high–Mach-number perpendicular shocks demonstrate that electron acceleration in the cross-shock electric field can produce downstream electron temperatures significantly higher than that expected for adiabatic compression.

PACS numbers: 52.35.Tc, 52.25.Sw, 52.65.+z, 97.60.Ew

The purpose of this Letter is to present fully kinetic simulations of high–Mach-number (HMN) perpendicular collisionless shocks that indicate that HMN shocks can significantly heat electrons. HMN is defined with respect to the electrons in that all of the shocks analyzed have shock speeds greater than the upstream electron thermal speed. A fully electromagnetic and relativistic particle code is employed to simulate HMN perpendicular shocks in one spatial dimension that exhibit periodic reformation.1 During the reformation cycle, the shock-transition-region thickness varies, thereby allowing us to investigate the electron response to the shock for different shock thicknesses. When the shocks are thinnest, the electrons are heated to significantly higher temperatures than that expected for adiabatic compression. Because the simulations are one dimensional, this heating cannot be due to cross-field instabilities, but is due to direct electron acceleration in the cross-shock electric field.

In the reference frame of the simulations, the plasma flows along the x direction into a shock that occupies the y-z plane, with the background and shock magnetic fields in the z direction. In the steady state, the plasma creates a motional electric field in the y direction, $E_y$, that is independent of the x coordinate, and within the shock transition region there is a cross-shock electric field in the $-x$ direction, $-E_x(x)\hat{x}$. To gain insight into the electron energization mechanism that is operating in the HMN shock, it is useful to consider a single electron trajectory in the $x$-$y$ plane and the macroscopic fields. Here, we discuss the dominant electron drifts in this problem, the $E \times B$ drifts, and derive a critical Alfvén Mach number, $M_A$, above which we expect the nonadiabatic electron behavior to occur.

Upstream of the shock an electron undergoes $E \times B$ drift in the x direction in the fields $E_y$ and $B_z$. When the electron encounters the shock it is deflected in the y direction, because of the presence of the cross-shock $E_x$ field, and if $E_y \ll E_x$, the electron drift is approximately along the y axis. As the electron drifts in this direction, its spatial position x oscillates, and if the upstream boundary of the shock $E_x$ field is at $x=0$, it achieves a maximum x displacement equal to about $2A/\omega_{ce}$, where $A$ is a constant that is determined by $v_y(t=0)$ and $\omega_{ce}$ is a local value of the electron gyrofrequency. If the shock speed is greater than the electron thermal speed, as is true for all the shocks analyzed in this paper, then it is a good approximation to treat the electrons as cold. Under these circumstances an electron has $v_y(t=0)=0$, yielding $A=cE_x/B_z$. Therefore, as the electron drifts it oscillates between $x=0$ and $x_{max}=2E_xc/\omega_{ce}B_z$. Because the fields are functions of position in the shock, $x_{max}$ is not precisely defined. To obtain an order-of-magnitude estimate, we evaluate $x_{max}$ using an $E_x$ corresponding to an electrostatic potential energy jump across the shock equal to the ion ram energy and a $B_z$ corresponding to the downstream $B_z$, yielding $x_{max}=\left(M_A^2/L\right)(c/\omega_{pe})$. In this equation, L is the shock thickness and $\omega_{pe}$ is the upstream electron plasma frequency.

As the electron undergoes $E \times B$ drift along y, it gains energy as it moves from $x=0$ to $x=x_{max}$ and it loses energy as it moves from $x=x_{max}$ back to $x=0$. If the shock thickness is much larger than $x_{max}$, nearly all of the shock electrostatic potential-energy jump is lost via work done against the motional electric field. However, if the shock thickness becomes comparable to $x_{max}$, the electron will be nonadiabatically energized as it traverses the shock. Equating $x_{max}$ to the shock thickness, L, we obtain the critical Alfvén Mach number

$$M_A^* = L(c/\omega_{pe})/c,$$

above which we expect the nonadiabatic electron behavior to occur. It is important to note that the shock thickness is an unknown function of $M_A$. For a shock thickness of an upstream ion inertial length, $c/\omega_{pi}$, $M_A^* = 43$, while a shock as thin as an upstream electron inertial length, $c/\omega_{pe}$, has $M_A^* = 1$. Therefore, unless perpendicular shocks in the solar-terrestrial environment with upstream plasma $\beta < 1$ achieve ramp thicknesses on the order of $10c/\omega_{pe}, M_A$ is usually less than $M_A^*$. However, significant electron heating may be occurring at, for example, HMN.
shocks near supernovas.4

We now present simulations of HMN perpendicular shocks that exhibit significant electron energization, consistent with the above discussion. The simulation results were obtained using the Los Alamos code WAVE, a two-dimensional, fully relativistic and electromagnetic code that self-consistently solves the Newton-Lorentz equation of motion and Maxwell's equations for ions and electrons with the particle-in-cell technique.5 The results in this paper are one-dimensional, as we assume spatial variations along only the x direction, but retain all three components of the electromagnetic fields and the particle positions and velocities. The shocks studied have upstream parameters $\omega_{pe}/\omega_{ce} = 1$, plasma $\beta_p = \beta_i = 10^{-2}$, and one of three Alfvén Mach numbers, $M_A = 2$, 4, or 6, corresponding to shock speeds of 0.2c, 0.4c, and 0.6c, respectively. It should be noted that because of the mass ratio chosen for the simulations, $m_i/m_e = 100$, all three Mach numbers correspond to shock speeds greater than the upstream electron thermal speed, 0.1c, and in this sense are HMN. The simulation length in the x direction is 150c/$\omega_{pe}$ and in the y direction is 0.2c/$\omega_{pe}$. There are 1500 cells in x, 2 in y, and initially 150,000 total particles. Plasma is continuously injected into the left of the simulation box and the shock is formed by reflecting this plasma off the right-hand side of the box.6 To suppress numerical Cherenkov noise, the simulation time step is 0.025$\omega_{pe}^{-1}$.

Figure 1 illustrates simulation results for the $M_A = 6$ shock. In the figure, time is normalized to upstream $\omega_{pe}^{-1}$, length to upstream c/$\omega_{pe}$, relativistic particle momentum P to $m_e c$, fields to upstream $m_e c \omega_{pe}/e$, and temperature to $m_e c^2/k$. Shown in the figure at three times during the simulation are ion momentum components $P_x$ and $P_y$ and magnetic field magnitude $B_z$ as functions of position x in the simulation box. This HMN perpendicular shock exhibits periodic reformation like that observed at lower Mach numbers.1 In the first column ($t = 325 \omega_{pe}^{-1}$), the shock is located at about $x = 100$ and is more than an ion gyroradius into the box. At this time the shock is near the beginning of a reformation cycle, with ion reflection off the upstream shock boundary commencing. At about this time the shock achieves its minimum thickness, a few times upstream c/$\omega_{pe}$, and has a large magnetic field magnitude at the overshoot. Also visible in the $B_z$ panel are weak pulses of extraordinary- (X-) mode radio noise that are propagating away from the shock. The second column of Fig. 1 is about one-half way into the reformation cycle and exhibits a broad shock transition, while the third column is just prior to the beginning of the next cycle.

At the beginning of the reformation cycle, the $M_A = 6$ shock ramp thickness is less than six times

\[
\begin{array}{ccc}
\text{t = 325} & \text{t = 425} & \text{t = 500} \\
\text{P_x} & \text{P_y} & \text{B_z} \\
& & \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{x (c/\omega_{pe})} & \text{x (c/\omega_{pe})} & \text{x (c/\omega_{pe})} \\
& & \\
\end{array}
\]

FIG. 1. Ion phase space and $B_z$ as functions of position in the simulation box for the $M_A = 6$ shock at the beginning, middle, and near the end of a typical reformation cycle.

1060
momentum along y, \( p_y = m_y v_y - eA_y / c \), have been followed across the \( M_A = 6 \) shock. The particles are given the same \( p_y \) because in the one-dimensional system \( p_y \) is a constant of the motion. They are initialized with Maxwellian velocity distributions in \( x \) and \( y \) and at all times have a spread in \( x \) position of only a few electron gyroradii.

Figure 2 illustrates \( P_y \) vs \( P_x \) for the test electrons at four positions near the \( M_A = 6 \) shock. The test particles encounter the shock at about \( t = 325 \omega_{pe}^{-1} \), with conditions as shown in the first column of Fig. 1. The frames in Fig. 2 correspond to upstream of the shock, just into the shock ramp, at about the magnetic overshoot, and downstream of the shock. In frame A the test particles are convecting into the shock. The distribution was isotropic at \( t = 0 \), but has now taken on a slight anisotropy due to weak interactions with fluctuating fields upstream of the shock. At the time of frame B, the test electrons encounter the shock \( (t = 325 \omega_{pe}^{-1}) \) experiencing a large drift in the \( y \) direction, due to \( E_x \), and exhibiting a “quarter moon”-like momentum-space signature produced by the large \( E_x \). When the electrons reach the overshoot, frame C, the high-energy portion of the “quarter moon” in frame B has wrapped around the low-energy portion many times, producing a spiral structure. Downstream of the shock, frame D, most of these features have been mixed away as a result, in part, of interactions with fluctuating fields.

The momentum-space signatures in Fig. 2 suggest that the electrons are energized significantly when they traverse the shock. A quantitative measure of the bulk electron heating is given in Fig. 3, where we plot electron \( P_x \) and electron temperature along \( x \) divided by \( B_z \), both as functions of \( x \) for the three Mach numbers \( M_A = 2, 4, \) and 6.
numbers $M_A = 2$, $4$, and $6$. For all three Mach numbers, the simulation time is $525 \omega_{pe}^{-1}$, with the $M_A = 2$ upstream shock boundary at about $x = 100$, the $M_A = 4$ boundary at about $x = 75$, and the $M_A = 6$ boundary at about $x = 50$. It can be shown that in the simulations conservation of the quantity $T_e/B_e$ is equivalent to conservation of the electron’s first adiabatic invariant. Because downstream of the shock the electrons are responding adiabatically, changes of the adiabatic invariant occurred across the shock as it swept through the box. It is clear that for $M_A = 2$ the invariant is conserved, while for $M_A = 4$ and $6$ it is not, with the violation severity increasing with increasing $M_A$. It can be verified that the maximum violations are at the beginning of a shock reformation period, when the shock is thinnest and $M_A$ becomes comparable to or greater than $M_A^*$.

The preceding discussion illustrates that electrons can be nonadiabatically heated at supercritical, perpendicular shocks without the need to invoke cross-field current-driven microturbulence. For sufficiently high Mach number, the thinning of the shock ramp during the reformation cycle results in the direct acceleration of the electrons by the cross-shock electrostatic field. An important question remains, however, concerning the limit of the heating mechanism. In the simulation with $M_A = 6$, roughly 1% of the downstream energy density is deposited into the electrons. While this is a small fraction of the total energy density, Fig. 3 demonstrates that the electrons are heated about an order of magnitude more than expected for adiabatic compression. It is plausible that as the Mach number increases the electron temperature will continue to increase, with no a priori reason for dismissing the possibility that $T_e$ can approach $T_i$ immediately downstream of the shock. If $T_e = T_i$ downstream, the shock jump conditions in the HMN limit yield downstream $kT_e = \frac{3}{16}(m_i v_i^2/2)$, and therefore the electrons would be energized to a significant fraction of the upstream ion ram energy. Unfortunately, an investigation of this hypothesis for the ultra-HMN regime is difficult with present computational resources.

Finally, we discuss distinctions between the fully kinetic WAVE simulations and an earlier hybrid simulation study of HMN perpendicular shocks and discuss the extension of the simulation results to two spatial dimensions. When the HMN shocks are thinnest, there is an important difference between the hybrid and fully kinetic simulations. In the hybrid simulations electrons are treated as a massless fluid, exhibiting either adiabatic behavior or heating via resistivity included in the electron energy equation. Because the inertial $c/\omega_{pe}$ scales are neglected in the hybrid study, the shock will thin down to the spatial cell size before overturning. In the case of adiabatic electron response, no irreversible heating occurs. With the inclusion of electron inertia in the fully kinetic WAVE simulations, the minimum shock thickness is now physically determined and, more importantly, the electrons are accelerated and significantly heated. If the simulations are extended to two spatial dimensions by allowance of variations along $y$, microturbulence driven by currents along $y$ is expected to result in shocks at least 2 to 3 times thicker than those observed in the one-dimensional simulations. This turbulent thickening and associated dissipation will compete with the direct electron energization mechanism discussed in this Letter. At the present time, the dominant process as a function of shock parameters is unknown.

This work was performed under the auspices of the United States Department of Energy and was supported in part by the NASA solar-terrestrial theory program.